Lateral Stresses in Soil
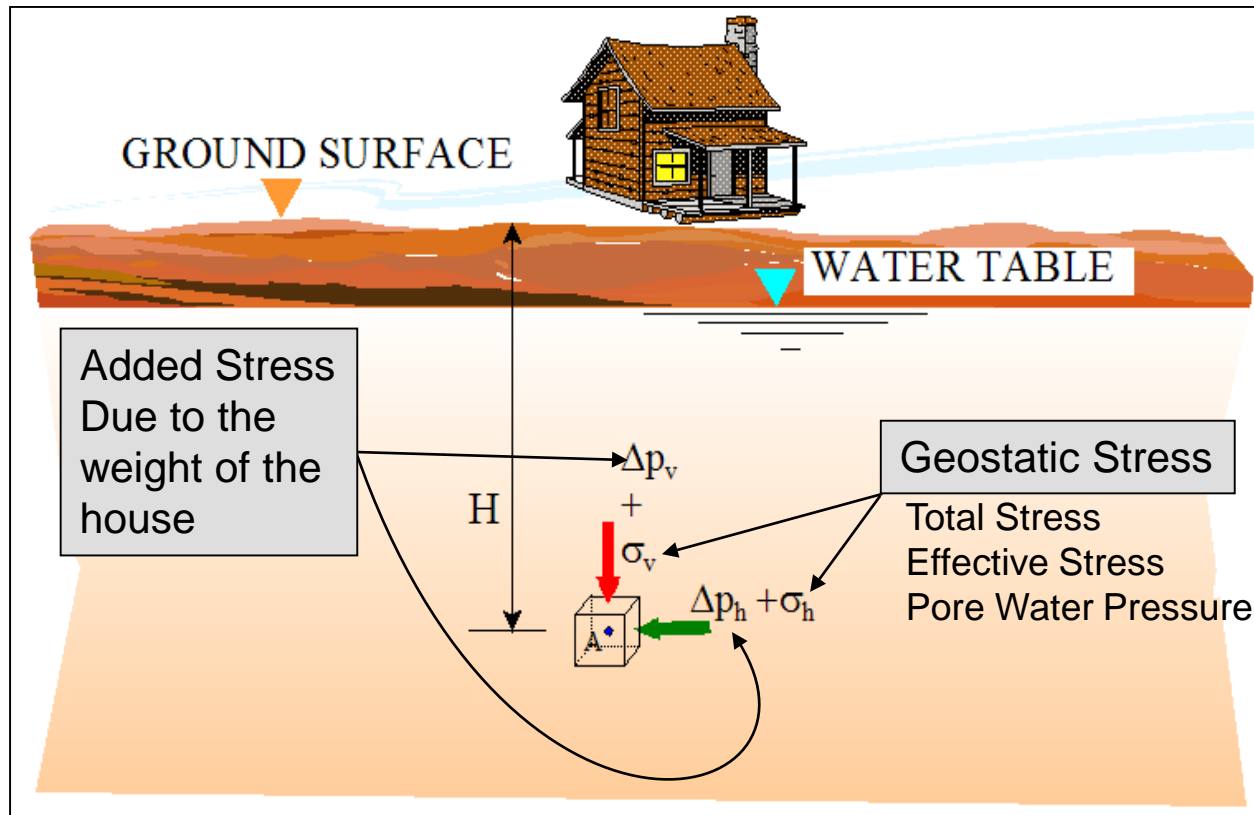
By

Kamal Tawfiq, Ph.D., P.E.

Spring 2017

There are two types of lateral stresses in soil.

By: Kamal Tawfiq, Ph.D., P.E

Added Stress
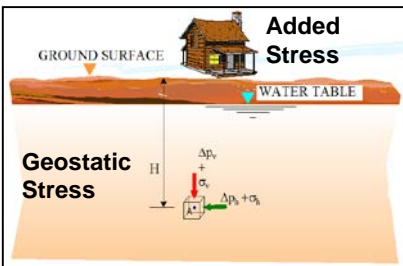
Geostatic Stress

Stress Distribution in Soils

Geostatic Stresses

Total Stress
Effective Stress
Pore Water Pressure

Total Stress= Effective Stress+ Pore Water Pressure

$$\sigma_{total} = \sigma_{eff} + u$$

Added Stresses (Point, line, strip, triangular, circular, rectangular)

Westergaard's Method
(For Pavement)

Bossinisque Equations
1. Point Load
2. Line Load
3. Strip Load
4. Triangular Load
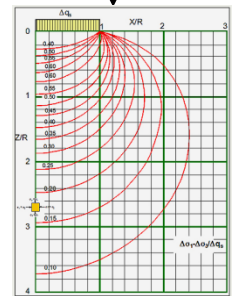5. Circular Load
6. Rectangular Load

σ_y
σ_x
τ_{xy}

Approximate Method
1:2 Method
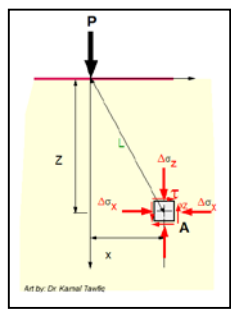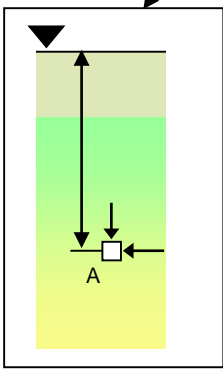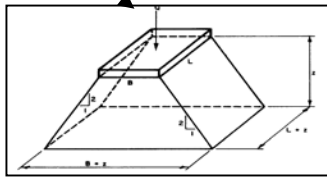
Influence Charts

$$\Delta \sigma_z = I_\sigma \bullet q$$

Stress Bulbs

Newmark Charts

Influence value=0.005

Point Load

Line Load

Wall

Continuous Load

Circular Load

Triangular Load

Rectangular Load

P

GROUND SURFACE

Y

X

Z

L

Principal Stresses

$\Delta P = \sigma_1$

σ_2

σ_3

$\Delta P = \sigma_v$

τ

σ_h
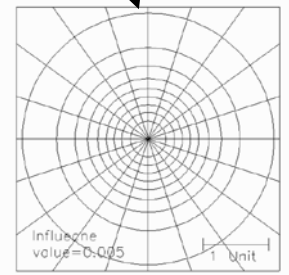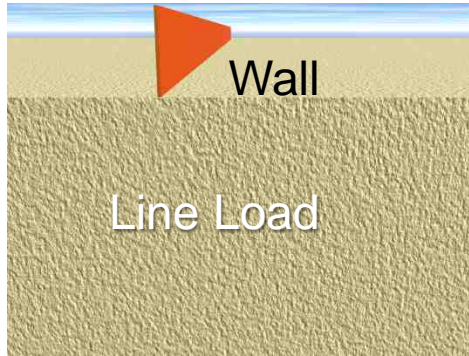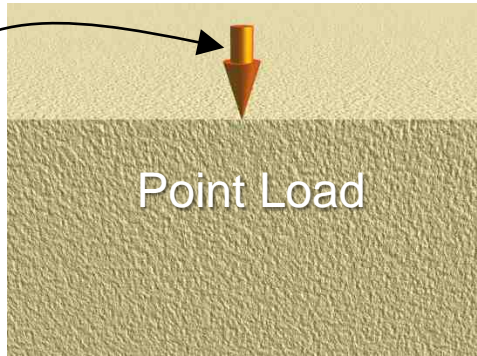
1. STRESSES CAUSED BY A POINT LOAD

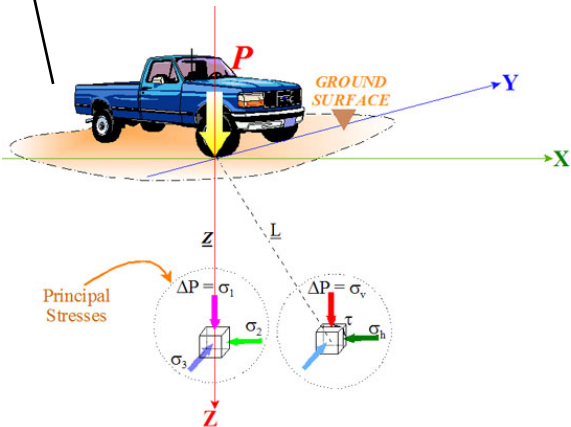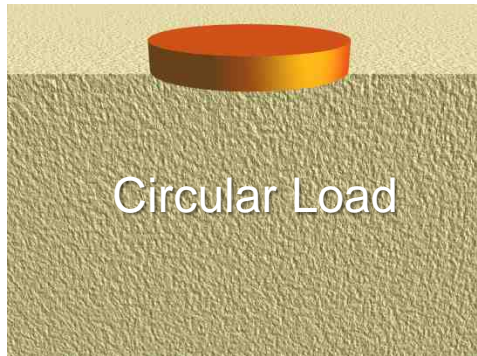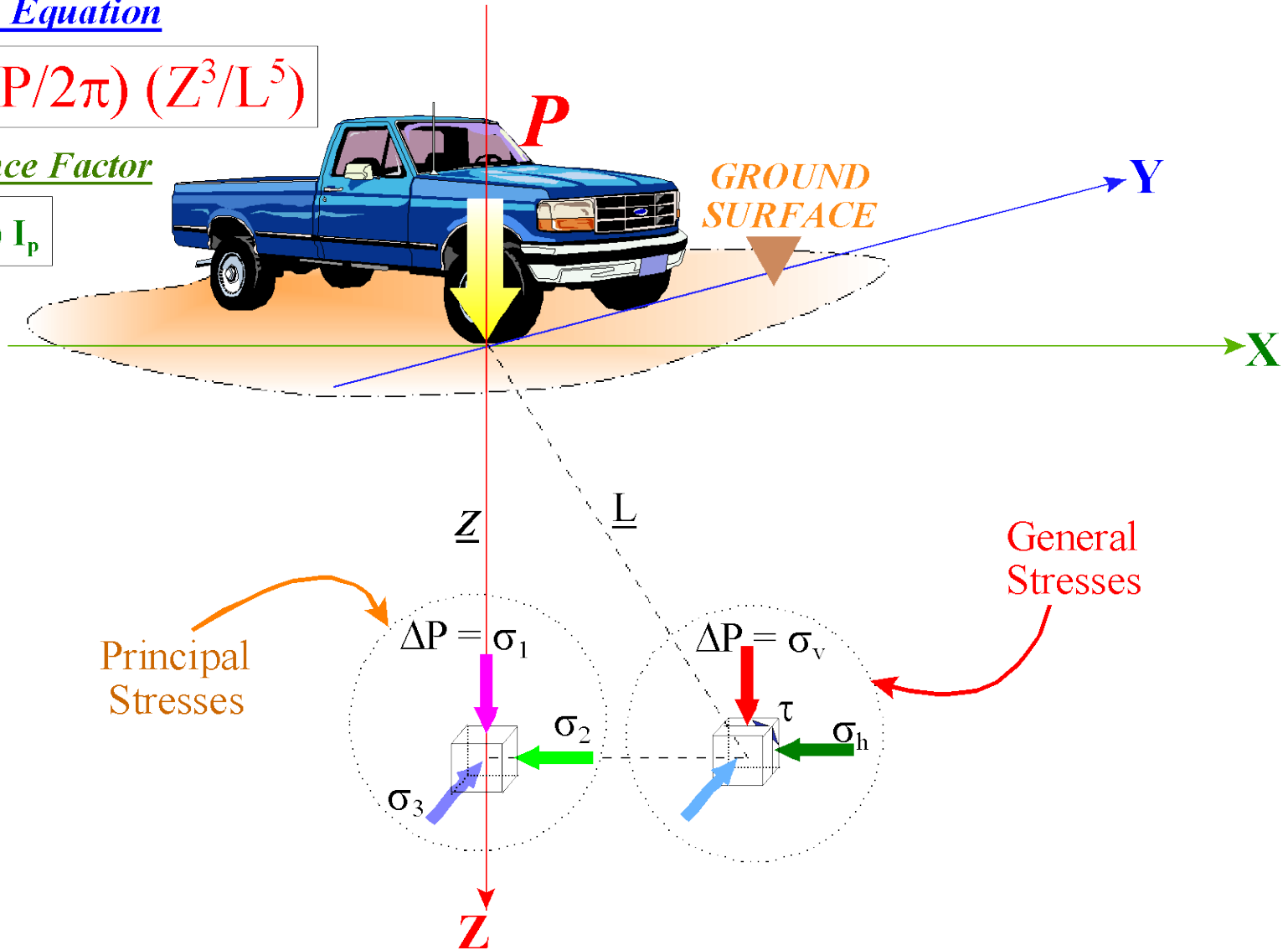**Boussinesq's Equation**

$$\Delta P_z = (3P/2\pi) (Z^3/L^5)$$

Using Influence Factor

$$\Delta P = (P/Z^2) \, I_p$$

P

GROUND SURFACE

Y

X

Z

L

General Stresses

Principal Stresses

$\Delta P = \sigma_1$

σ_2

σ_3

$\Delta P = \sigma_v$

τ

σ_h

Z

2. STRESSES CAUSED BY A LINE LOAD

Q_O (Load/Unit Length)

X

Z

Q_O **(Load/Unit Length)**

Y

WALL

X

Z

Z
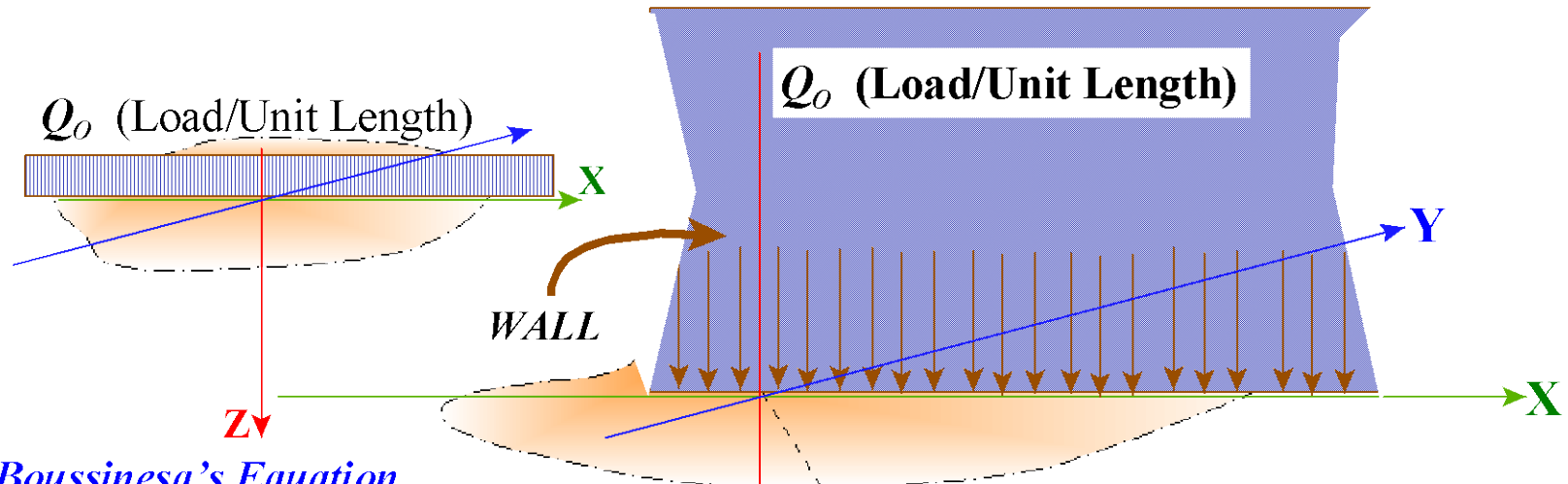
Boussinesq's Equation

$$\Delta P_z = \{2\ Q_o\ Z^3/\pi\ (X^2 + Z^2)^2\}$$

Using Influence Factor

$$\Delta P = (P/Z)\ I_L$$

General Stresses

$\Delta P = \sigma_v$

σ_1

σ_2

σ_3

τ

σ_h

Principal Stresses

3. CONTINUOUS LOAD

$$\Delta P = q/\pi \, [\beta + \sin \beta \cos (\beta + 2\,\delta)]$$

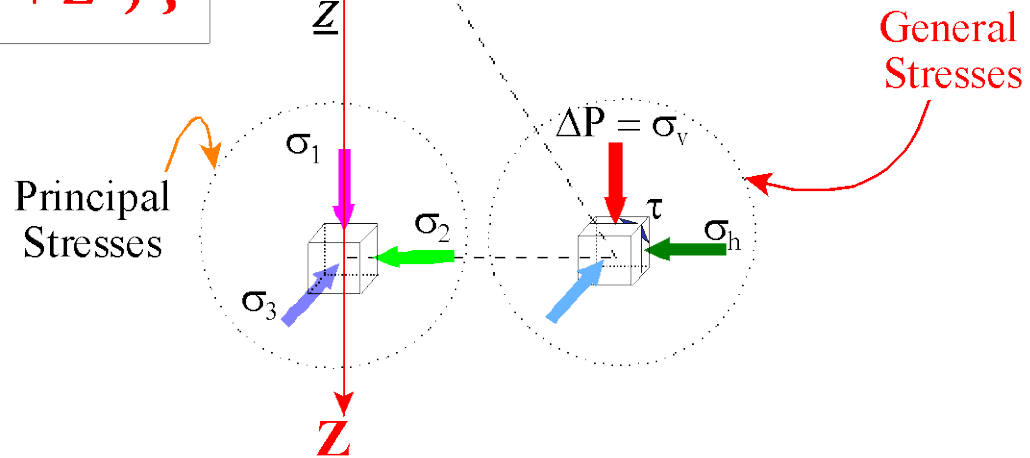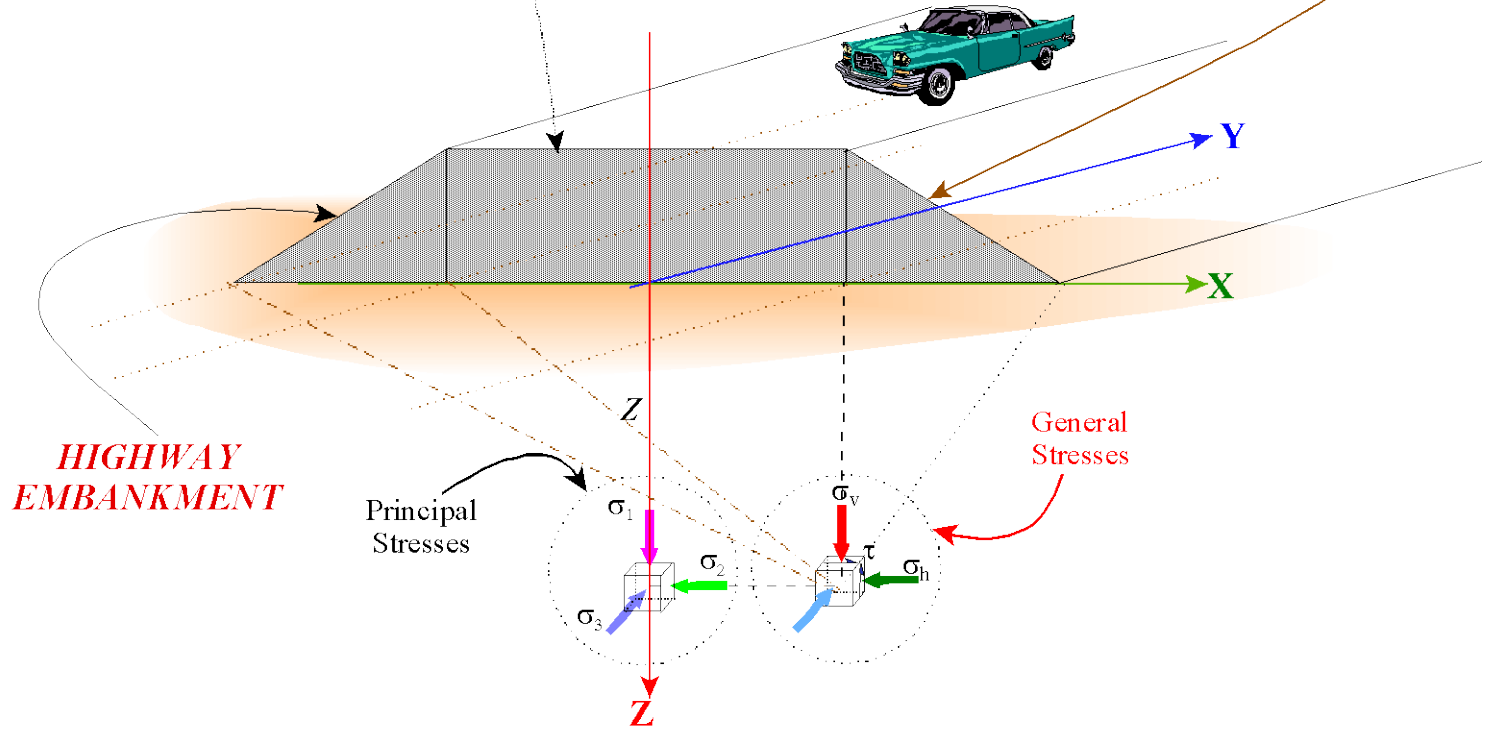**4. TRIANGULAR LOAD**

$$\Delta P = (q/\pi) \, \{(X/B)\, \alpha - (\sin 2\delta/2)\}$$

Y

X

Z

HIGHWAY EMBANKMENT

Principal Stresses

General Stresses

σ_1

σ_2

σ_3

σ_v

τ

σ_h

Z

By Kamal Taufiq, PhD; P.E.

5. VERTICAL STRESS DUE TO CIRCULAR LOAD

Q_o

X

Z

q

GROUND SURFACE

Y

X

$$\Delta P= q \{ 1- 1/[(R/Z)^2 + 1]^{3/2} \}$$

$\underline{z}$

Principal Stresses

σ_1

σ_2

σ_3

$\Delta P = \sigma_v$

τ

σ_h

General Stresses

Z
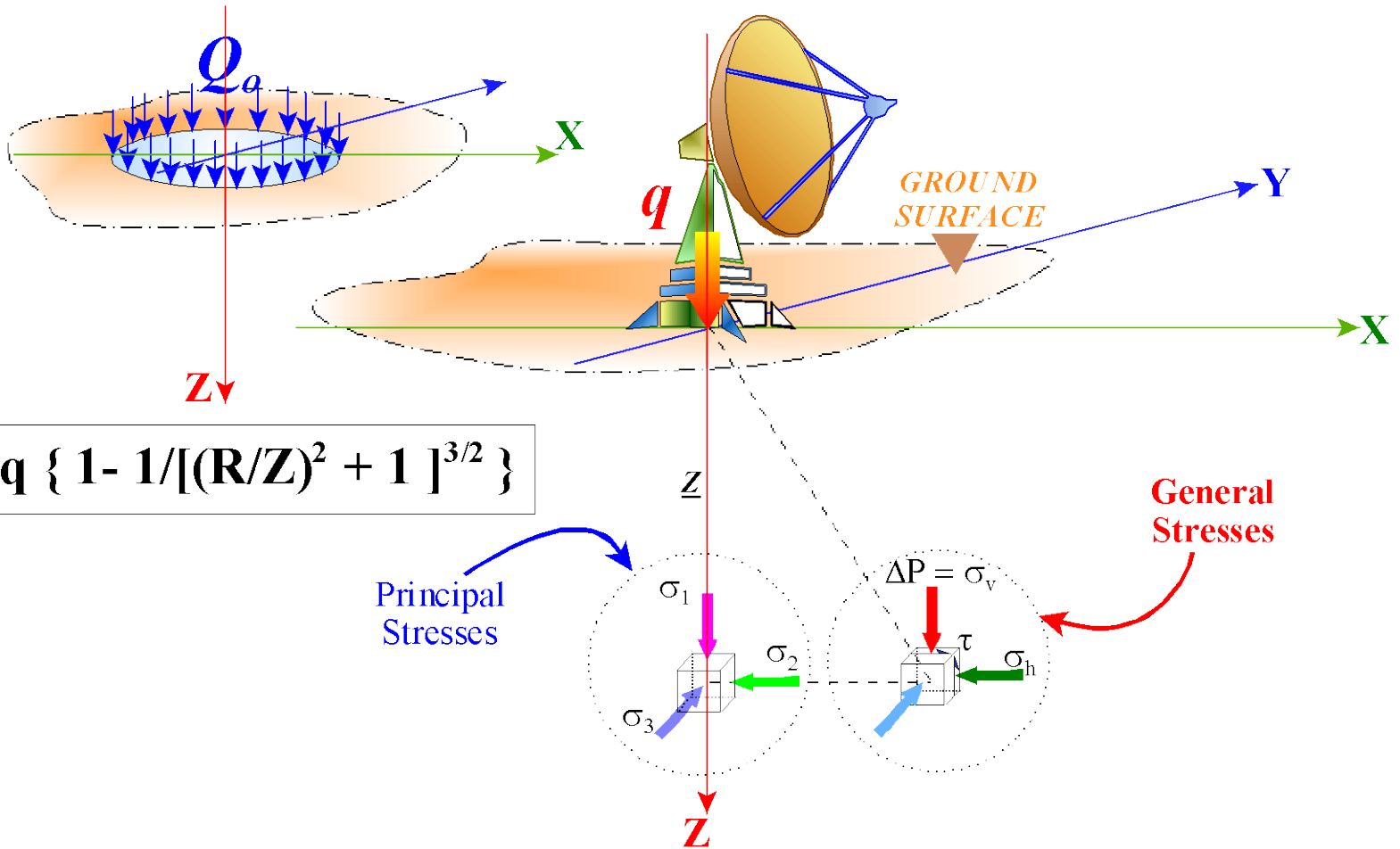
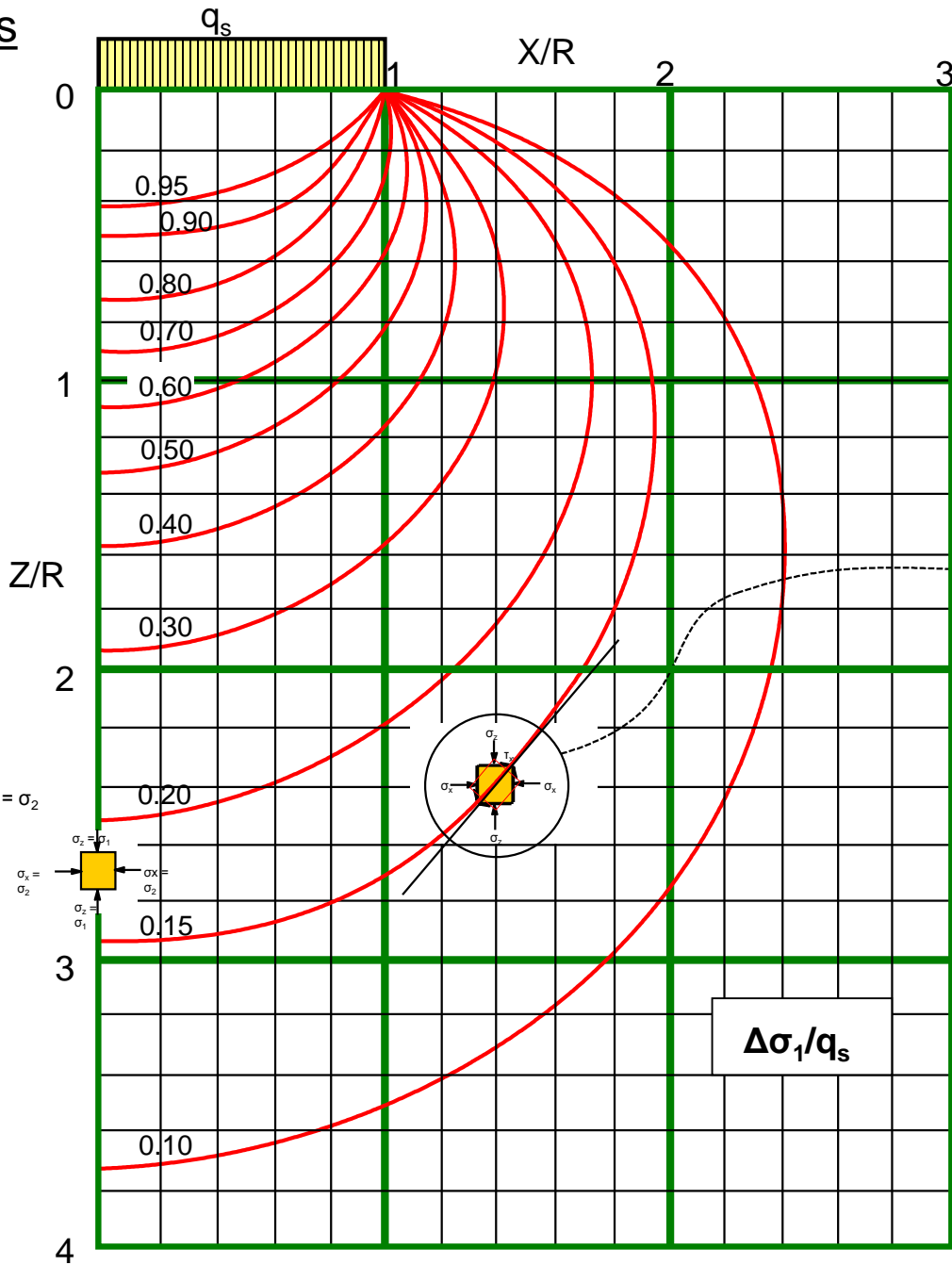# 6. VERTICAL STRESS DUE TO RECTANGULAR LOAD

$$\Delta P = q\,I_R$$

$I_R = 1/4\pi\,\{[\,(2.\,m.n\,(m^2 + n^2 + 1)^{1/2}\,)\,/\,(m^2 + n^2 + m^2.\,n^2 + 1)\,]\,[(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] +$
$\tan^{-1}\,(2.m\,.n\,(m^2 + n^2 + 1)^{1/2}\,/\,(m^2 + n^2 - m^2\,.\,n^2 + 1)\}$

$I_R = f\,(m,n)$

L

B

$q = Load\,/Area$

Loaded Area
Foundation Level

L

B

$m = B/Z$

$n = L/Z$

$m\,\&\,n\ \ from\ Charts$

Corner of
the Building

Z

$\Delta P = \sigma_v$

τ

σ_h
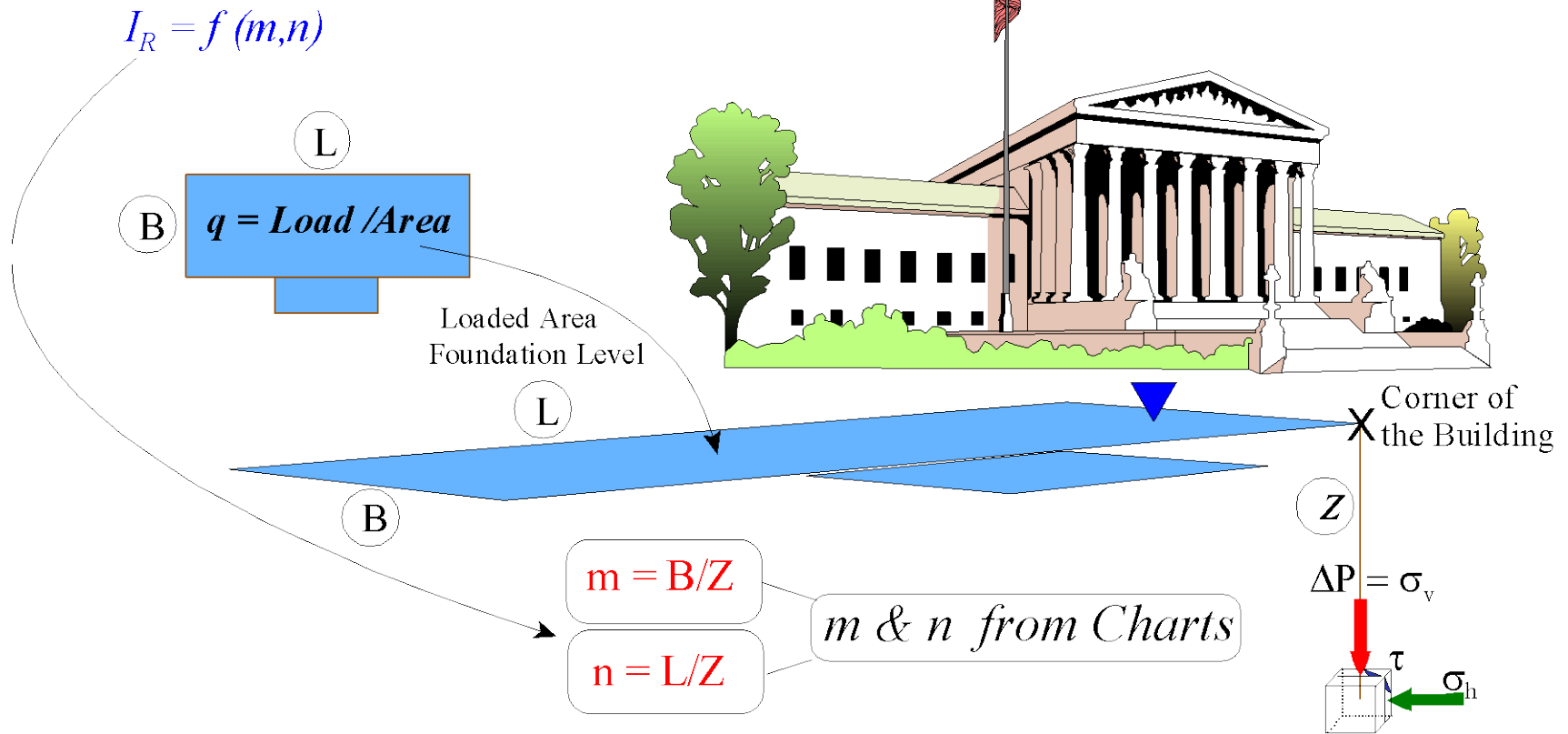
Stress Bulbs
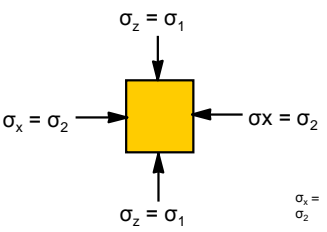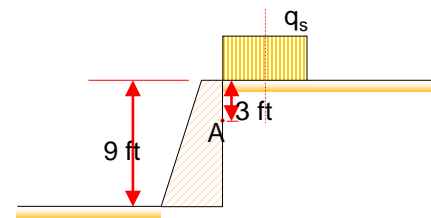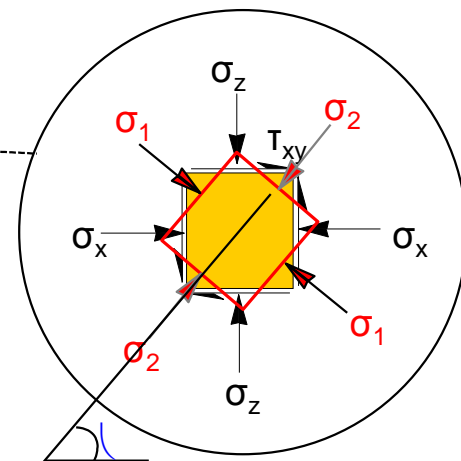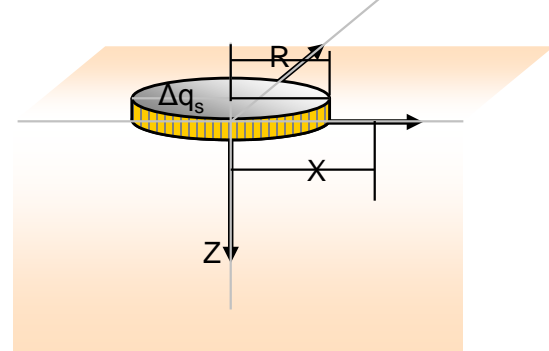
Circular Load: (Major Principal Stress)/(Surface Stress)

Geostatic Stresses
&
Stress Distribution in Soils

Newmark's Charts

Find:
Vertical Stress at Point A

11 ft

$q = 800$ lb/ft^2

15 ft

12 ft

Z = 10 ft

Vertical Stress

Point A

15'

11'   **q** = 800 psf

12'

Z= 10'

A

Influence Value = 0.005

a   b

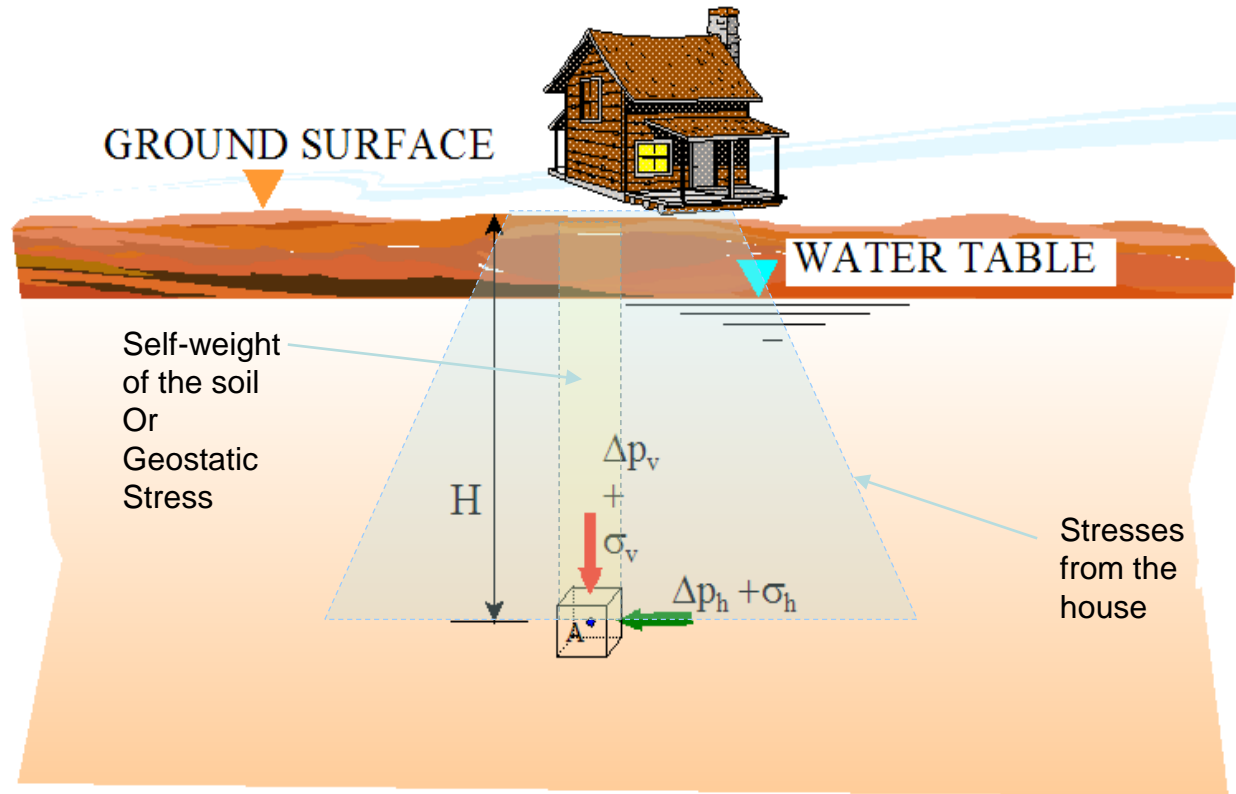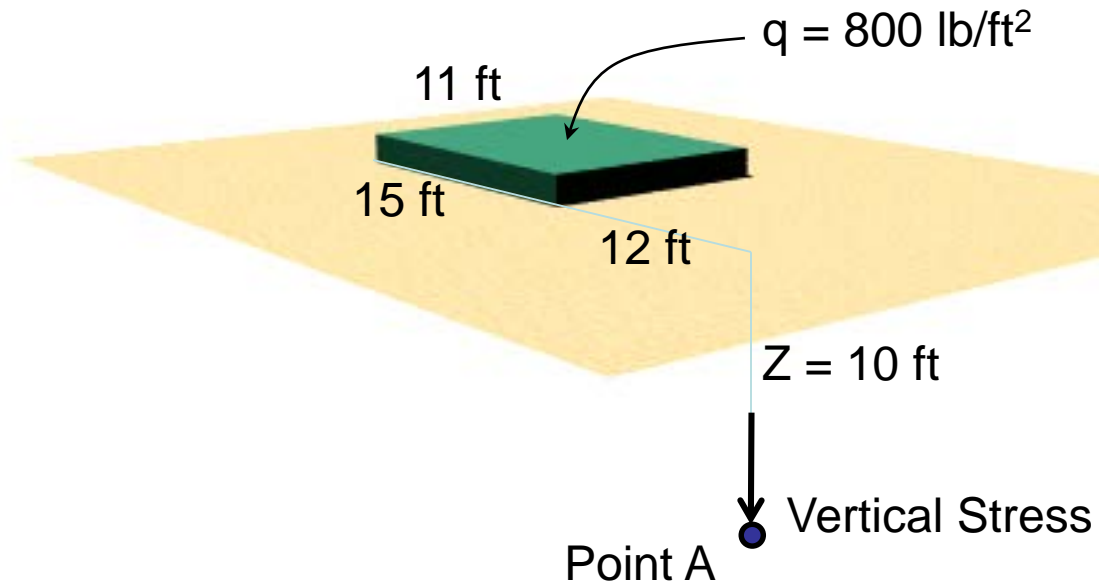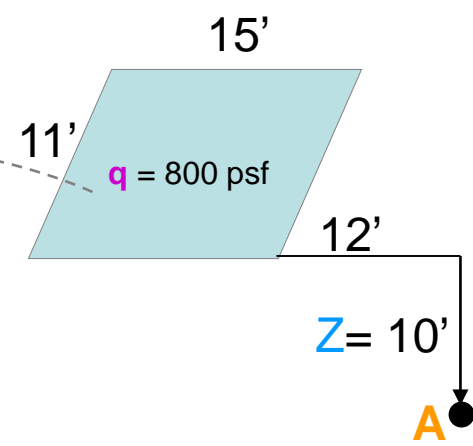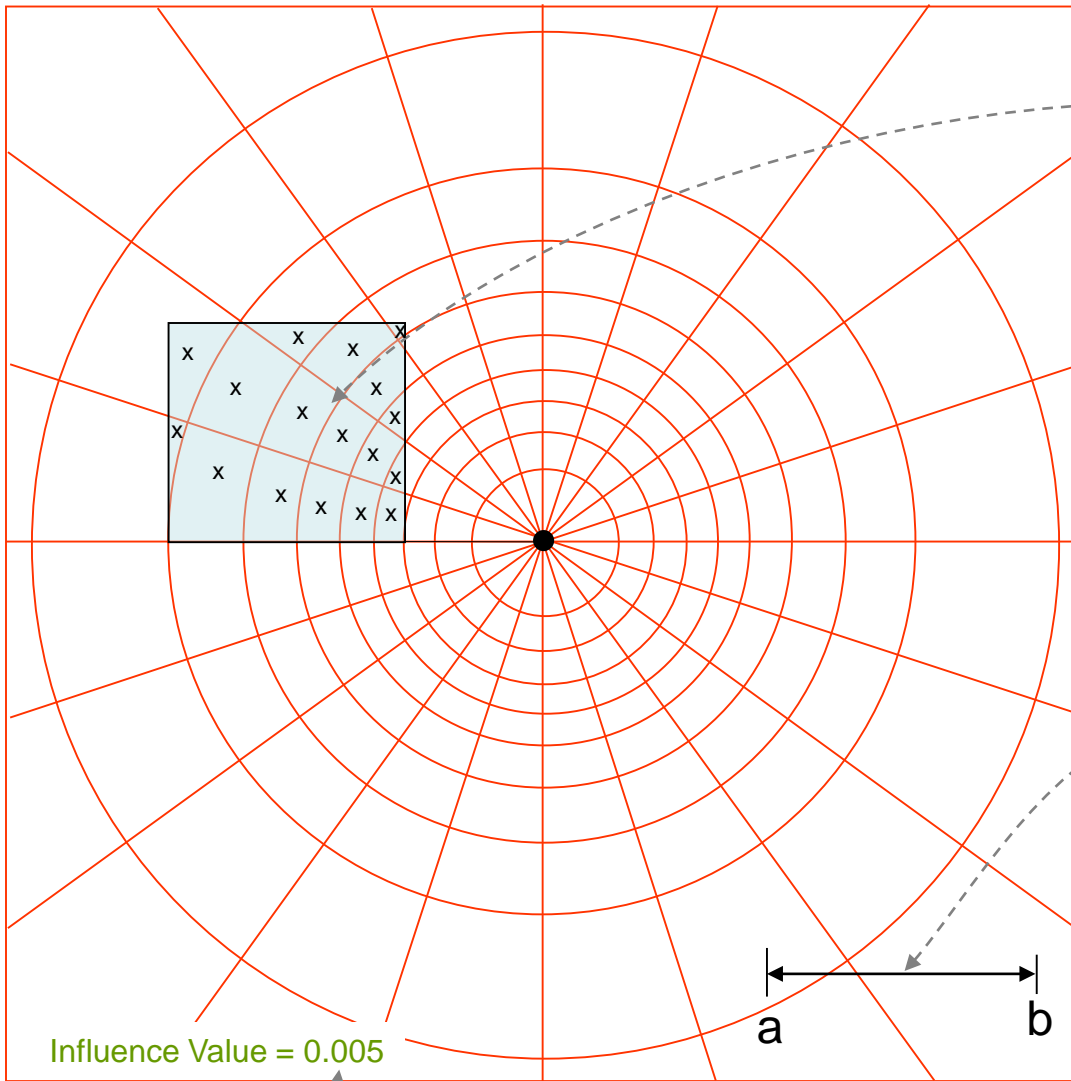**Steps to determine Vertical Stress Using Newmark's Charts:**

1- Select a scale to draw the shape shown above

2- The scale is
    Z = length of line **ab**

3- Draw the shape using the selected scale

4- Place the shape on Newmark chart so that point **A** is at the center

5- Count the stress fields **n**

6- The stress at point **A** = **n** x **q** x 0.005
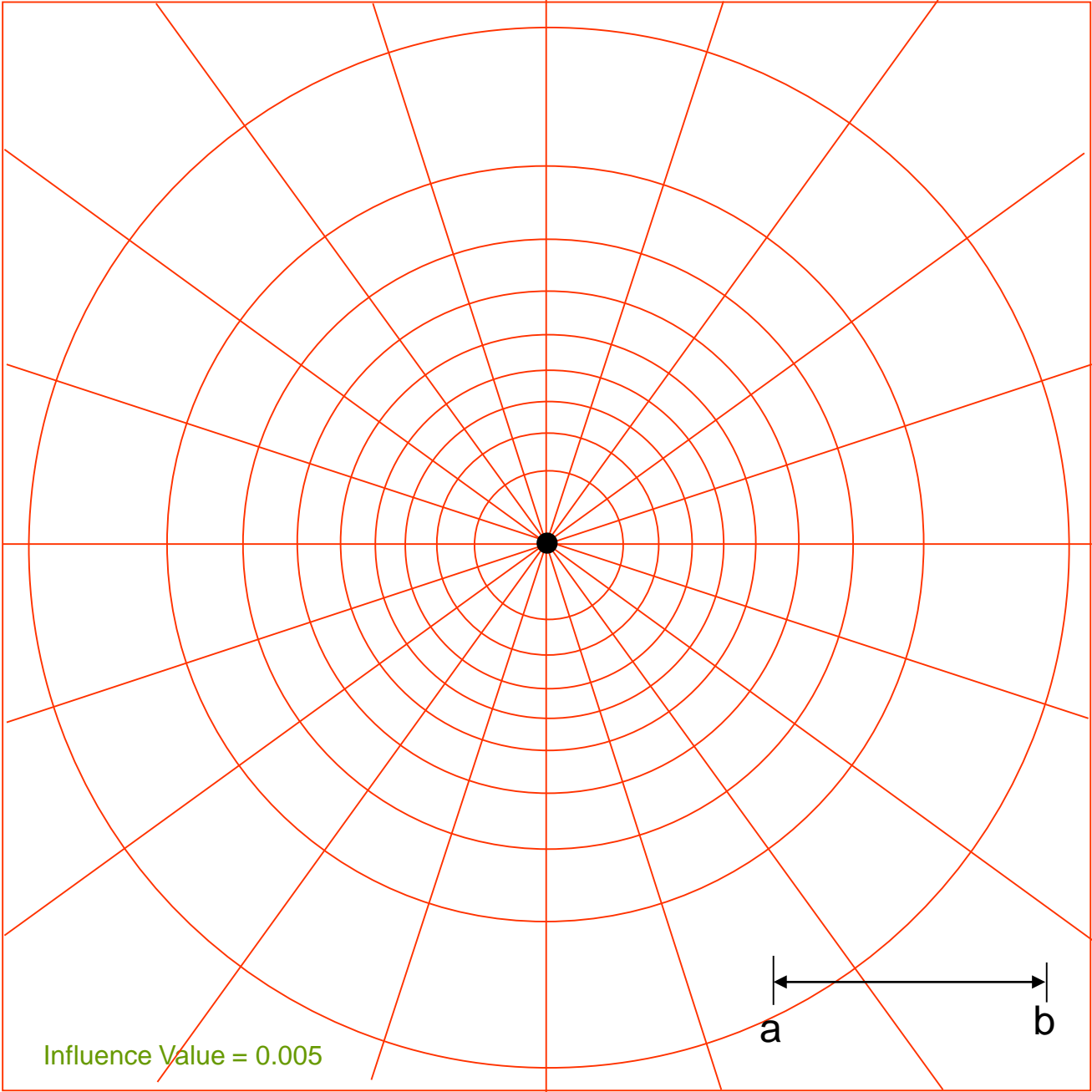
Influence Value = 0.005

Lateral stresses due to self-weight of soil

Vertical and Horizontal Stress in Water

Hydrostatic Stress or Pressure
Isotropic Stress

Water Table

$$\sigma_v = d\, \gamma_{water}$$

d

$$\sigma_h = d\, \gamma_{water} = \sigma_v$$
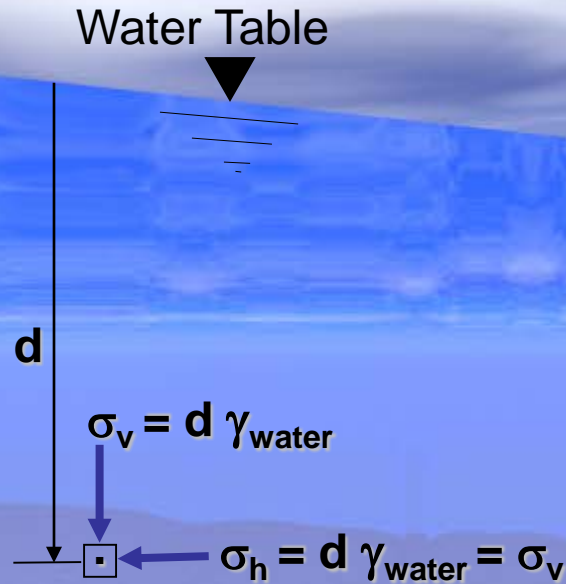
Vertical and Horizontal Stress in Water

Assume a massive wall retaining water. What is the pressure on this wall??

Is the wall stable???

Water Table

d

$\sigma_v = d\ \gamma_{water}$

$\sigma_h = d\ \gamma_{water} = \sigma_v$

$P_w = H\ \gamma_{water}$

Water pressure on the wall

By: Kamal Tawfiq, Ph.D., P.E.
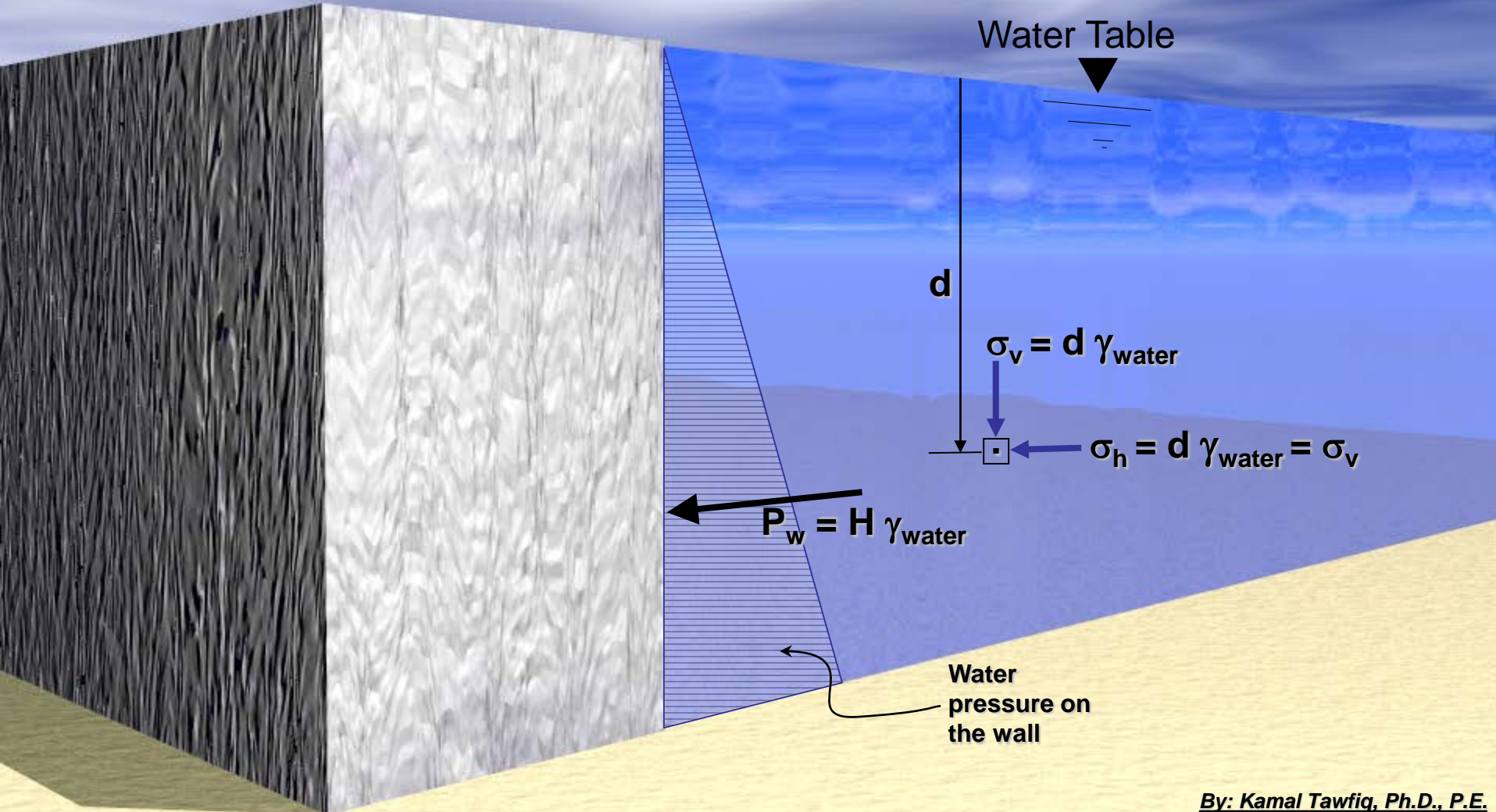
Vertical and Horizontal Stress in Soil

Geostatic Stress
Anisotropic Stress

Ground Surface

d

$\sigma_v = d\ \gamma_{soil}$

$\sigma_h = (d\ \gamma_{soil})\ K_o = \sigma_v\ K_o$

By: Kamal Tawfiq, Ph.D., P.E.
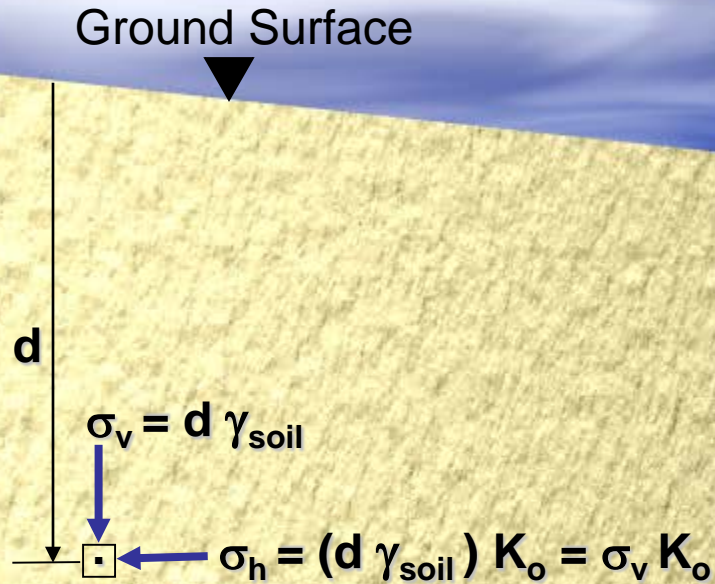
Vertical and Horizontal Stress in Soil

Assume a vertical cut in sand soil. Is this cut safe????

Soil Displacement = ??

Ground Surface

d

$\sigma_v = d\ \gamma_{soil}$

$\sigma_h = (d\ \gamma_{soil})\ ?? = \sigma_v\ ??$

By: Kamal Tawfiq, Ph.D., P.E.
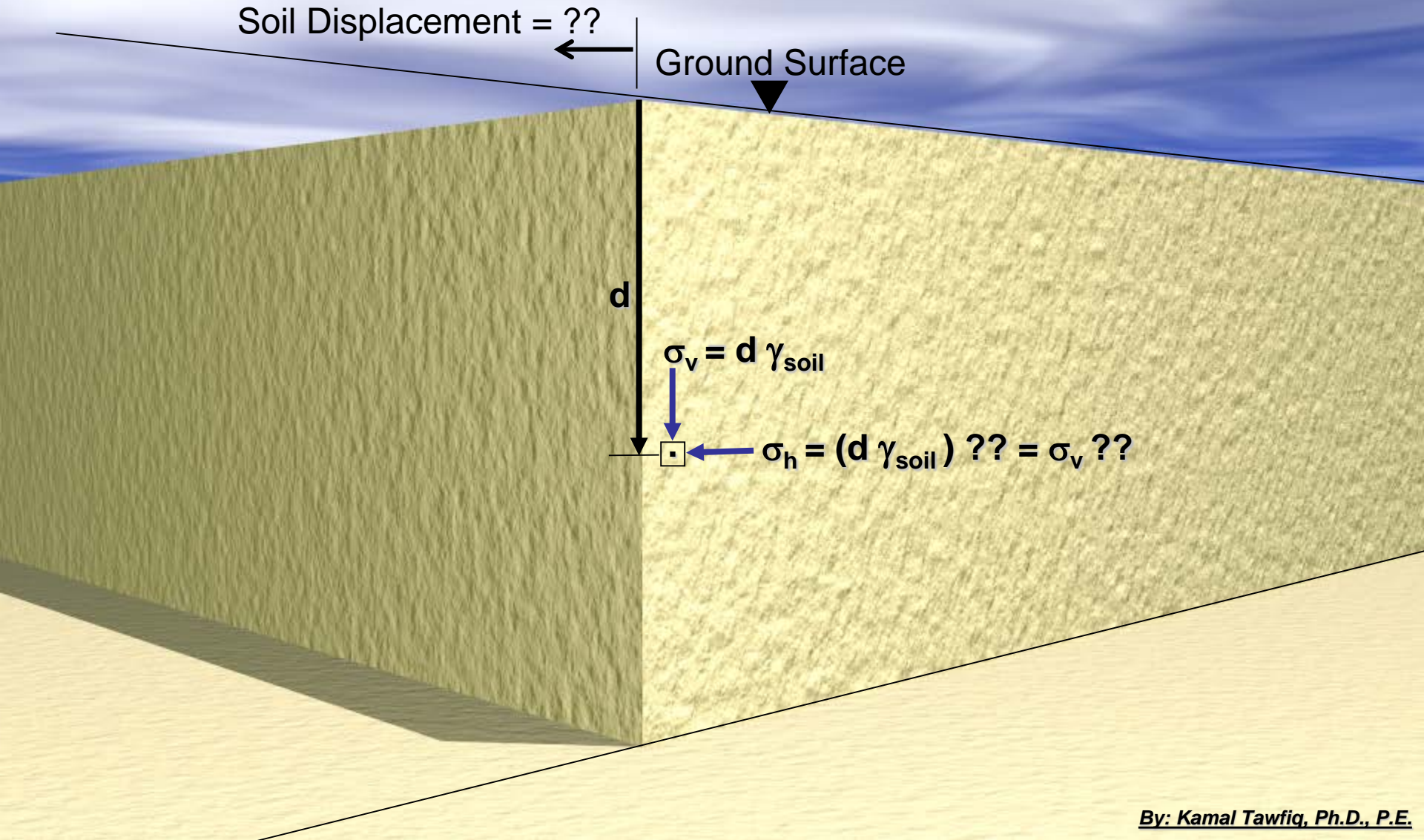
AT REST CONDITIONS

$\Delta = 0$
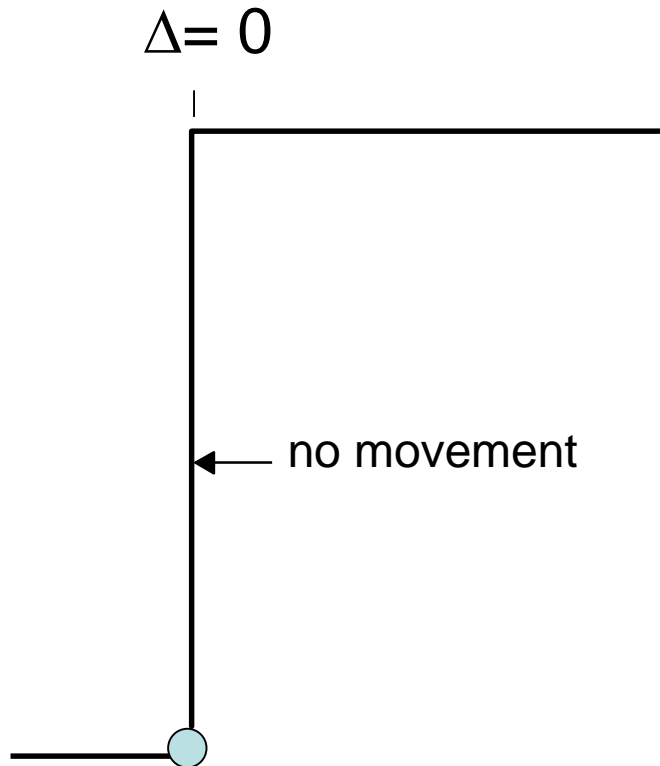
no movement ←

The soil remains undisturbed.

Structure is rigid, does not move and can be placed in the soil without allowing any lateral soil movement.

Lateral pressures existing in the soil before installations are still prevailing

ACTIVE CASE
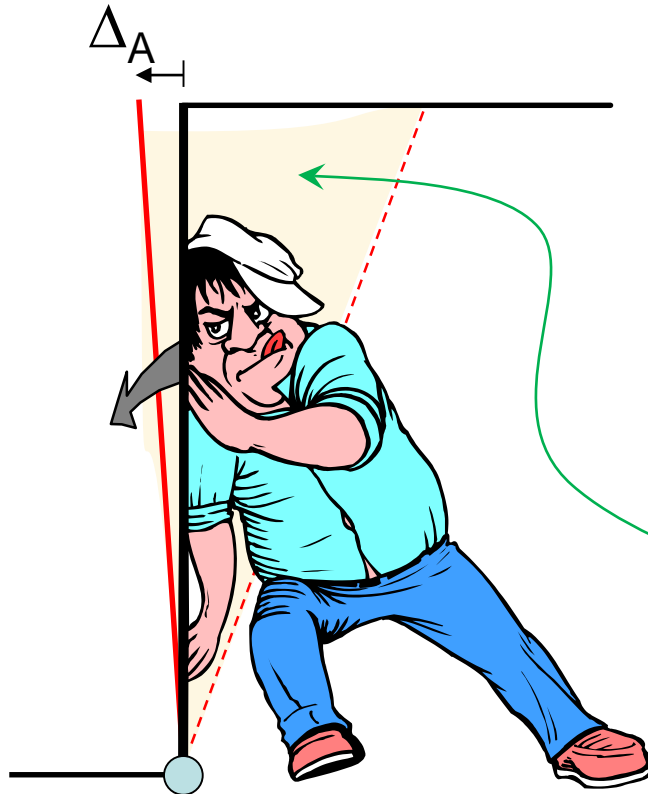
The structure moves away from the soil about its base.

The soil movement is assisted by gravity.

Active Wedge

PASSIVE CASE

Δ_P
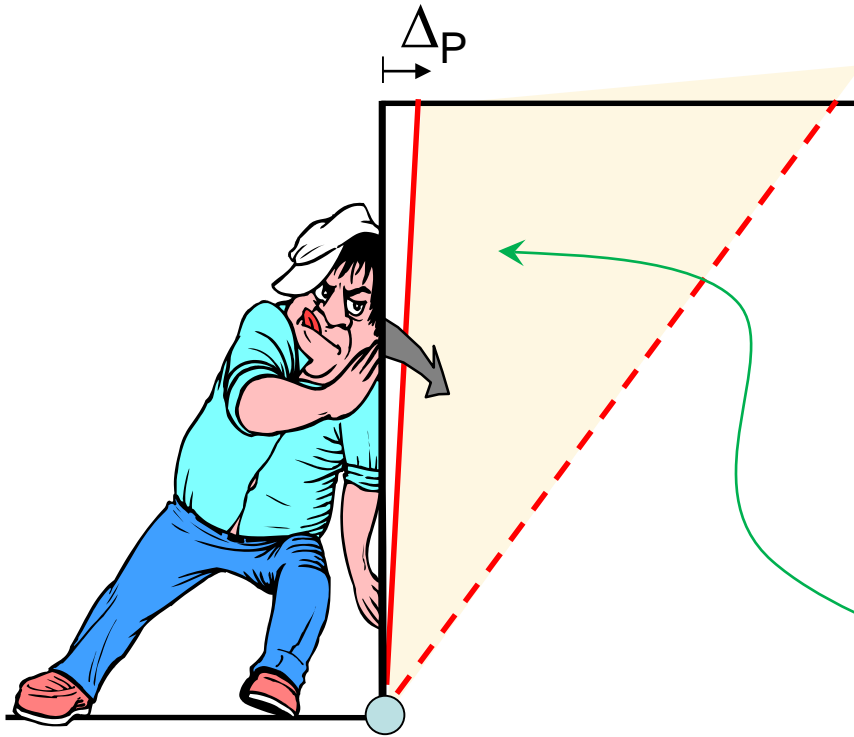
The structure moves towards the soil about its base.

The soil movement is against gravity

Passive Wedge

Vertical and Horizontal Stress in Soil

Stability of a retaining wall.

Deformation

Active Wedge.
Exerting pressure
on the wall

d

$\sigma_v = d \, \gamma_{soil}$

$\sigma_h = (d \, \gamma_{soil}) \, K_a = \sigma_v \, K_a$

Active Wedge

Active

Passive Wedge
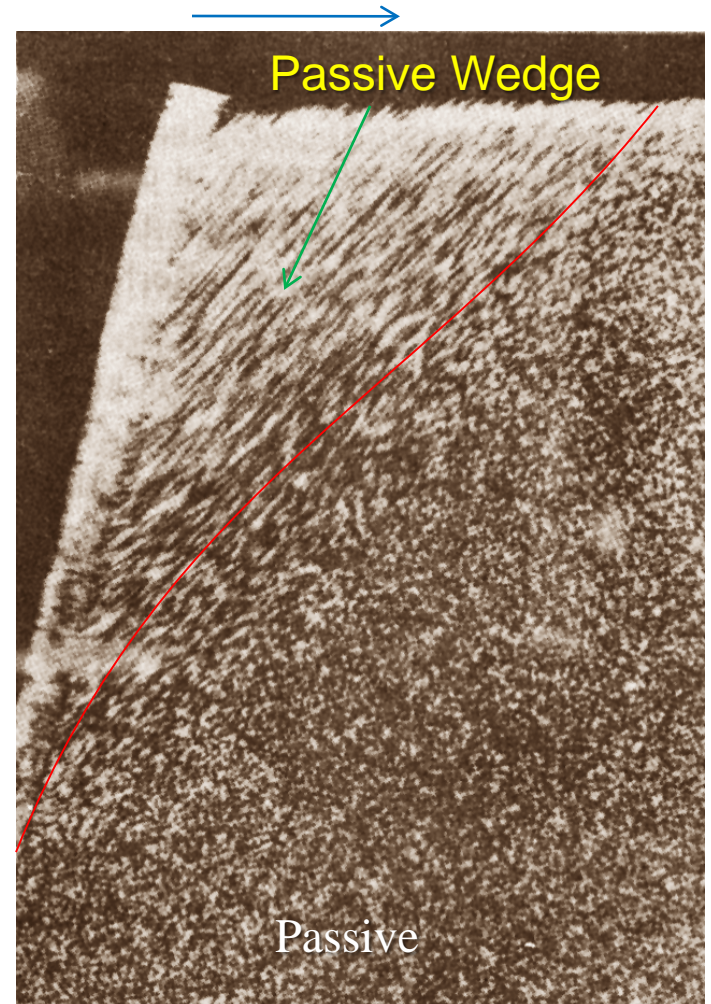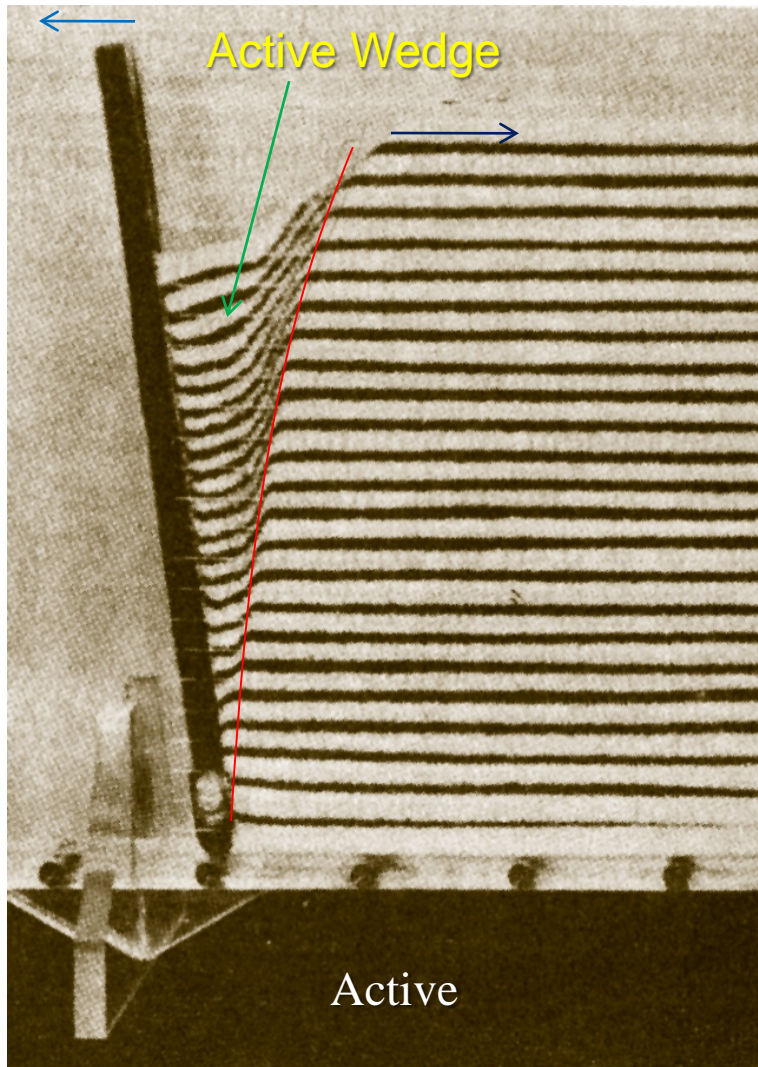
Passive

Earth Pressure Behind Retaining Wall

Rankine's Active Earth Pressure in ϕ – Soil
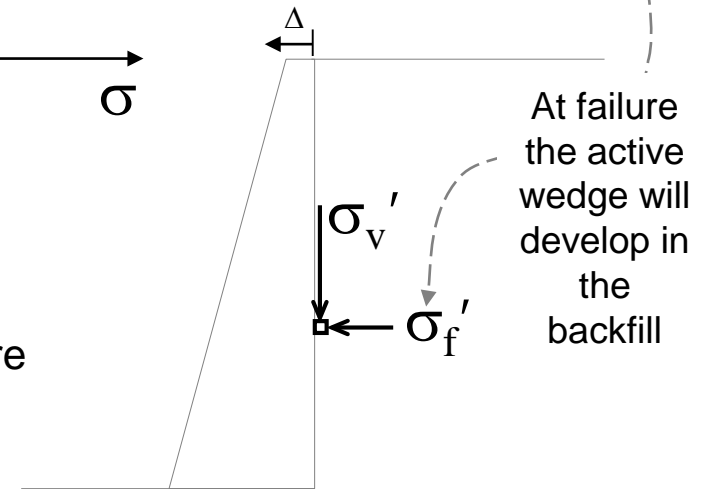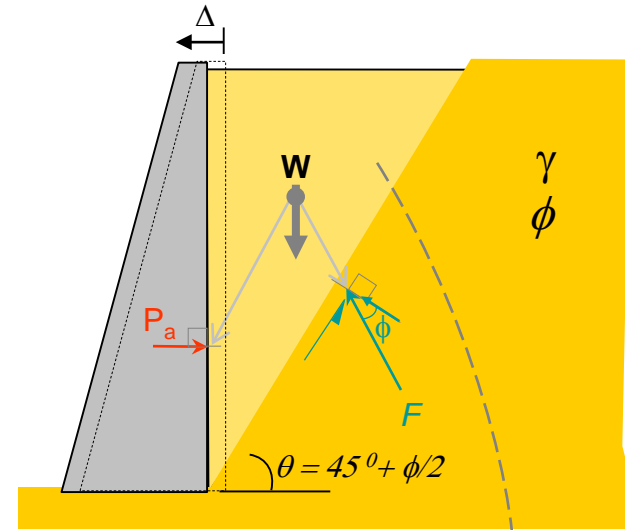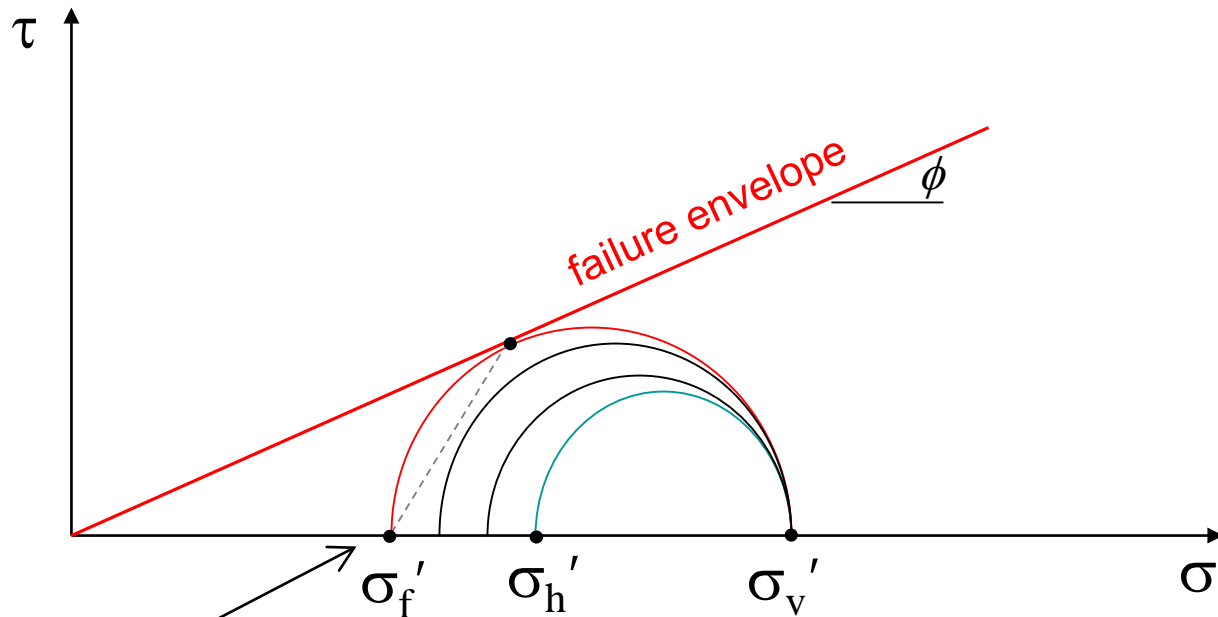
active earth pressure

σ_h is decreasing until failure

Coefficient of active earth pressure

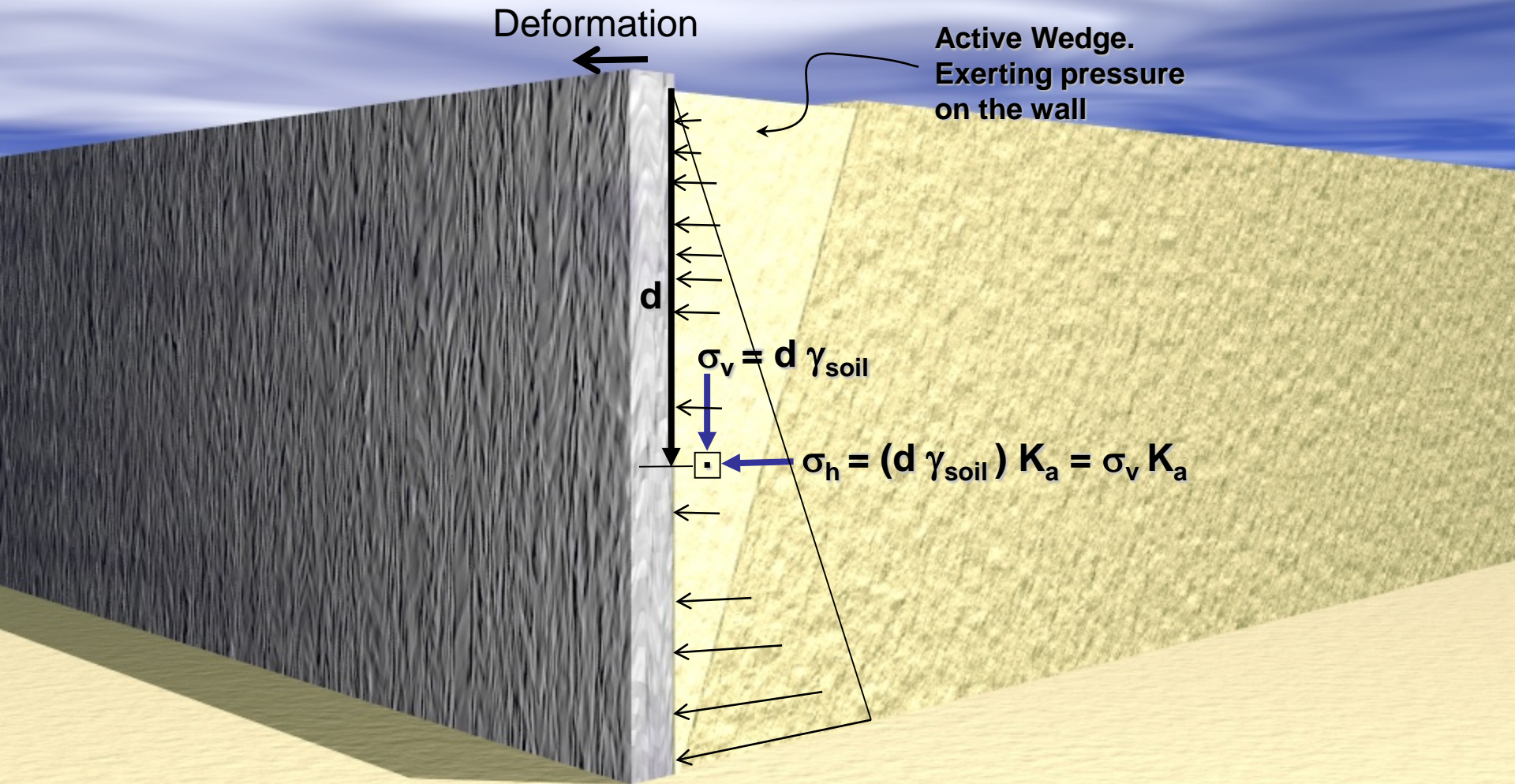$$\sigma_f' = \sigma_v' \, \tan^2 (45 - \phi/2)$$

$$K_a = \tan^2 (45 - \phi/2) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

At failure the active wedge will develop in the backfill

$\theta = 45^0 + \phi/2$

By: Kamal Tawfiq, Ph.D., P.E.

Rankine's Passive Earth Pressure in ϕ – Soil

$$\sigma_f' = \sigma_v' \, \tan^2 (45 + \phi/2)$$

Coefficient of passive earth pressure

$$K_p = \tan^2 (45 + \phi/2) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

By: Kamal Tawfiq, Ph.D., P.E.

Rankine's Active Earth Pressure in ϕ − Soil

$\alpha = 0$

$\delta = 0$

Active Wedge

σ_v'

σ_f'

$\beta = 0$

$$K_a = \tan^2 (45 - \phi/2) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

G.S.

γ_{soil}
ϕ

W.T.

h_1

h_2

$\sigma_h = \gamma_{soil} h_1$

$+$

$\sigma_h = \gamma_{soil} h_1 + (\gamma_{soil} + \gamma_w) h_2$

$\sigma_w = \gamma_w h_2$
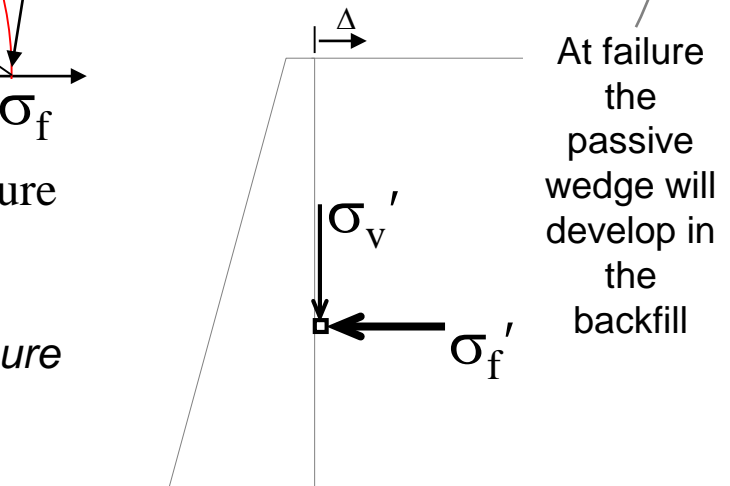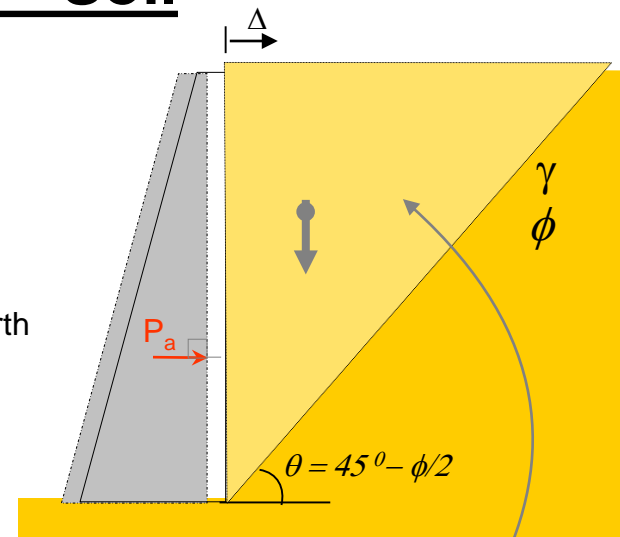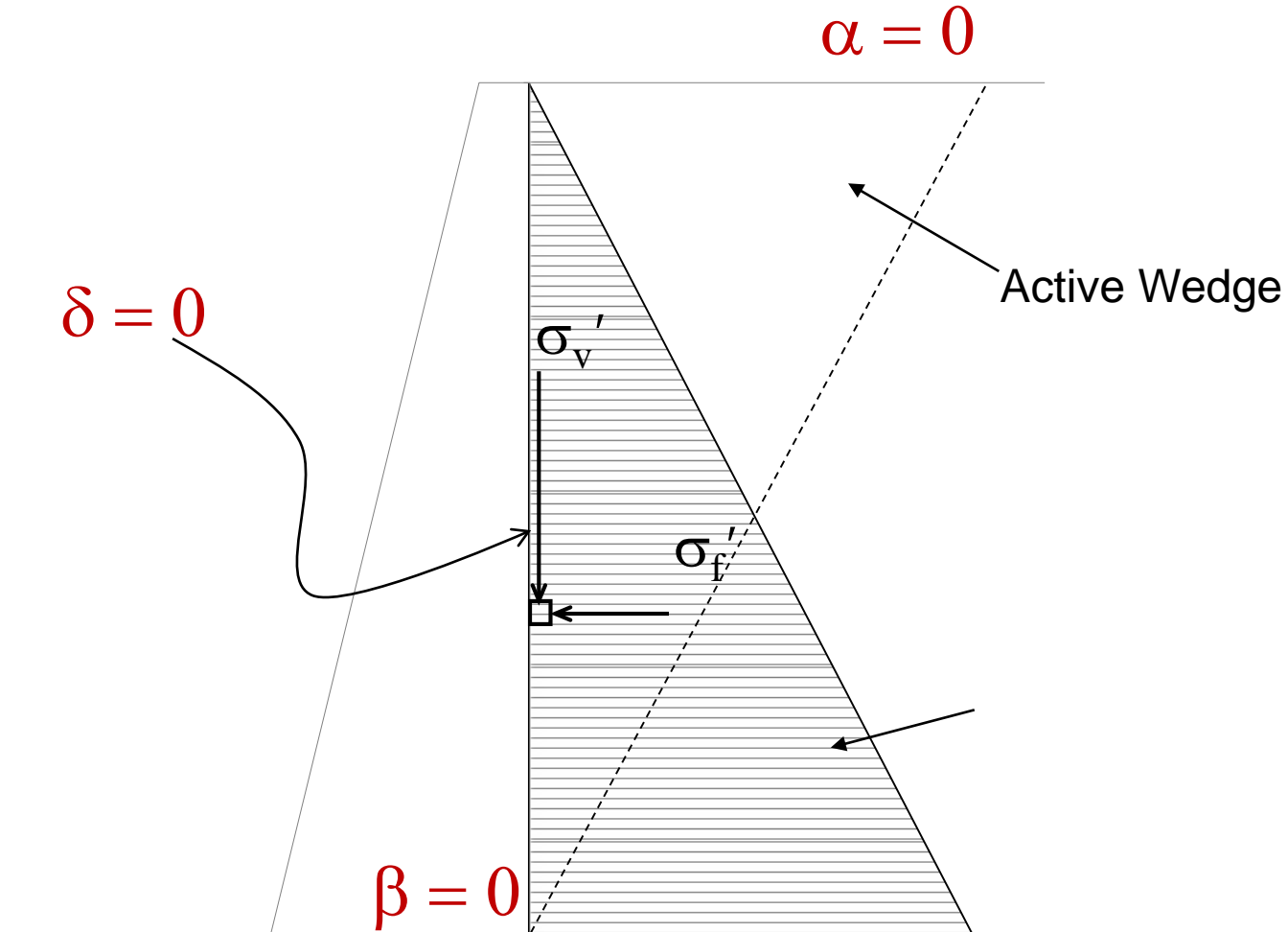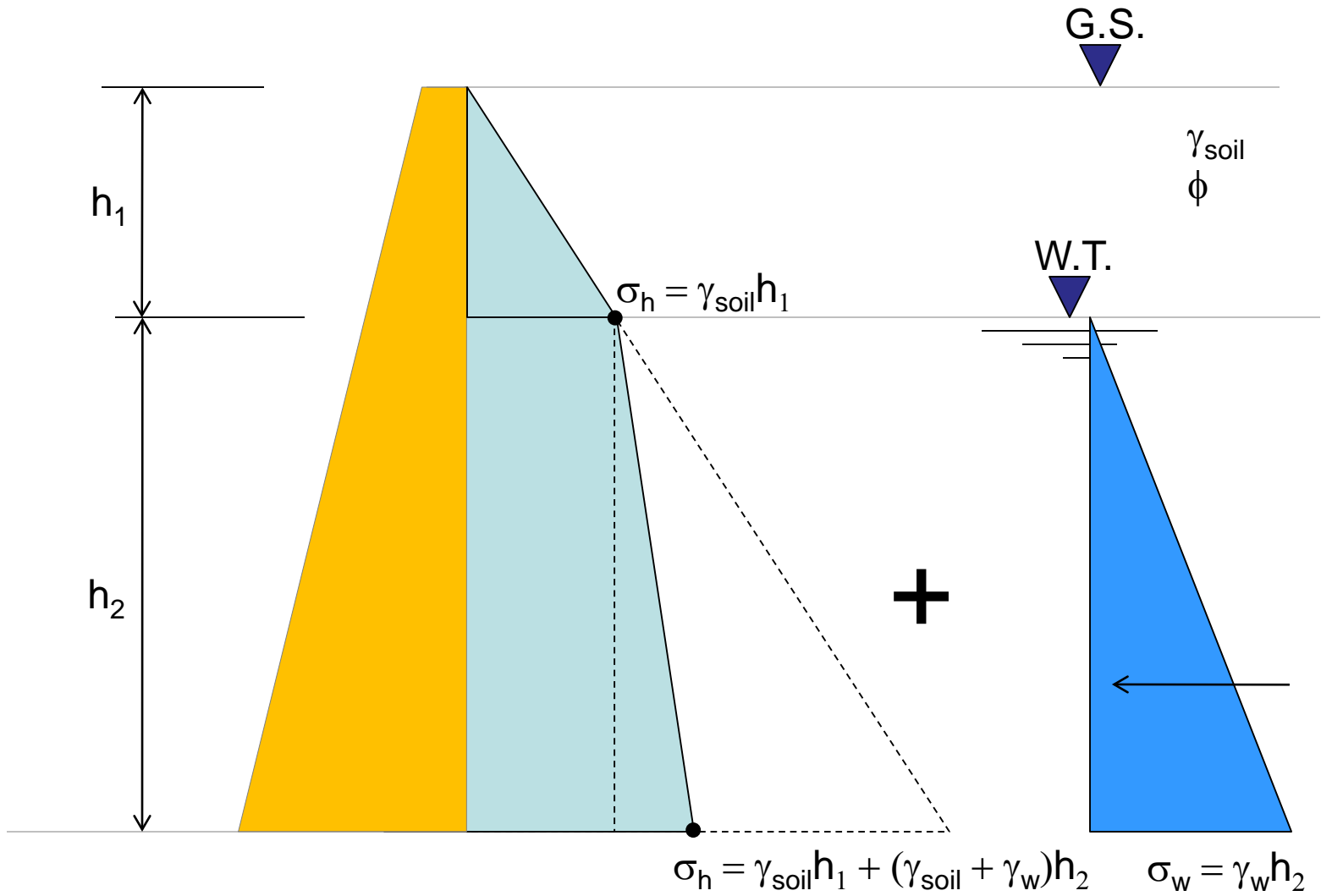
Rankine's Active Earth Pressure

Case 2: c- ϕ Soil
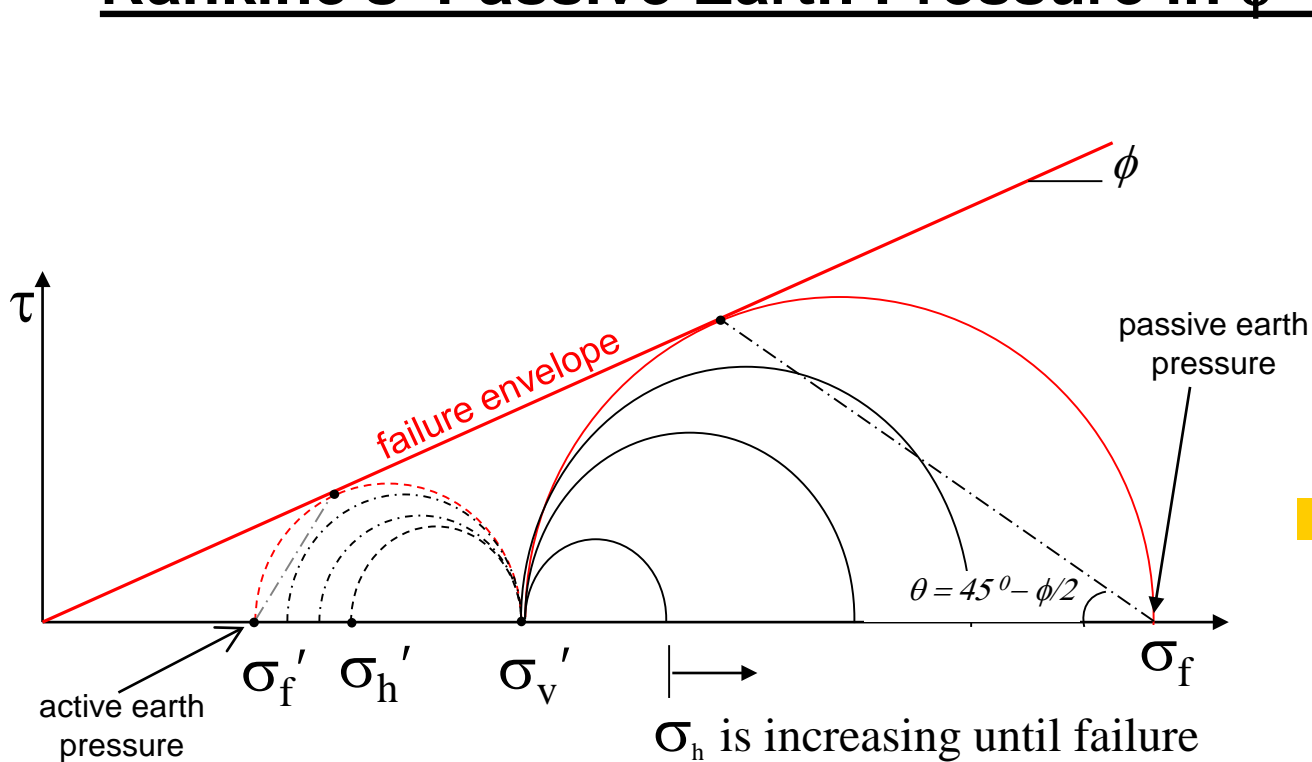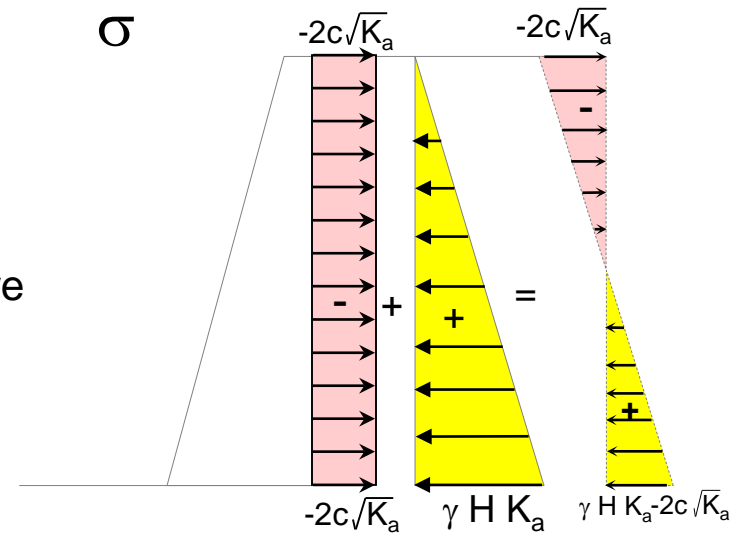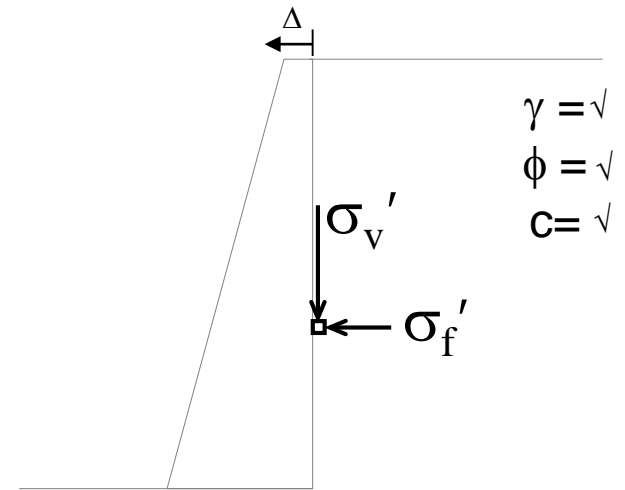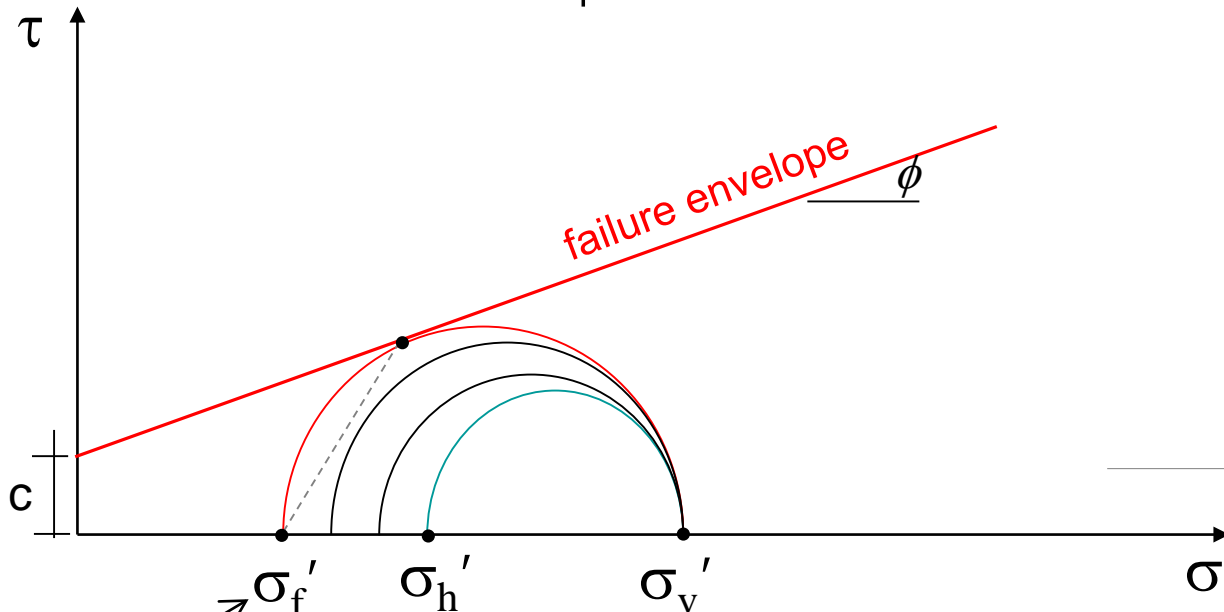

active earth pressure

σ_h is decreasing until failure

Coefficient of active earth pressure

$$\sigma_f{}' = \sigma_v{}' \ tan^2 \ (45 - \phi/2) - 2 \ c \ tan \ (45 - \phi/2)$$

$$where \ K_a = tan^2 \ (45 - \phi/2) = \frac{1 - sin\phi}{1 + sin\phi}$$

$$\sigma_f{}' = \sigma_v{}' \ K_a - 2 \ c \ \sqrt{K_a}$$

By: Kamal Tawfiq, Ph.D., P.E.

Rankine's Passive Earth Pressure in c- ϕ Soil

failure envelope ϕ

τ

passive earth pressure

$\theta = 45^0 - \phi/2$

σ_f'

active earth pressure

σ_h' σ_v' σ_f

σ_h is increasing until failure

Δ

$\gamma = \sqrt{}$
$\phi = \sqrt{}$
$c = \sqrt{}$

σ_v'

σ_f'
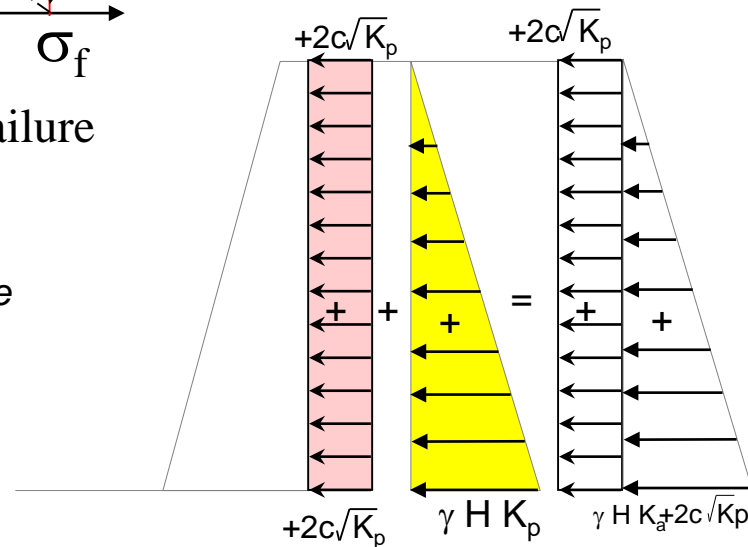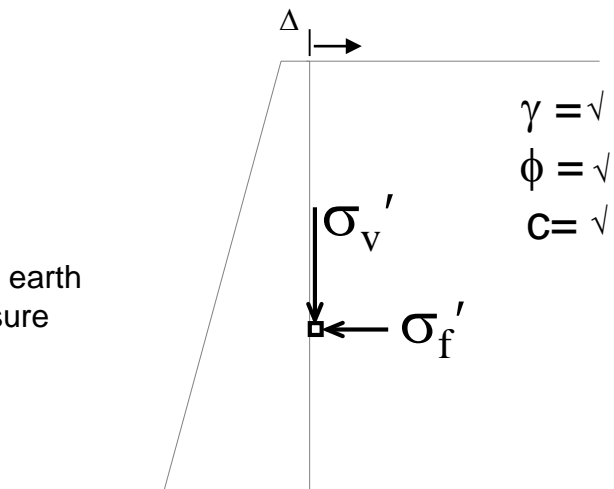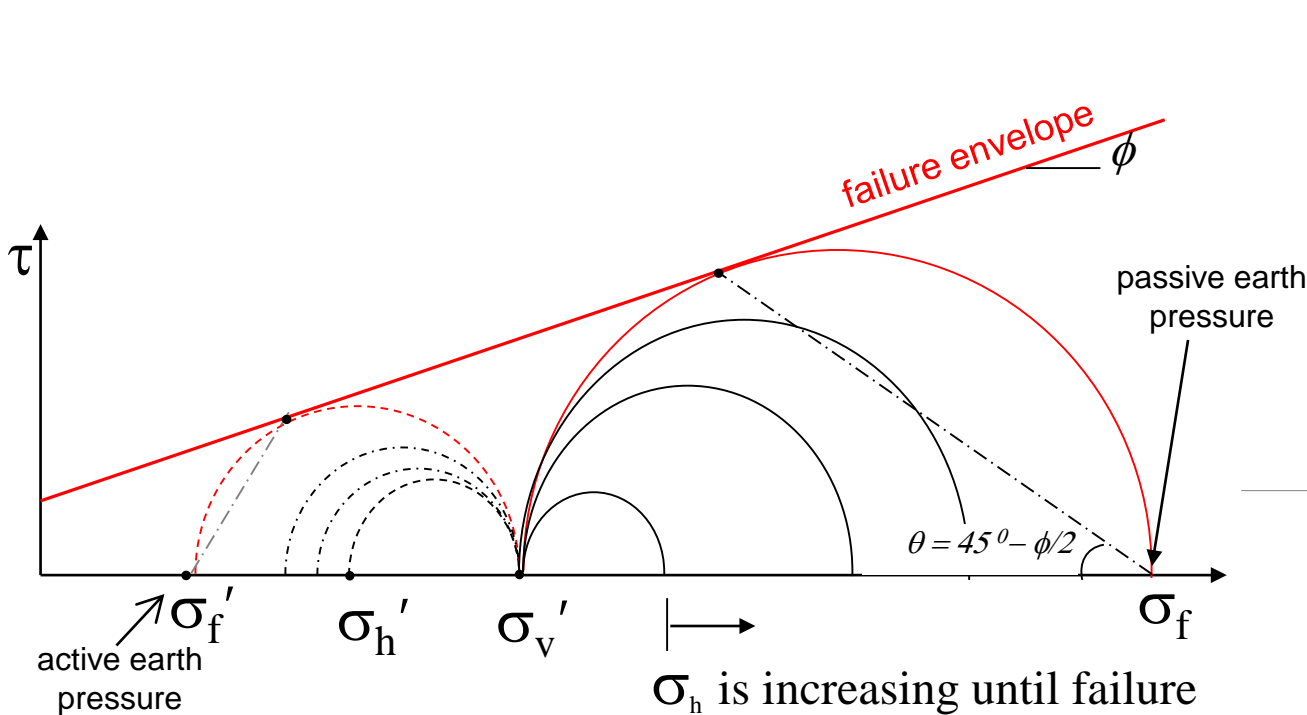
Coefficient of passive earth pressure

$$\sigma_f' = \sigma_v' \tan^2 (45 + \phi/2) + 2 c \tan (45 + \phi/2)$$

$$\text{where } K_p = \tan^2 (45 + \phi/2) = \frac{1 + sin\phi}{1 - sin\phi}$$

$$\sigma_f' = \sigma_v' K_p + 2 c \sqrt{K_p}$$

$+2c\sqrt{K_p}$ $+2c\sqrt{K_p}$

$+$ $+$ $+$ $=$ $+$ $+$

$+2c\sqrt{K_p}$ γ H K_p γ H K_a $+2c\sqrt{Kp}$

By: Kamal Tawfig, Ph.D., P.E.

Rankine's Active Earth Pressure in ϕ – Soil

Case 1: For Inclined Surface

$$K_a = \cos\alpha \; \frac{\cos\alpha - (\cos^2\alpha - \cos^2\phi)^{1/2}}{\cos\alpha + (\cos^2\alpha - \cos^2\phi)^{1/2}}$$

$$P_a = 0.5\,\gamma\,H^2\,K_a$$
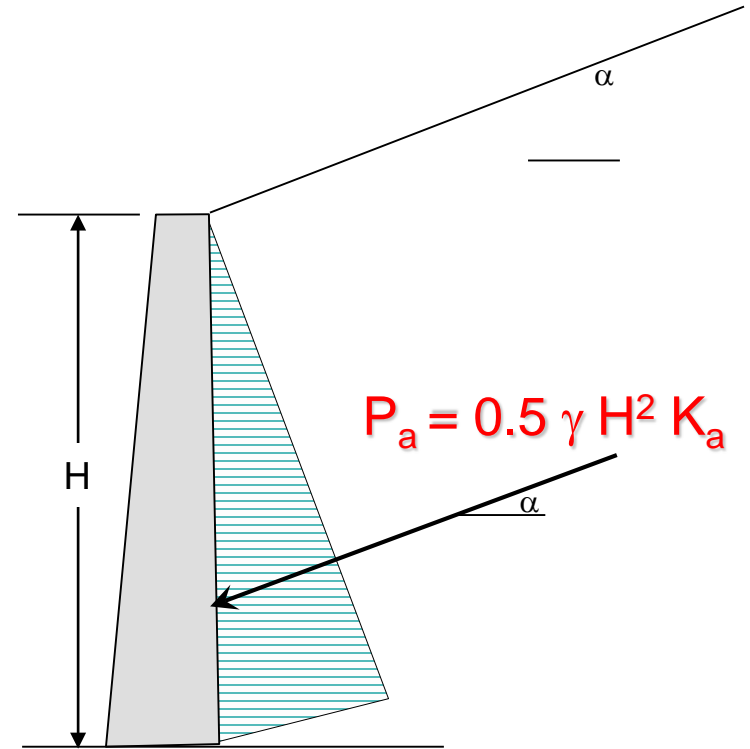
Fig.1

Coulomb Earth Pressure Method (1776)

Coulomb Earth Pressure Method (1776)

Forces acting on the wall.

Wall Displacement = Δ

This active wedge will slide along the failure surface and will exert pressure on the wall. Also, the sliding wedge will push the wall away from the backfill.

W

Soil-Wall Friction

Soil-Soil Friction

Failure surface

Pressure from the sliding wedge

By: Kamal Tawfiq, Ph.D., P.E.
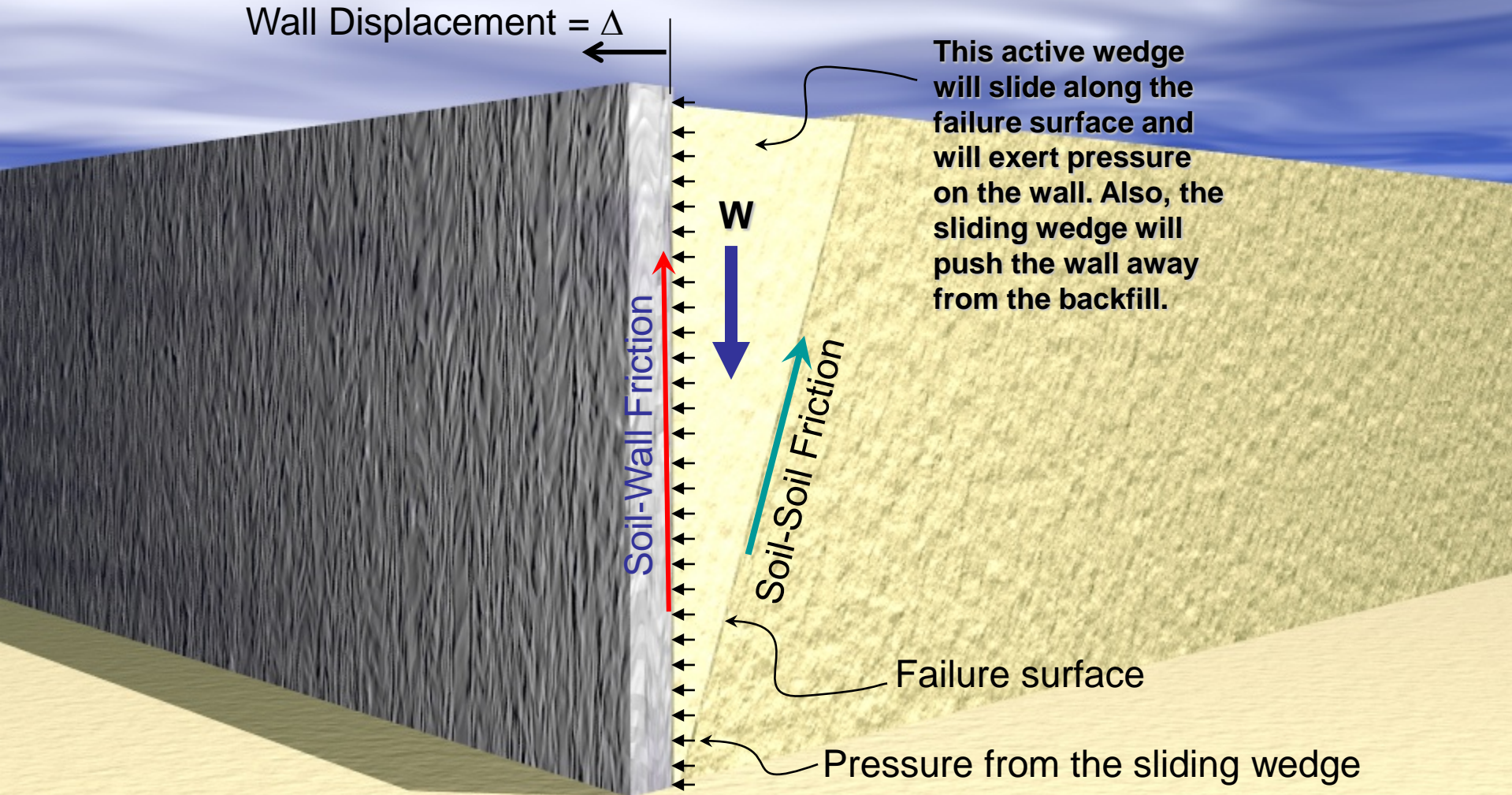
Coulomb Earth Pressure Method (1776)

Forces acting on an inclined wall.
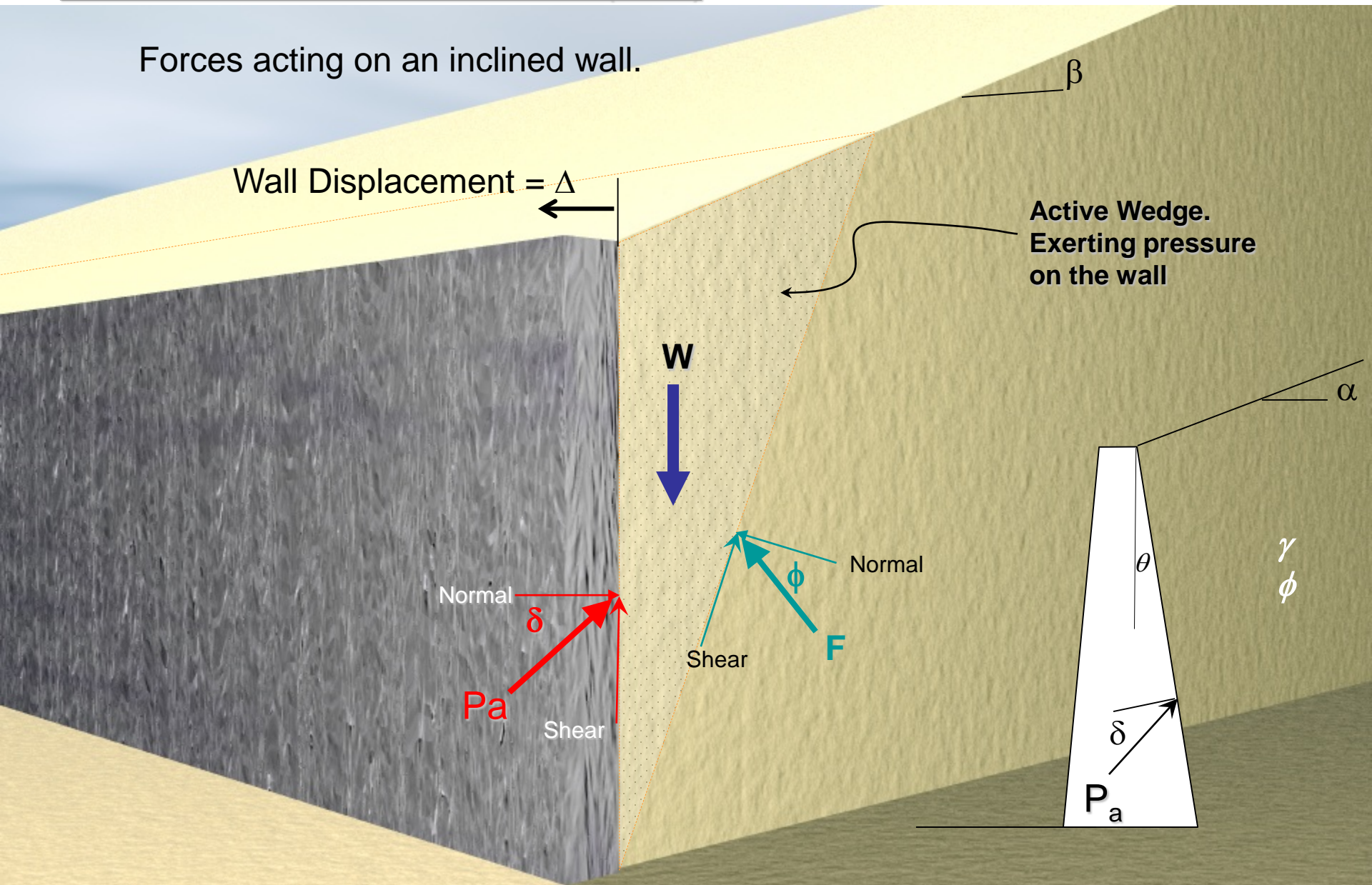
β

Wall Displacement = Δ

**Active Wedge.
Exerting pressure
on the wall**

α

W

Normal

δ

Normal

ϕ

Pa

Shear

F

Shear

θ

γ
ϕ

δ

P_a

By: Kamal Tawfiq, Ph.D., P.E.

Coulomb Earth Pressure Method (1776)

Forces acting on the wall.

Wall Displacement = Δ

**Active Wedge.
Exerting pressure
on the wall**

W

Normal

δ

Pa

Shear

Normal

ϕ

Shear

F
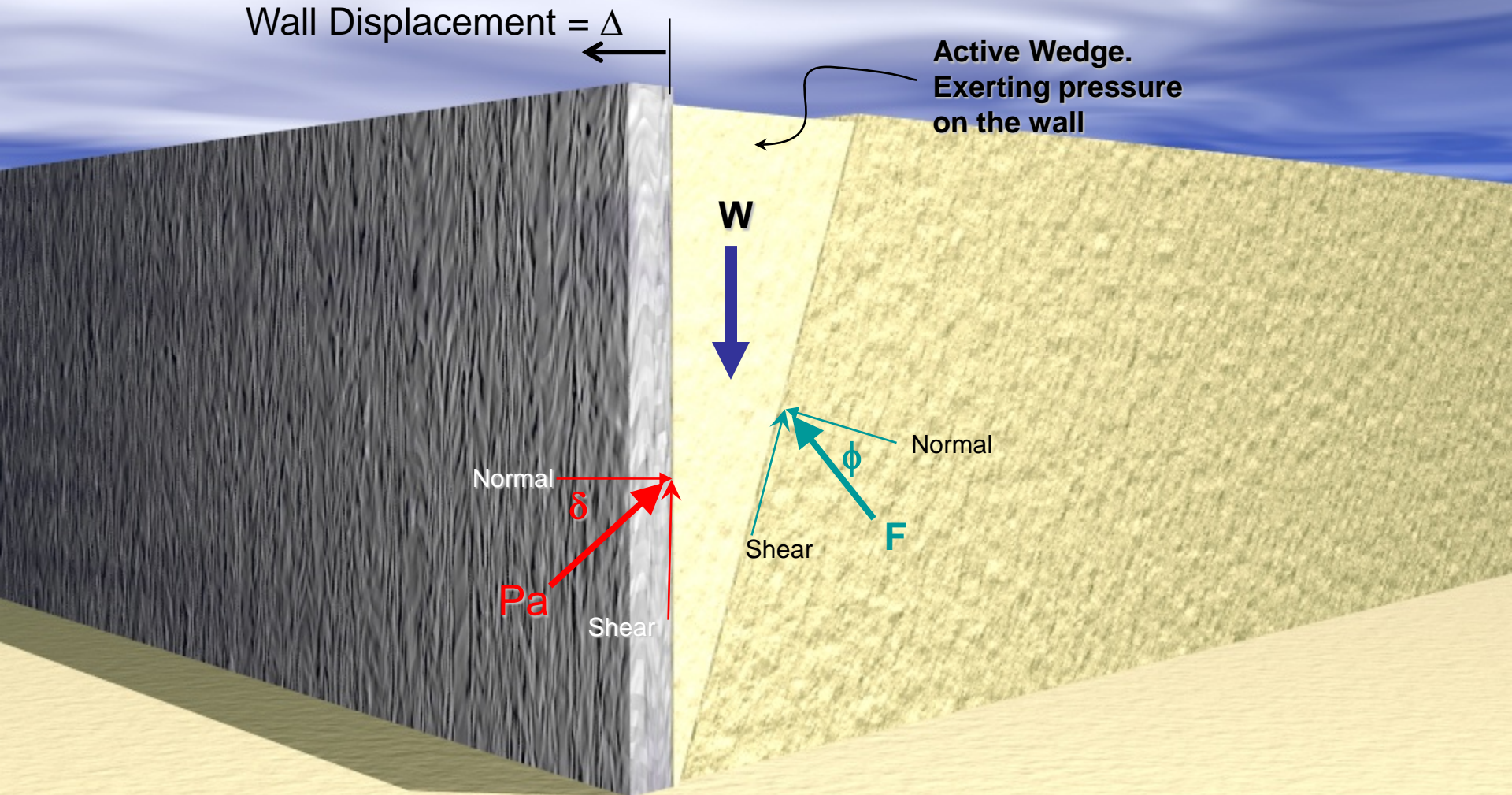
Coulomb Earth Pressure Method (1776)

W = weight of the soil wedge
R = resultant of the shear and normal forces on the failure surface BC
P_a = the active force per unit length of the wall. The direction of Pa is inclined at an angle d to the normal drawn and the face of the wall that supports the soil

Δ = the angle of friction between the soil and the wall

The force triangle for the wedge is shown in the Figure 2. From the sine law, the forces can be set as follows:

$$\frac{W}{Sin\,(90 + \theta + \delta - \beta + \phi)} = \frac{P_a}{Sin\,(\beta - \phi)} \qquad (1)$$

$$P_a = \frac{Sin\,(\beta - \phi)\,W}{Sin\,(90 + \theta + \delta - \beta + \phi)} \qquad (2)$$

Substitute for W, Eq. 2 can be written as

$$P_a = 0.5\,\gamma\,H^2\left[\frac{\cos\,(\beta - \phi)\,\cos\,(\phi - \alpha)\,\sin\,(\beta - \phi)}{\cos^2\theta\,\sin(\beta - \alpha)\,\sin\,(90 + \theta - \delta - \beta + \phi)}\right] \qquad (2)$$

To determine the critical value for β for maximum P_a

$(dP_a/d\beta) = 0$

$$P_a = \tfrac{1}{2}\,K_a\,\gamma\,H^2$$

Where K_a = Coulomb's active earth pressure coefficient

$$K_a = \frac{\cos^2\,(\phi - \theta)}{\cos^2\theta\,\cos(\delta - \theta)\left[1 + \sqrt{\dfrac{\sin(\delta + \phi)\,\sin\,(\phi - \alpha)}{\cos\,(\delta + \theta)\,\cos\,(\theta - \alpha)}}\right]^2}$$

Fig.1

Fig.2

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Coulomb's Earth Pressure

$$\phi = \sqrt{}$$

$$\boxed{\begin{array}{l} \theta = 0 \\ \delta = 0 \\ \alpha = 0 \end{array}}$$

$$K_a = \frac{\cos^2 (\phi - \theta)}{\cos^2 \theta \, \cos(\delta - \theta) \left[1 + \sqrt{\frac{\sin(\delta + \phi) \, \sin(\phi - \alpha)}{\cos(\delta + \theta) \, \cos(\theta - \alpha)}} \right]^2}$$
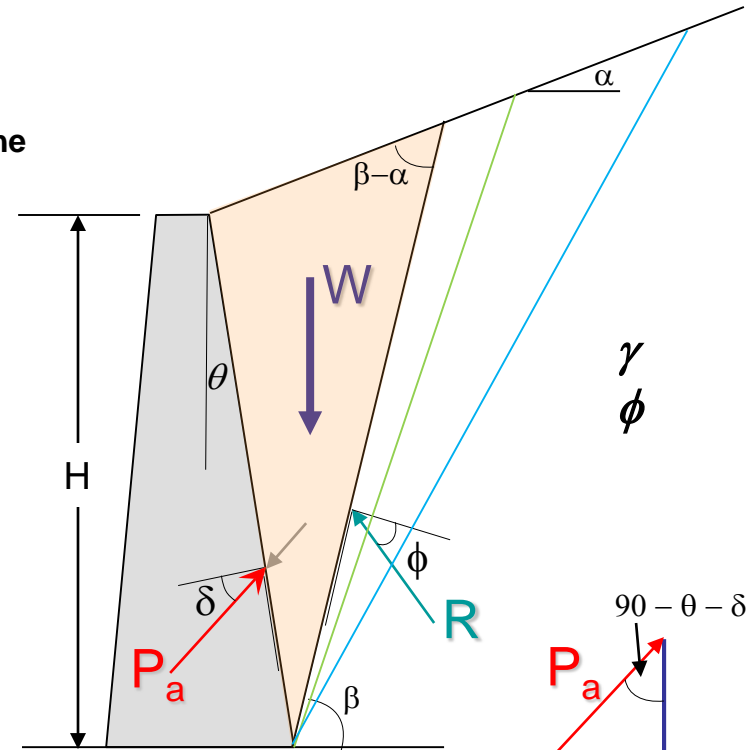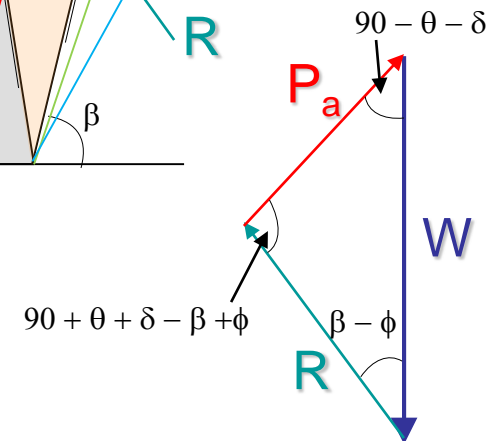
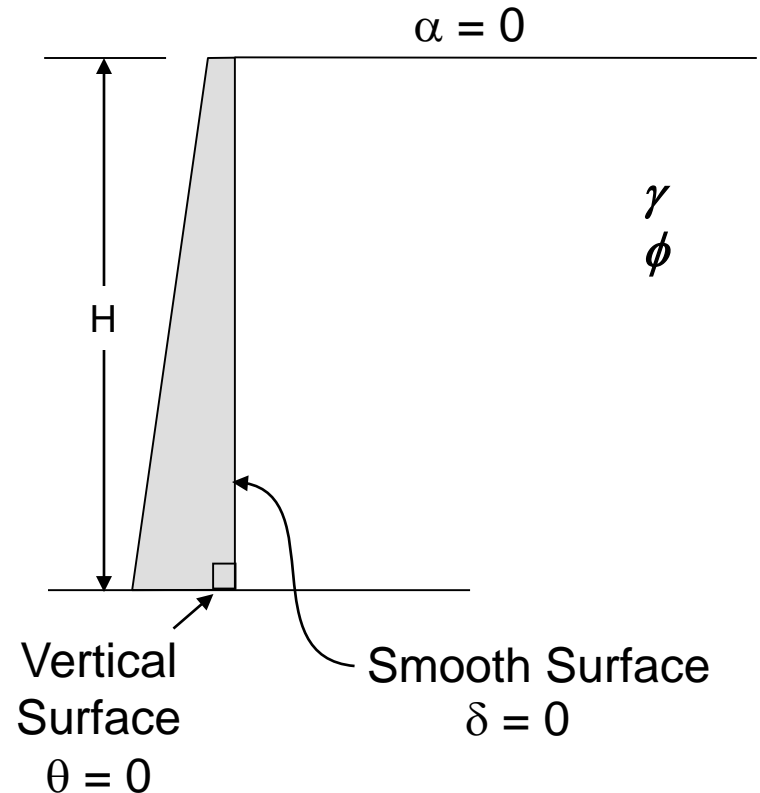Under the given wall and backfill conditions, K_a of Coulomb's active earth pressure becomes equivalent to K_a of Rankine's

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$P_a = \frac{1}{2} \, K_a \, \gamma \, H^2$$

$\alpha = 0$

γ
ϕ

H

Vertical
Surface
$\theta = 0$

Smooth Surface
$\delta = 0$

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Active Earth Pressure in ϕ – Soil

Example -1

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth

Find:
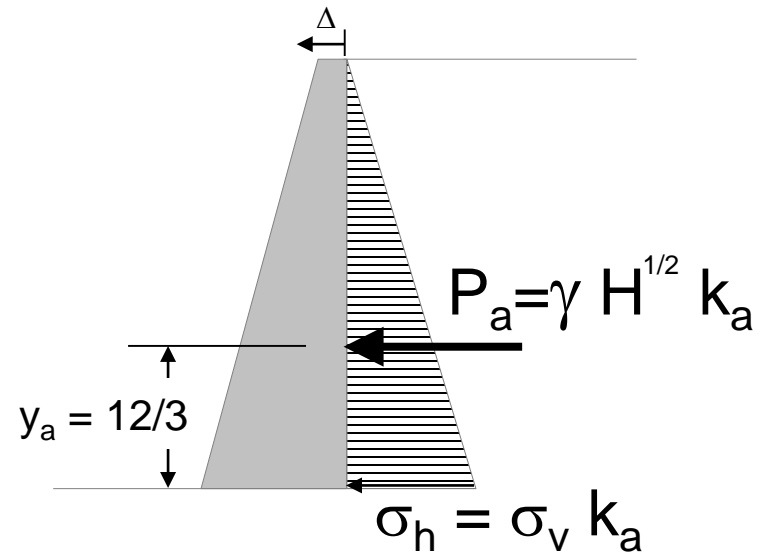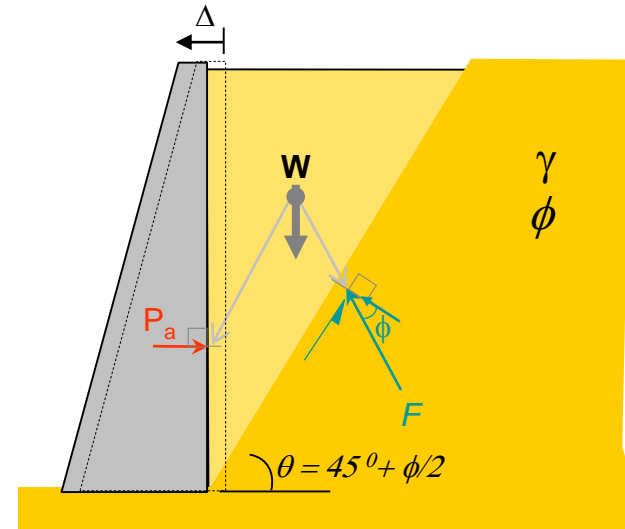- Lateral force P_a acting on the wall
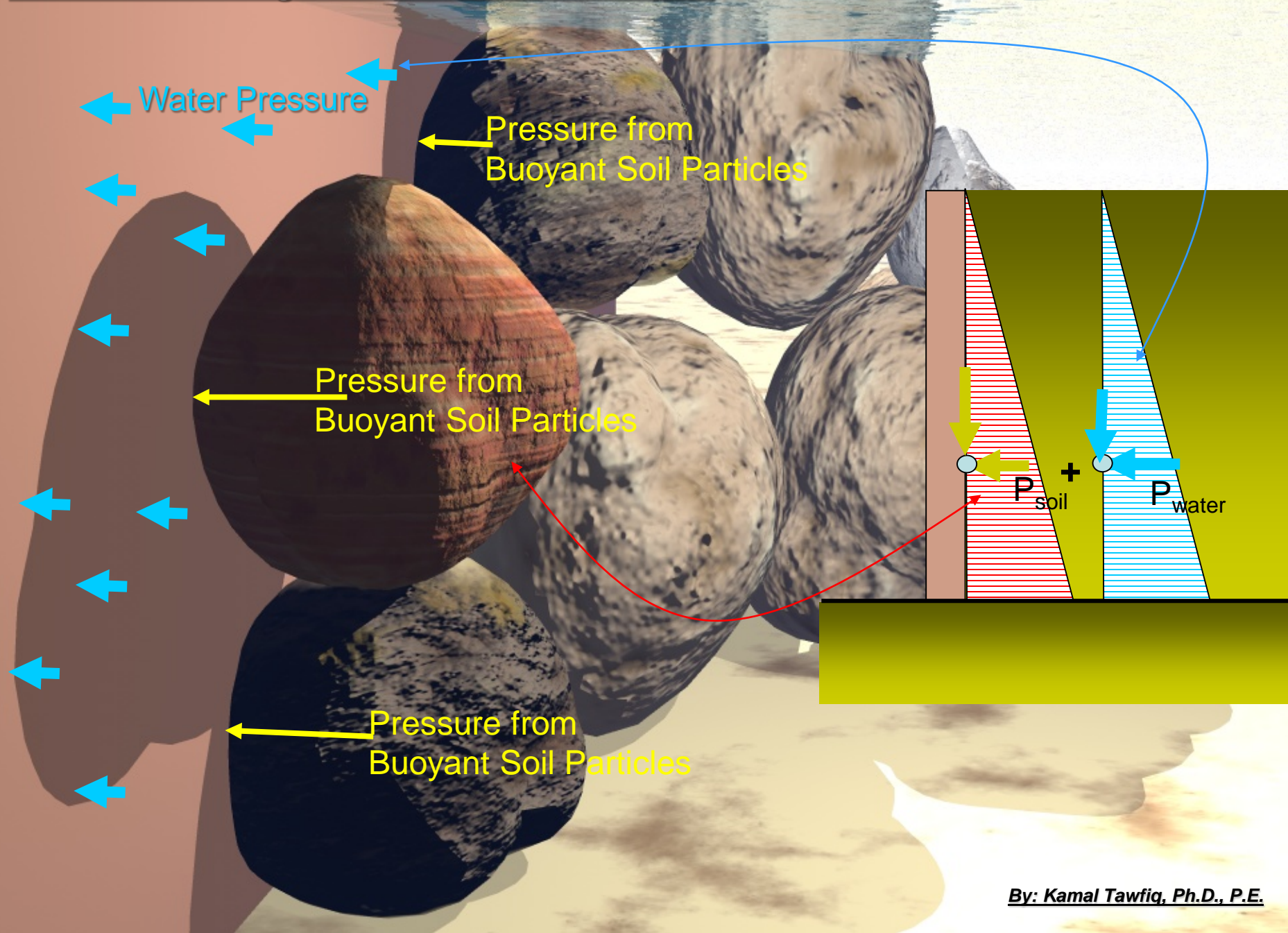
Solution:

$\sigma_h = \sigma_v\, k_a$

$P_a = \gamma\ H^{0.5}\, k_a$

$K_a = \dfrac{1 - \sin\phi}{1 + \sin\phi}$

$P_a = 115 \times 12^2 \times 0.5$
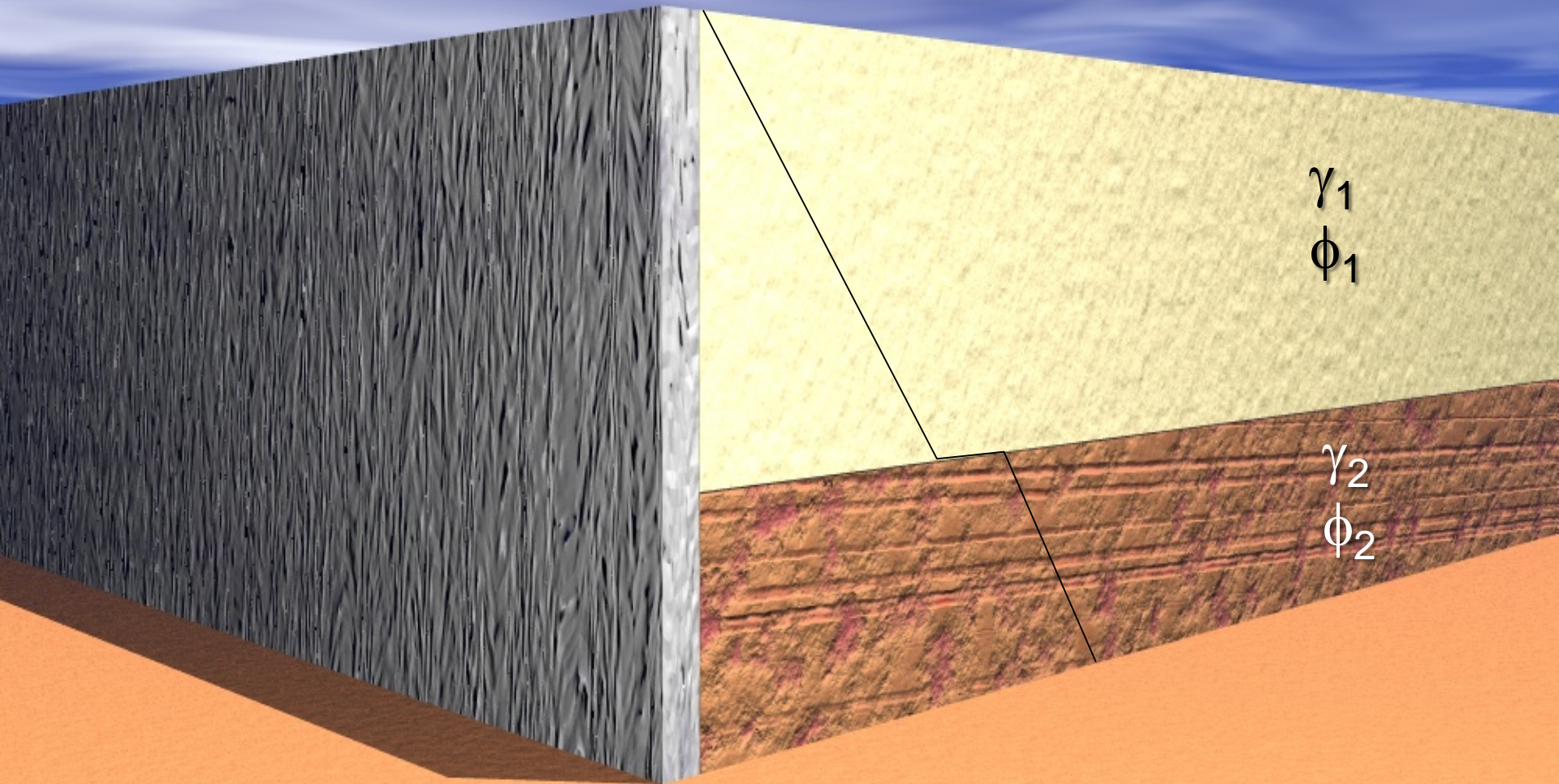

W γ ϕ

P_a

F

$\theta = 45^0 + \phi/2$

$P_a = \gamma\, H^{1/2}\, k_a$

$y_a = 12/3$

$\sigma_h = \sigma_v\, k_a$

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Effect of Submerged Soil of Lateral Pressure

Water Pressure

Pressure from
Buoyant Soil Particles

Pressure from
Buoyant Soil Particles

Pressure from
Buoyant Soil Particles

P_{soil} + P_{water}

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Effect of Submerged Soil of Lateral Pressure

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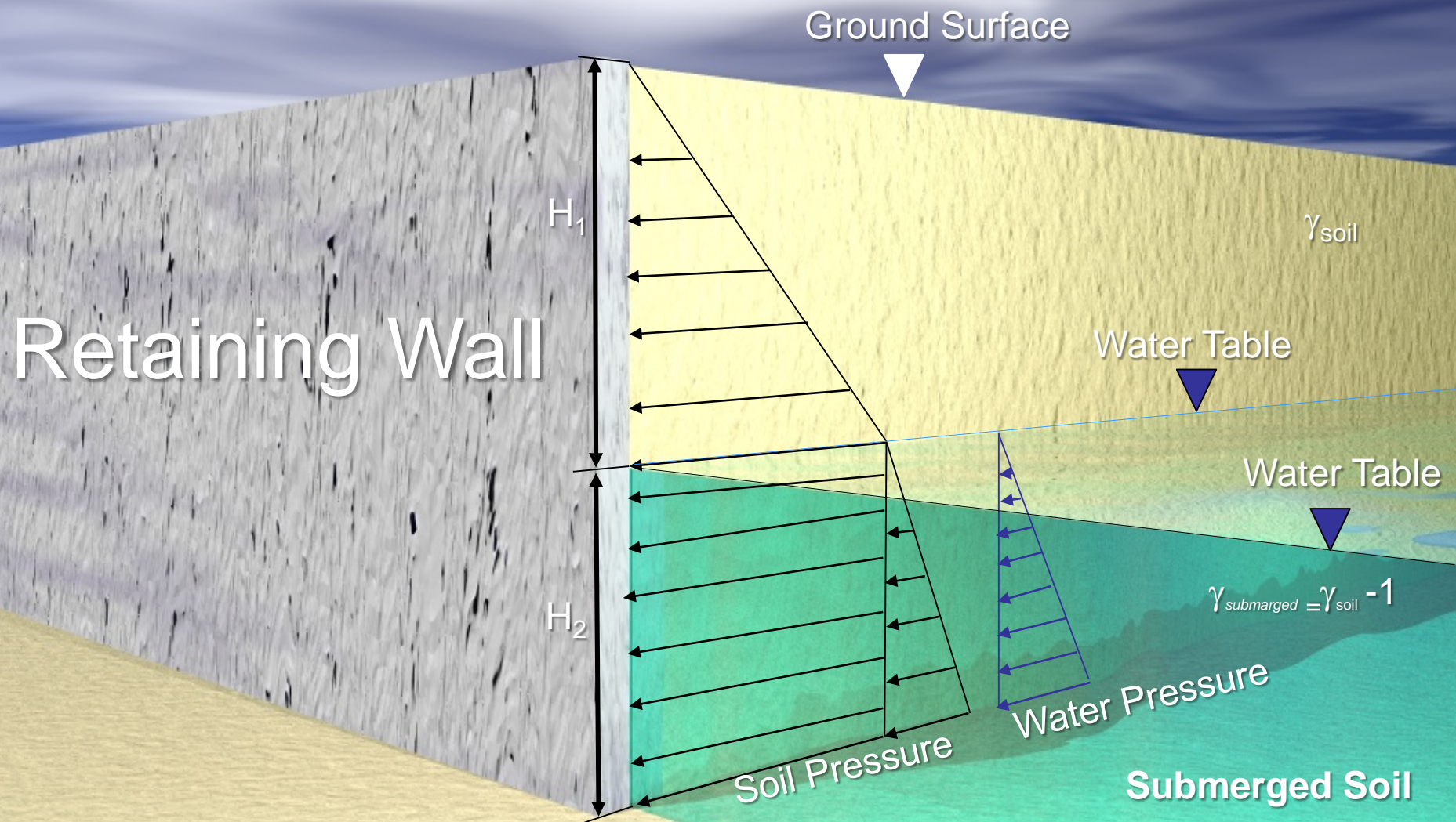
Effect of Soil Layers on Lateral Pressure

γ_1
ϕ_1

γ_2
ϕ_2

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Effect of Clay on Lateral Pressure

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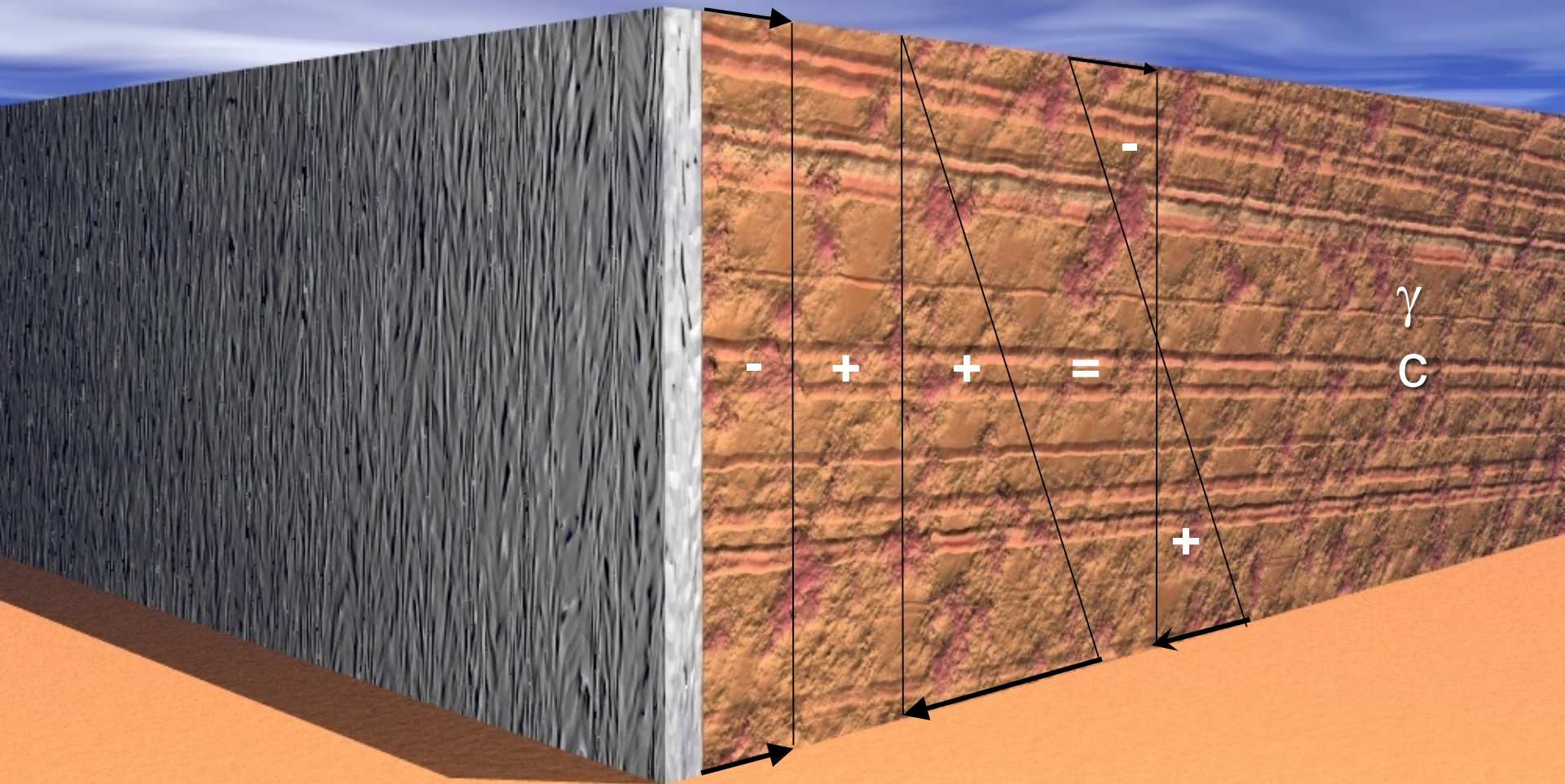
Effect of Surcharge Load on Earth Pressure