Special case : no moving parts

Move to previous lecture

$$\int_i^e v\,dp + \frac{1}{2} Vel_2^2 + g Z_2 - \frac{1}{2} Vel_1^2 - g Z_1 = 0$$

Special special case : incompressible $v = \text{constant}$

$$p_2 v + \frac{1}{2} Vel_2^2 + g Z_2 = p_1 v + \frac{1}{2} Vel_1^2 + g Z_1$$

Bernoulli law.

Also have $\dot{m} = \cancel{\rho} \frac{A\,Vel}{v} \rightarrow \cancel{\rho} \frac{A_2 Vel_2}{v_2} = \frac{A_1 Vel_1}{v_1}$

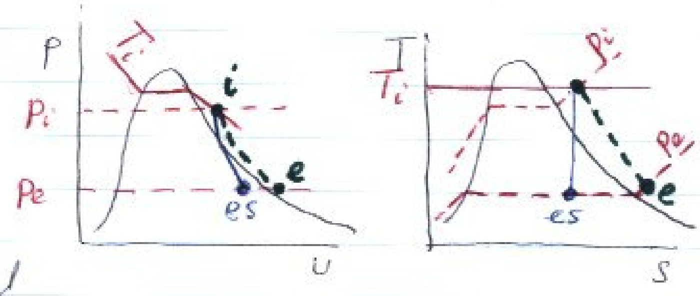
Pipe flows

Flow fields

S9.5 Efficiencies

Turbine:

--- actual — ideal



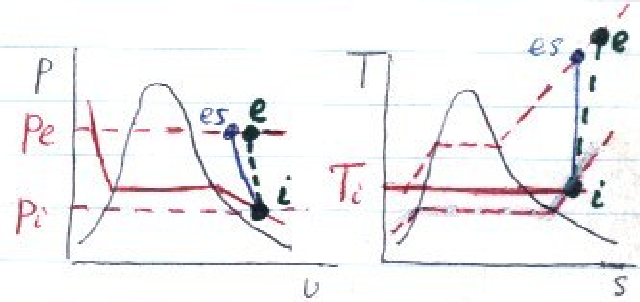
Ideal turbine, indicated with "s"*, has the same entrance conditions and exit pressure, but is isentropic.

$$\boxed{\eta_{turbine} \equiv \frac{w}{w_s}}$$

(*: Note that "s" is the 10th letter ~~of it~~ after the "i" of ideal, and next after the "r" of reversible)

Note that we will assume that $\Delta KE \approx 0$, $\Delta PE \approx 0$
So if adiabatic, $w = h_e - h_s$

Compressor

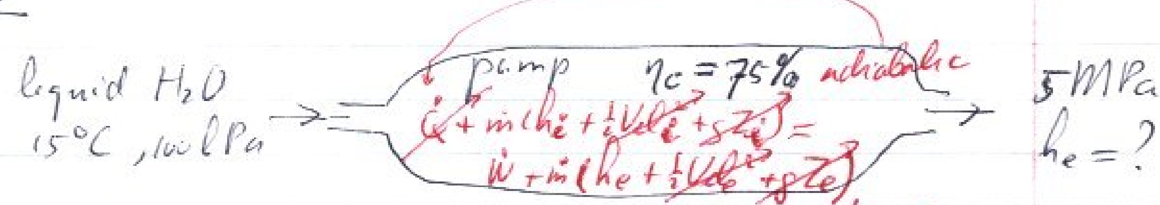


Ideal compressor has the same entrance conditions and exit pressure, but is isentropic

$$\boxed{\eta_{comp} \equiv \frac{w_s}{w}}$$

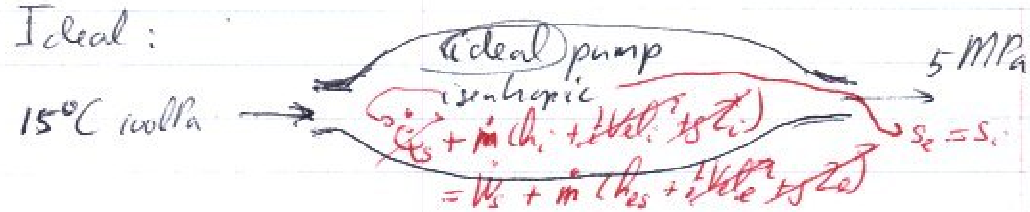
SKIP? ↓

P9.97



Asked h_e: (ignore KE, PE) $\quad \eta = \frac{w_s}{w} \quad \dot{W} = \dot{m} w$

Ideal:



Answer:

A-4 since no 100 kPa in B.1.4

Table B.1.1 @ 15°C: $h_i = u_i + p_i v_i = 62.98 \frac{kJ}{kg} + 100\,kPa\ 0.001001 \frac{m^3}{kg}$

$= \overset{62.982}{63.08}\ kJ/kg$

A-6 $\quad s_i = 0.2245\ kJ/kg\,K$

Table B.1.4 @ 5 MPa and $s_e = s_i = 0.2245\ kJ/kg\,K$ interpolated:

$$h_{e,s} = 5.02 + \frac{0.2245 - 0.0001}{0.2958 - 0.0001}(88.64 - 5.02) = 68.54\ \frac{kJ}{kg}$$

$$w_s = h_i - h_{e,s} = -5.462\ kJ/kg$$

Efficiency: $\eta = \frac{w_s}{w} \quad 0.75 = \frac{-5.462}{w} \rightarrow w = -7.282\ \frac{kJ}{kg}$

1st law: $\cancel{q}^{\,0} + h_i = w + h_e \quad 63.08\ kJ/kg = -7.282\ \frac{kJ}{kg} + h_e$

$\rightarrow h_e = 70.36\ kJ/kg$

4/18/06 end →

9/19 '6

Note: Could have approximated as incompressible

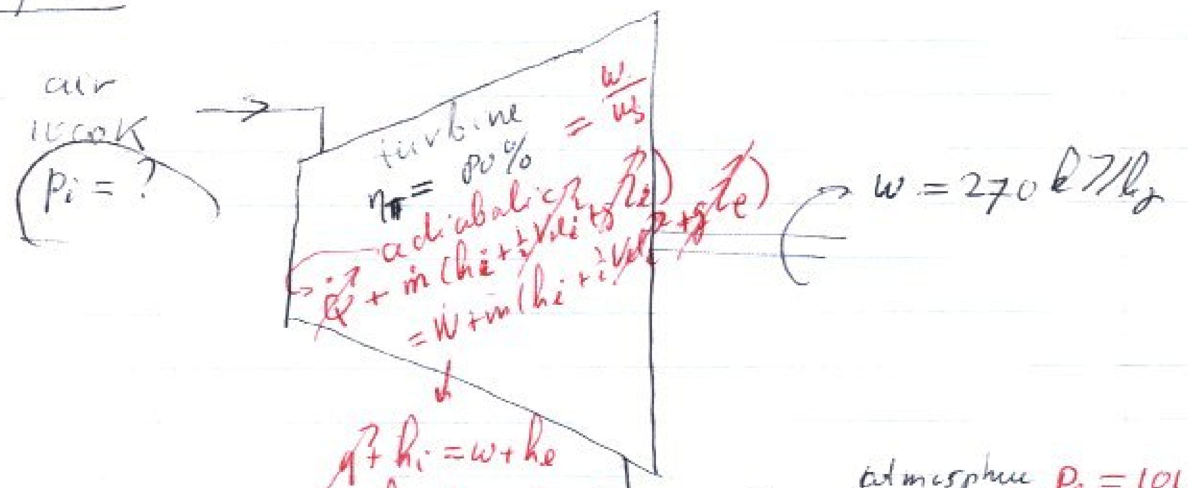
$$w_s = -\int_i^e v\,dp \approx -v(p_e - p_i) = -4.905 \ \mathrm{kJ/kg}$$

~~The error in this approximation is probably less than the interpolation error in B.1.4.~~

With this value, $h_e = 69.6$

yes, it is far less

Pg. 101



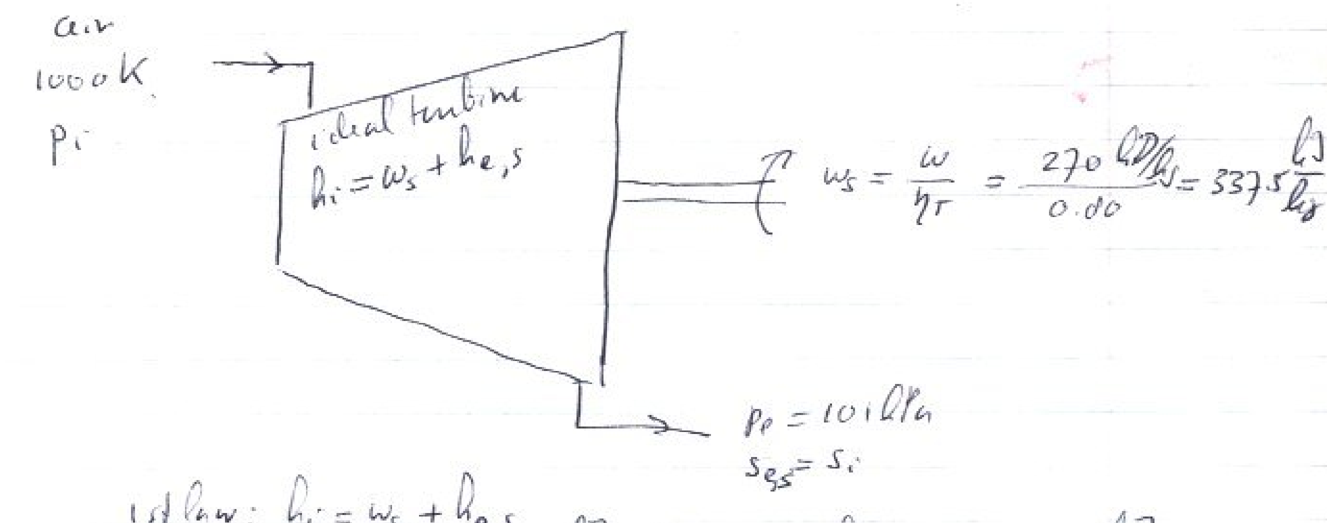
$h_i = w + h_e$

since h_i depends only on temperature, I can find it in A7.1 even though I only know T_i : $h_i = 1046.22$

So 1st law will give h_e, and that will give T_e

Answer:

ideal turbine first



1st law: $h_i = w_s + h_{e,s}$

$1046.22 \frac{\text{kJ}}{\text{kg}} = 337.5 \frac{\text{kJ}}{\text{kg}} + h_{e,s} \longrightarrow h_{e,s} = 708.72 \frac{\text{kJ}}{\text{kg}}$

from A17 / from A7 @ 1000K

2nd law: $s_{e,s} - s_i = 0 = s^o_{T_{e,s}} - s^o_{T_i} - R \ln \frac{p_e}{p_i}$

$s^o_{T_{e,s}}$ from A7.1 @ $h = 708.72 \frac{\text{kJ}}{\text{kg}}$, interpolated:

702.52 kJ/kg, 2.57277, 2.55731, 2.5662

$s^o_{T,e,s} = 7.70903 + \frac{708.72 - 692.12}{713.56 - 692.12}(7.74010 - 7.70903) = 7.733 \frac{\text{kJ}}{\text{kgK}}$

2.55731

713.27 − 702.52

$s^0_{T_i}$ from $\cancel{A\,7.1}$ A 17 @ 1000 K: $s^0_{T_i} = \cancel{8.13493}\ 2.96770 \frac{kJ}{kg\,K}$

Then 2nd law becomes

$$0 = \cancel{7.733}\ 2.5662 \frac{kJ}{kg\,K} - \cancel{8.13493}\ 2.9677 \frac{kJ}{kg\,K} - \underset{0.287 \frac{kJ}{kg\,K}}{R} \ln \frac{101\,kPa}{p_i}$$

$$\ln \frac{101\,kPa}{p_i} = \frac{\cancel{7.733}\ 2.5662 \frac{kJ}{kg\,K} - \cancel{8.13493}\ 2.9677 \frac{kJ}{kg\,K}}{0.287 \frac{kJ}{kg\,K}} = \cancel{-1.39895}\ -1.40045$$

$$\frac{101\,kPa}{p_i} = e^{-1.40045} = .246 \qquad p_i = \underline{\underline{409.8\,kPa}}$$

actual turbine to find T_e

1st law: $h_i = w + h_e$

$$1046.\cancel{22}\,04 \frac{kJ}{kg} = 270 \frac{kJ}{kg} + h_e \rightarrow h_e = 776.22 \frac{kJ}{kg}$$

Table $\cancel{A\text{-}7.1}$ A-17 @ $h_e = 776.22 \frac{kJ}{kg}$ interpolated:

$$T_e = \cancel{740}\ 750 + \frac{\cancel{776.22 - 756.73}\ \ 776.22 - 767.29}{\cancel{778.46 - 756.73}\ \ 778.18 - 767.29}(\cancel{760}\ 760 - \cancel{740}\ 750) = \underline{\underline{758\,K}}$$

Other way: If I approximate $c_p = \text{constant} = 1.005$, $k = 1.4$ (with some error)

ideal turbine:

$$h_i = w_s + h_{e,s} \rightarrow w_s = c_p(T_i - T_{e,s}) \rightarrow T_{e,s} = 663.0\,K$$

(or equivalently $w_s = \frac{n(p_e v_e - p_i v_i)}{1-n} = \frac{n(RT_e - RT_i)}{1-n}$ with $n = k$)

$$\frac{p_i}{p_e} = \left(\frac{T_i}{T_{e,s}}\right)^{\frac{n}{n-1}} \rightarrow p_e = \underline{\underline{423.0\,kPa}}$$

actual turbine

$$\underset{270}{w} = \underset{1.004}{c_p}(\underset{1000K}{T_i} - T_e) \rightarrow T_e = \underline{\underline{731\,K}}$$