Formulae Collection

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Contents

1 Introduction

These formulae are not guaranteed to be complete and/or correct, though we tried to make them so. The *student* has the final reponsibility for knowing all formulae covered in both class lectures and homework correctly. So, if you see an error, or miss a formulae that you think was covered, (if you are mistaken about that, it could get you in trouble!), let us know immediately.

This document can *not* be taken to the exams.

2 Units

Basic SI units are kilogram (kg), meter (m), and second (s). Many derived units exist (table A.1). The book uses kPa, kJ, kW instead of the "normal" units Pa, J, W. Conversions may be needed (table A.1)

Definitions (table A.1): 1 bar = 10^5 Pa. 1 atm = 101,325 Pa.

Absolute temperature (required in most formulae and some tables): add 273.15 to convert Centrigrade into Kelvin, substract 273.15 to convert Kelvin into centrigrade. However, temperature *differences* are the same in Kelvin as in Centrigrade; the two temperatures in the difference do not need to be converted.

3   Properties

The intensive specific properties are lowercase, the corresponding extensive properties are uppercase:

$$v = \frac{V}{m} \quad u = \frac{U}{m} \quad h = \frac{H}{m} \quad s = \frac{S}{m} \qquad V = mv \quad U = mu \quad H = mh \quad S = ms$$

Definitions of enthalpy and entropy:

$$h \equiv u + pv \qquad s \equiv \int_{\mathrm{ref}} \left(\frac{\delta q}{T} \right)_{\mathrm{rev}}$$

Specific volume and density:

$$v = \frac{1}{\rho}$$

Pressure and force:

$$F = pA$$

Static pressure differences versus height H (height also indicated by Z) differences:

$$p_A - p_B = -\rho g \left(H_A - H_B \right)$$

Number of kilomoles:

$$\bar{n} = \frac{m}{M}$$

where M is the molecular mass (table A.5.)

4   Formulae for substances

How you treat a substance depends on the type of substance.

4.1   Substances with B-tables

Tables of specific properties such as tables B.1.1-B.1.4 for water are available. Use requires normally drawing pv- and Tv- or Ts-diagrams, which students must master, and which is covered elsewhere[1].

In the two phase region:

$$v = v_f + x \left(v_g - v_f \right) \quad u = u_f + x \left(u_g - u_f \right) \quad h = h_f + x \left(h_g - h_f \right) \quad s = s_f + x \left(s_g - s_f \right)$$

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[1] http://www.eng.fsu.edu/~dommelen/courses/eml3100/aids/tables

where f stands for the saturated liquid ($x = 0$) value at the same pressure and same temperature, and g for the saturated vapor ($x = 1$) value. These can be found in the Appendix B tables. The quality x is the ratio of the vapor mass to the total mass of vapor and liquid. The quality x is undefined if the substance is not two-phase or saturated.

For compressed liquids, often there are no suitable tables available. Fortunately, good approximate values for v, u, s, and h can usually be taken from the saturated tables at the correct given temperature. The fact that the given pressure is not the same as the saturated pressure must then be ignored. (For h, it is more accurate to write $h = u + pv$ and then take u and v from the saturated tables, instead of h itself.)

4.2 Ideal Gases

Specific gas constant R in terms of the universal gas constant $\bar{R}$ (table A.1):

$$R = \frac{\bar{R}}{M}$$

where M is the molecular mass (table A.5, but this table already includes R).

Forms of the ideal gas law:

$$pv = RT \qquad p\bar{v} = \bar{R}T$$

$$pV = mRT \qquad pV = \bar{n}\bar{R}T$$

To correct for real gas effects, replace R (or $\bar{R}$) by ZR ($Z\bar{R}$), where Z is the compressibility factor (figure D1, with $P_r = p/p_c$, $T_r = T/T_c$.)

For an ideal gas, u, h, C_p, C_v, and k only depend on temperature. Also,

$$u_2 - u_1 = \int_1^2 C_v \, dT \qquad h_2 - h_1 = \int_1^2 C_p \, dT$$

(Using table A.6, these can actually be integrated.) Note that for ideal gasses we often do not have u, h, or s themselves, and we have to make do with differences. (For a mixing chamber you will need to replace $\dot{m}_3$ with $\dot{m}_1 + \dot{m}_2$ before you can use differences.)

Specific heat relation:

$$C_p - C_v = R$$

Hence if we know C_p (tables A.5 or A.6) we can compute C_v and vice-versa. Also, if C_p is constant, then so is C_v and vice-versa,

Definition of specific heat ratio:

$$k = \frac{C_p}{C_v}$$

If k is constant, then so are C_p and C_v, and vice-versa.

Note that *isothermal* ideal gasses are also polytropic with $n = 1$.

4.2.1 Ideal gasses in A.7.1 or A8

The internal energy u and enthalpy h can be read off in the table as a function of temperature. Or vice-versa, for that matter.

The formula that gives you the entropy s is, (in terms of differences),

$$s_2 - s_1 = s_{T2}^0 - s_{T1}^0 - R \ln \left(\frac{p_2}{p_1} \right)$$

where s_{T1}^0, also written as $s_T^0(T_1)$, is whatever you find in the table's s_T^0 column at the temperature T_1. (It is what the entropy would be if the pressure would have been one atmosphere.)

4.2.2 Ideal gasses with constant specific heats

If no A.7.1/A.8 table is available, you will have to assume that the specific heats C_p and C_v, as well as their ration k are constant. Specific heats that can be assumed to be approximately constant can be computed from the average process temperature using table A.6. Less accurately, near room temperature they can be taken from A.5. Use A.6 wherever possible unless you are very close to room temperature.

Internal energy:

$$u_2 - u_1 \approx C_{v_{\text{ave}}} (T_2 - T_1)$$

Enthalpy:

$$h_2 - h_1 \approx C_{p_{\text{ave}}} (T_2 - T_1)$$

Entropy:

$$
\begin{aligned}
s_2 - s_1 \ &= \ C_{p_{\text{ave}}} \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) \\[2mm]
&\approx \ C_{v_{\text{ave}}} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{v_2}{v_1} \right) \\[2mm]
&\approx \ C_{v_{\text{ave}}} \ln \left(\frac{p_2}{p_1} \right) + C_{p_{\text{ave}}} \ln \left(\frac{v_2}{v_1} \right)
\end{aligned}
$$

If the process is isothermal or reversible adiabatic (isentropic), the following "polytropic" relations also apply (they are equivalent to the equations above, but often much more convenient):

$$
\begin{aligned}
\frac{p_2}{p_1} \ &= \ \left(\frac{v_1}{v_2} \right)^n \ = \ \left(\frac{T_2}{T_1} \right)^{\frac{n}{n-1}} \\[2mm]
\frac{T_2}{T_1} \ &= \ \left(\frac{v_1}{v_2} \right)^{n-1} \ = \ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \qquad \left(\begin{array}{rl} \text{isothermal:} & n = 1 \\ \text{isentropic and } k \text{ constant:} & n = k \end{array} \right) \\[2mm]
\frac{v_2}{v_1} \ &= \ \left(\frac{p_1}{p_2} \right)^{\frac{1}{n}} \ = \ \left(\frac{T_1}{T_2} \right)^{\frac{1}{n-1}}
\end{aligned}
$$

4.3 Solids and Liquids without B-tables

Approximate formulae if no better tabulated values are available, or to simplify things.

Heat added:

$$_1Q_2 = m \int_1^2 C_{(p)} \, dT \approx m C_{(p)_{\text{ave}}} (T_2 - T_1)$$

Enthalpy:
$$h_2 - h_1 \approx C_{(p)_\text{ave}} (T_2 - T_1) \quad [+v\,(p_2 - p_1)]$$

The term in the square brackets can often be neglected.

Entropy:
$$s_2 - s_1 \approx C_{(p)_\text{ave}} \ln\left(\frac{T_2}{T_1}\right)$$

5 Processes

5.1 Control Mass Processes

Specific heat and work:
$$_1q_2 = \frac{_1Q_2}{m} \quad _1w_2 = \frac{_1W_2}{m} \quad _1Q_2 = m\,_1q_2 \quad _1W_2 = m\,_1w_2$$

Continuity (mass conservation):
$$m_1\,(+m_\text{added}) = m_2$$

The first law of thermo (energy conservation):
$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 \qquad (E = U + m\tfrac{1}{2}\mathbf{V}^2? + mgZ?)$$

Work:
$$_1W_2 = \int_1^2 p\,\mathrm{d}V = \begin{cases} \text{incompressible/constant volume (isochoric):} & 0 \\[4pt] \text{constant pressure (isobaric):} & p(V_2 - V_1) \\[4pt] \text{pressure linear in V:} & \dfrac{p_1 + p_2}{2}(V_2 - V_1) \\[6pt] \text{polytropic for } n \neq 1\ (pV^n = \text{constant}): & \dfrac{p_2 V_2 - p_1 V_1}{1 - n} \\[6pt] \text{polytropic for } n = 1\ (pV = \text{constant}): & pV \ln\left(\dfrac{V_2}{V_1}\right) \end{cases}$$

In case of an ideal gas, note that pV can be replaced by mRT.

Heat added:
$$_1Q_2 = \begin{cases} \text{Adiabatic:} & 0 \\[4pt] \text{reversible:} & \displaystyle\int_1^2 T\,\mathrm{d}S \\[4pt] \text{reversible isothermal:} & T(S_2 - S_1) \end{cases}$$

The second law for reversible adiabatic (isentropic) processes:
$$S_2 = S_1$$

The second law of thermo for general processes:

$$_1S_{2,\text{gen}} = \Delta S_\text{net} = S_2 - S_1 - \frac{_1Q_2}{T_\text{surr}} \geq 0$$

5.2 Rate Equations

The first law as a rate equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{Q} - \dot{W} \qquad \left(\frac{\mathrm{d}E}{\mathrm{d}t} = m\frac{\mathrm{d}u}{\mathrm{d}t} + m\frac{\mathrm{d}\frac{1}{2}\mathbf{V}^2}{\mathrm{d}t}? + mg\frac{\mathrm{d}Z}{\mathrm{d}t}? \right)$$

Work as a rate equation:

$$\dot{W} = p\dot{V}$$

For an ideal gas:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = C_v\frac{\mathrm{d}T}{\mathrm{d}t}$$

For liquids and solids:

$$\dot{Q} = mC_{(p)}\frac{\mathrm{d}T}{\mathrm{d}t}$$

5.3 Steady State Control Volume Processes

The control volume is assumed to be steady state in all formulae below.

Specific work output and heat added (i.e., per unit mass flowing through):

$$w = \frac{\dot{W}}{\dot{m}} \quad q = \frac{\dot{Q}}{\dot{m}} \qquad \dot{W} = \dot{m}w \quad \dot{Q} = \dot{m}q$$

In- and outflow velocities and pipe cross-sectional areas:

$$\dot{m} = \dot{V}/v = A\mathbf{V}/v \qquad A = \frac{\pi}{4}D^2$$

Continuity (mass conservation):

$$\sum \dot{m}_i = \sum \dot{m}_e$$

where $\sum$ means sum over all inflow/outflow points, if there is more than one.

The first law of thermo (energy conservation):

$$\dot{Q} + \sum \dot{m}_i \left(h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i \right) = \sum \dot{m}_e \left(h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e \right) + \dot{W}$$

The kinetic energy and potential energy terms are often ignored. Devices without moving parts do not do work, $\dot{W} = 0$. Adiabatic devices have no heat transfer, $\dot{Q} = 0$.

The second law of thermo for a reversible adiabatic (isentropic) process inside a single entrance and exit CV:

$$s_e = s_i$$

The second law of thermo for a reversible isothermal process inside a single entrance and exit CV:

$$q = T \left(s_e - s_i \right)$$

The second law for a general control volume:

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}} \qquad \dot{S}_{\text{gen}} \geq 0$$

Specific work done during a reversible process inside a single entrance and exit CV:

$$w + \tfrac{1}{2}\mathbf{V}_e^2? - \tfrac{1}{2}\mathbf{V}_i^2? + gZ_e? - gZ_i? = -\int_i^e v \, \mathrm{d}p$$

where

$$-\int_i^e v \, \mathrm{d}p = \begin{cases} \text{constant pressure (isobaric):} & 0 \\[2mm] \text{incompressible (constant volume/isochoric):} & -v\left(p_e - p_i\right) \\[2mm] \text{polytropic for } n \neq 1 \ (pv^n = \text{constant}): & \dfrac{n\left(p_e v_e - p_i v_i\right)}{1 - n} \\[2mm] \text{polytropic for } n = 1 \ (pv = \text{constant}): & -pv \ln\left(\dfrac{p_e}{p_i}\right) \end{cases}$$

In case of an ideal gas, note that pv can be replaced by RT.

Adiabatic turbine and compressor/pump efficiencies:

$$\eta_{\text{turbine}} = \frac{w}{w_s} \qquad \eta_{\text{compressor}} = \frac{w_s}{w}$$

where s indicates an ideal isentropic device with the same entrance conditions and exit pressure.

6   Cycle Devices

For cycle devices, the normal sign conventions (heat in and work out are positive) are ignored. Instead, the signs of heat and work are defined such that $\dot{Q}_H$, $\dot{Q}_L$ and net work $\dot{W}$ are normally positive. The pictures below show the directions of positive heat fluxes and work rates for a heat engine as compared to a heat pump (or refrigeration cycle).
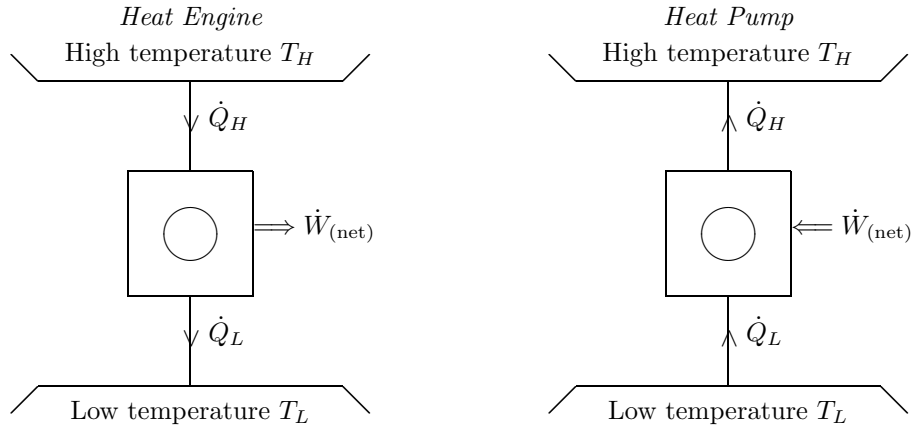
The first law for a cycle:

$$\text{Net work } {}^{\text{out}}_{\text{in}} = \text{Net heat } {}^{\text{in}}_{\text{out}}: \qquad W_{(\text{net})} = Q_H - Q_L$$

The Kelvin-Planck Statement: Any working heat engine must dump some waste heat to a lower temperature than the input heat is provided at.

The Clausius Statement: Any working heat pump must have positive work going in.

Thermal efficiency:

$$\eta_{\text{thermal}} \equiv \frac{\dot{W}_{\text{net}}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} \quad \leq \quad \eta_{\text{thermal, Carnot}} = \frac{T_H - T_L}{T_H}$$

Refrigeration coefficient of performance:

$$\beta \equiv \frac{\dot{Q}_L}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} \quad \leq \quad \beta_{\text{Carnot}} = \frac{T_L}{T_H - T_L}$$

Heating heat pump coefficient of performance $\beta' = 1 + \beta$:

$$\beta' \equiv \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} \quad \leq \quad \beta'_{\text{Carnot}} = \frac{T_H}{T_H - T_L}$$

Ideal gas Carnot cycle:

$$q_H = {}_1q_2 = RT_H \ln\left(\frac{v_2}{v_1}\right) \qquad q_L = -{}_3q_4 = RT_L \ln\left(\frac{v_3}{v_4}\right) \qquad \frac{v_2}{v_1} = \frac{v_3}{v_4}$$