EXERCISES and EXAMPLES concerning the Kelvin-Plank and Clausius Statements of the Second Law of Thermodynamics.

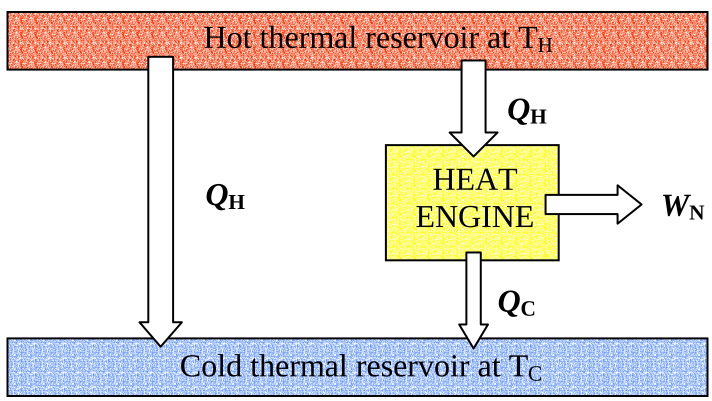
Example 1: heat transfer across a finite temperature difference.

Use the second law of thermodynamics to show that heat transfer across a finite temperature difference is irreversible.

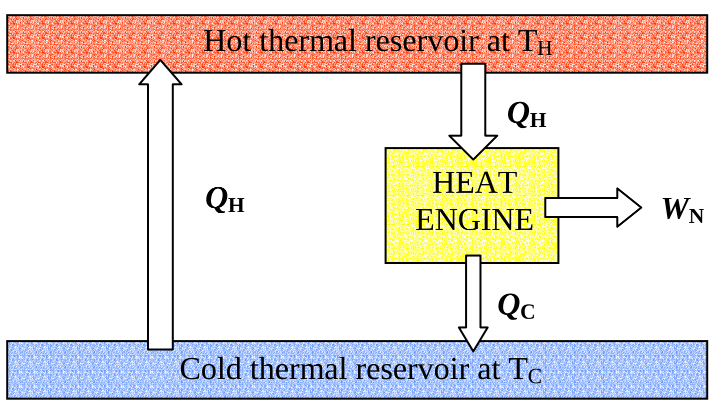
Procedure:

1) Assume that heat transfer across a finite temperature is reversible.
2) Show that this assumption leads to a violation of the second law of thermodynamics.

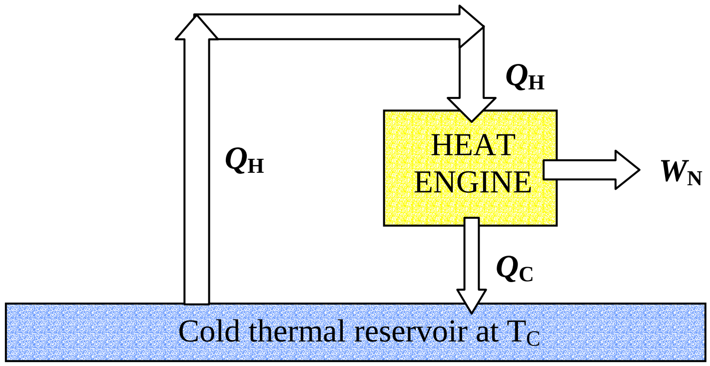
The following configuration is not a violation of the second law. Heat transfer flows Q_H directly from the hot thermal reservoir to the cold thermal reservoir.



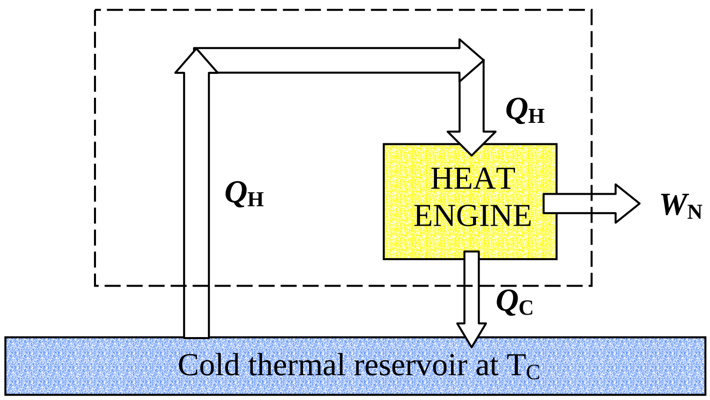
Assume that the heat transfer from hot to cold thermal reservoir, Q_H, is reversible and reverse the heat flow.



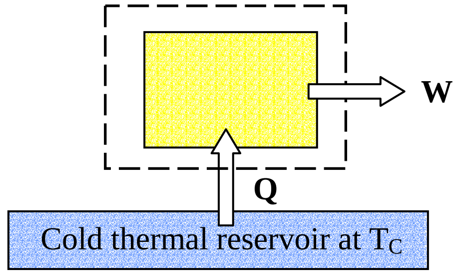
Let the reversed heat flow go directly to the heat engine, bypassing the hot thermal reservoir.



Take the reversed heat transfer and the heat engine as a single system.



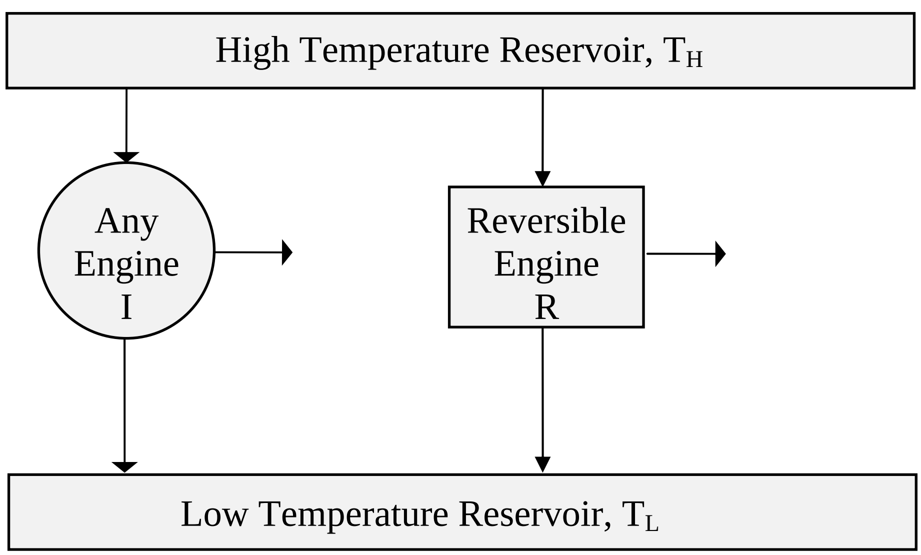
We now have a system that receives heat ($Q_H - Q_C$) from a single reservoir and does and equivalent amount of work on the surroundings.



This is a violation of the second law of thermodynamics. Thus, heat transfer across a finite temperature difference must be irreversible.

Example 2:

It is impossible to construct an engine, operating between two given thermal reservoirs, that is more efficient than a reversible engine operating between the same two reservoirs.



Proof: (See notes under Week 14 for the proof)

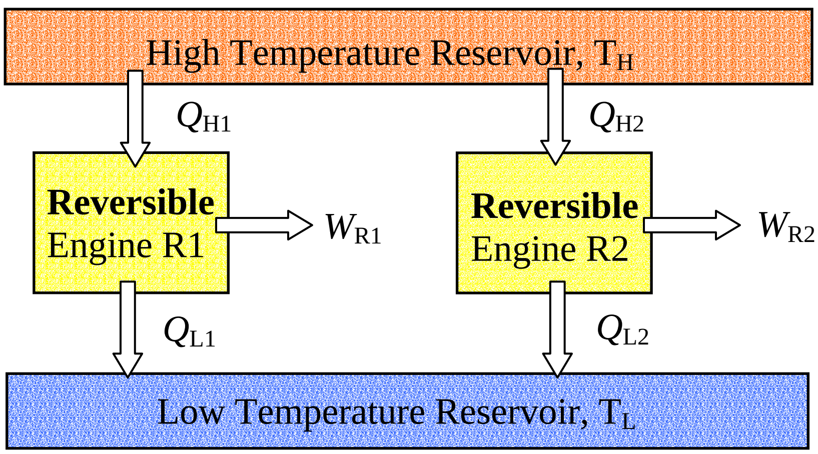
Exercise/ example 3:

Are all reversible engines operating between the same temperature limits have the same efficiency?

Answer: YES!

Proof: (sketched below, it is very similar to the proof we made during Dec 3rd lecture).

1) Take two reversible engines R1 and R2



2) Assume $\eta_{R1} > \eta_{R2}$

3) Reverse reversible engine R2 and let $Q_{H2} = Q_{H1} = Q_H$

4) This will lead to the conclusion that $\eta_{R1} \leq \eta_{R2}$

5) Next assume $\eta_{R2} > \eta_{R1}$

6) Reverse reversible engine R1 and let $Q_{H1} = Q_{H2} = Q_H$

7) This will lead to the conclusion that $\eta_{R2} \leq \eta_{R1}$

8) There is a contradiction unless $\eta_{R1} = \eta_{R2}$

NOTE: In the above no restrictions were placed upon the type of engine or system to be used or upon the details of the operation, hence the results are completely general.

Exercises:

1. Prove that a violation of the Clausius statement implies a violation of the Kelvin-Planck statement (class Dec. 3rd, 2003).
2. Prove that a violation of the Kelvin-Planck statement implies a violation of the Clausius Statement of the second law of thermodynamics.

Proof of (1):

1. Assume that a system that violates the Clausius statement exists:

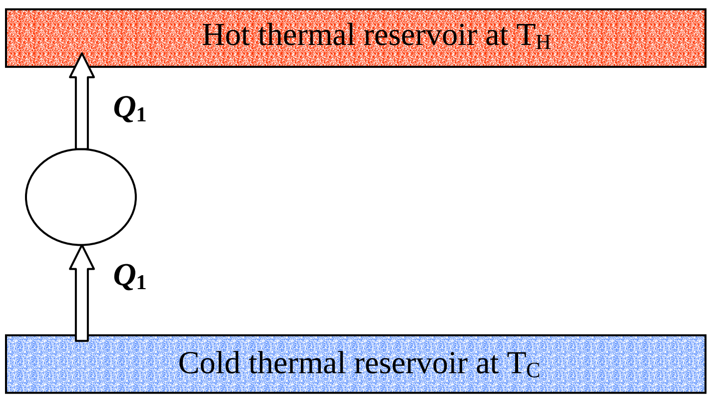
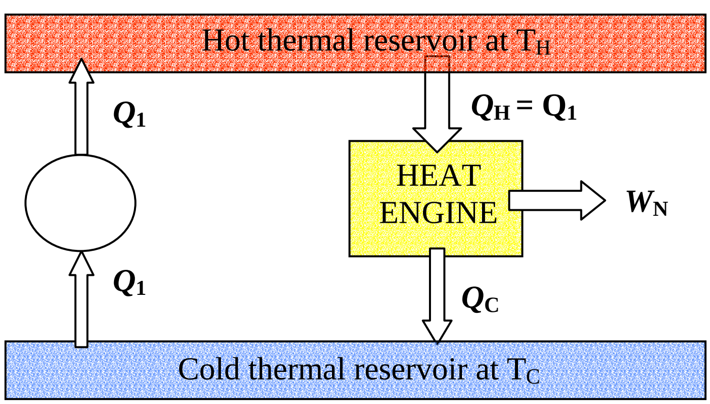
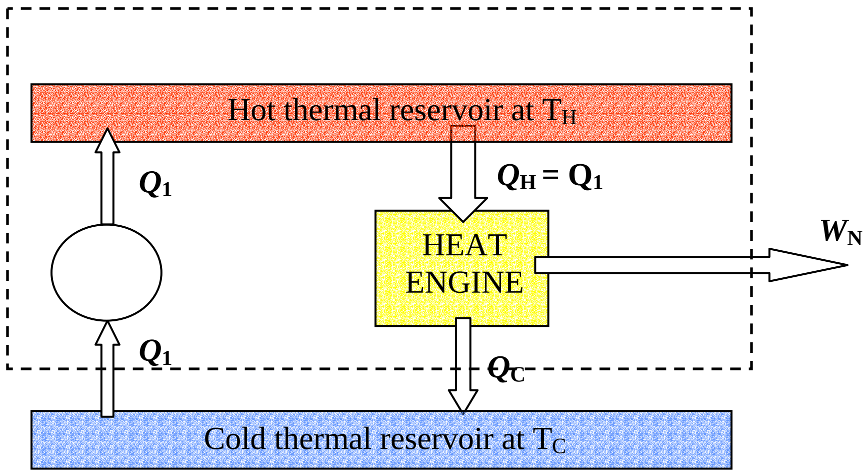


Figure A

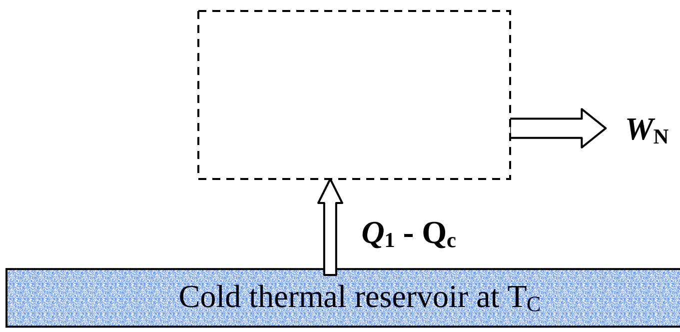
2. Consider a reversible heat engine using Q_1 as heat input operating between T_H and T_C:



2) Now consider the system shown with dashed lines:



It is equivalent to :



This system violates the Kelvin-Planck Statement. We have seen that IF we ASSUME that a system that violates the Clausius statement exists (figure A) then we can prove that a system that violates the Kelvin-Planck statement would also exist.

More exercises (similar to the one solved during Dec. 3rd lecture):

T-F Lecture 13 Some additional examples.

(Problems form: Thermodynamics, Van Wylen, Sontag, And Borgnakke, Wiley, 4th Ed.)

6.11 A sales person selling refrigerators and deep freezers will guarantee a minimum coefficient of performance of 4.5 year round. How would you evaluate that? Are they all the same? Discuss

6.13 A cyclic machine, shown in Fig. P6.13, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible? Impossible

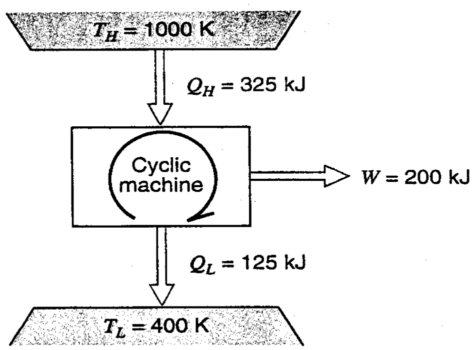


FIGURE P6.13

6.14 A household freezer operates in a room at 20°C. Heat must be transferred from the cold space at a rate of 2 kW to maintain its temperature at –30°C. What is the theoretically smallest (power) motor required to operate this freezer? 0.41 kW

6.16 An inventor has developed a refrigeration unit that maintains the cold space at –10°C, while operating in a 25°C room. A coefficient of performance of 8.5 is claimed. How do you evaluate this? Impossible

6.19 A house is heated by a heat pump driven by an electric motor using the outside as the low-temperature reservoir. The house loses energy directly proportional to the temperature difference as $Q_{loss} = K(T_H - T_L)$. Determine the minimum electric power to drive the heat pump as a function of the two temperatures. $\frac{K(T_H - T_L)^2}{T_H}$

6.25 We wish to produce refrigeration at –30°C. A reservoir, shown in Fig. P6.25, is available at 200°C and the ambient temperature is 30°C. Thus, work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir to the heat transferred from the –30°C reservoir, assuming all processes are reversible. 0.687

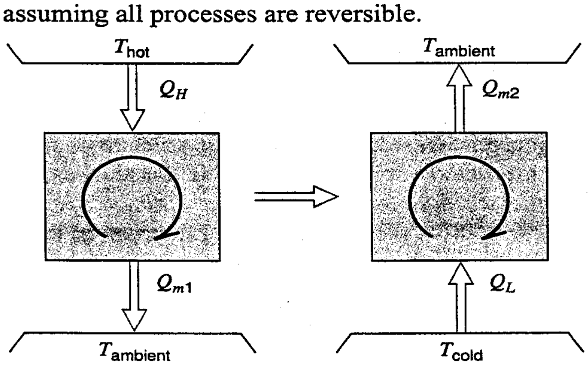


FIGURE P6.25