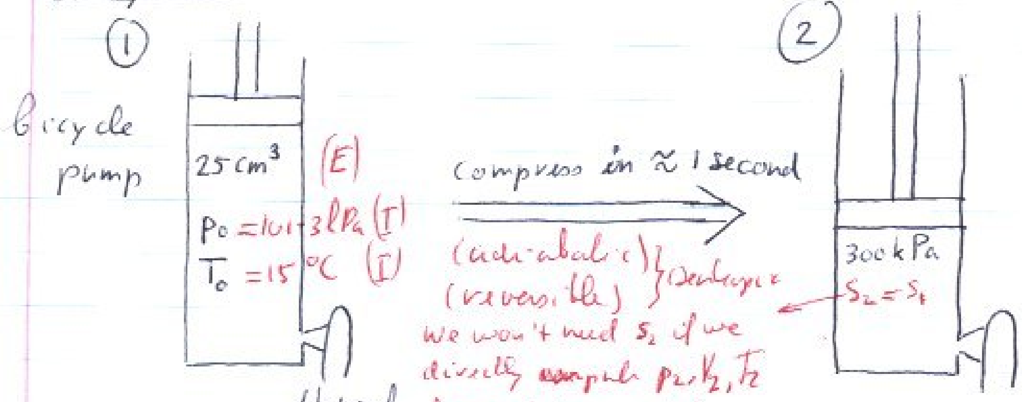
3/24 08

P 8.93 (1)



We won't need s_2 if we directly compute p_2, V_2, T_2 from polytropic relations, assuming k = constant

↳ otherwise like 8.90

Asked: V_2, T_2 $\{{}_1W_2\}$

Solution: "Normal" compressions of "normal" air typically adiabatic. We will also assume reversible ⟹ isentropic

Also, standard sea-level atmosphere: $p_0 = 101.3\ kPa$, $T_0 = 15°C$

Problem is exactly same as 8.90! But here, we will assume that $k = 1.4$ (i.e. c_p and c_v constant, 25°C, values from table A.5)

Polytropic relationships:

$$\frac{p_2}{p_1}\left[=\left(\frac{v_1}{v_2}\right)^n\right]=\left(\frac{V_1}{V_2}\right)^n=\left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} \quad \text{with } n = k = 1.4$$

$$\frac{300\ kPa}{101.3\ kPa}=\left(\frac{25\ cm^3}{V_2}\right)^{1.4}=\left(\frac{T_2}{273+15\ K}\right)^{\frac{1.4}{0.4}}$$

$$V_2=\left(\frac{101.3\ kPa}{300.0\ kPa}\right)^{\frac{1}{1.4}} 25\ cm^3 = \underline{\underline{11.518\ cm^3}}$$

$$T_2=\left(\frac{300\ kPa}{101.3\ kPa}\right)^{\frac{0.4}{1.4}} (273+15)K = 392.7\ K$$
$$= \underline{\underline{119.7\ °C}}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{101.3 \text{ kPa} \ 0.25\,10^{-6}\,m^3}{0.287 \frac{kJ}{kg\,K}\ (273+15)K} = 30.639\ 10^{-6}\ kg$$

Can now compute work as ${}_1W_2 = \frac{P_2V_2 - P_1V_1}{1-n} = \underline{-2.307\ J}$ 3/29 9

or as ${}_1W_2 = U_1 - U_2 = mc_v(T_2 - T_1) = \underline{\cancel{2}.300\ J}$

Pb.93 (2)

Same, except compress in about 1 hour instead of 1 second.

Solution: This slowly, the temperature will remain at 15°C, since any temperature difference has time to diffuse away.

$$\rightarrow T_2 = T_1 = \underline{15^\circ C}$$

Isothermal is polytropic with $n=1$:

$$\frac{p_2}{p_1} = \frac{V_1}{V_2} \quad \rightarrow V_2 = \frac{p_1}{p_2} V_1 = \underline{8.44}\ cm^3$$

~~${}_1W_2\ P_1V_1\ln$~~

$${}_1W_2 = P_1V_1 \ln\frac{V_2}{V_1} = +P_1V_1\ln\frac{P_1}{P_2} = \underline{-2.75\ J}$$

or using 1st law $\underbrace{U_2 - U_1}_{\to 0 \text{ if isothermal}} = {}_1Q_2 - {}_1W_2 \rightarrow$

$$\rightarrow {}_1W_2 = {}_1Q_2 = T(S_2 - S_1) = \overset{273+15}{T}\, m\left[\cancel{c_p \ln\frac{T_2}{T_1}}^{\,0} - R\ln\frac{P_2}{P_1}\right]$$

$$= \underline{-2.75\ J}$$

<u>Entropy generation</u> (Replaces § 8.6 – 8.8)

$$\boxed{\text{Irreversible effects always act to increase total entropy}}$$

Assume: control mass CM

${}_1Q_2$ transferred to CM from surroundings at temperature T_{surr}

Then:

$$\boxed{\underbrace{{}_1S_{2\,gen}}_{\substack{\text{entropy generated}\\ \text{by irreversible effects}}} \equiv \underbrace{\Delta S_{net}}_{\substack{\text{total change}\\ \text{in entropy}}} = \underbrace{S_2 - S_1}_{\substack{\text{entropy change}\\ \text{of CM}}} - \underbrace{\frac{{}_1Q_2}{T_{surr}}}_{\substack{\text{entropy change}\\ \text{of surroundings}}}} \geq 0$$

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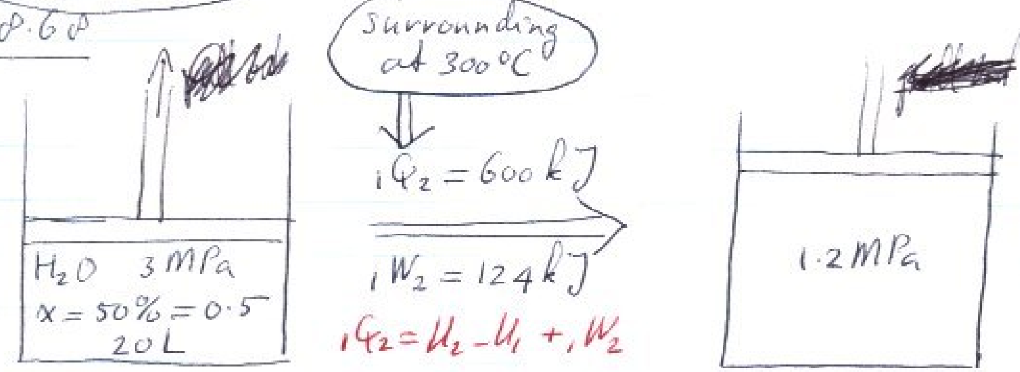
P 8.68



$_1Q_2 = U_2 - U_1 + {}_1W_2$

Asked: Does this look O.K.?

Answer:

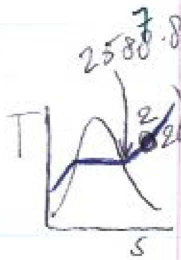
Table B.1.2 @ 3 MPa: T (A5)

$$v_1 = v_f + x v_{fg} = 0.001216\ (7) + 0.5 \cdot 0.06546 = 0.033946\ m^3/kg \qquad \overbrace{(0.066667 - .001217)}^{0.06545}$$

$$u_1 = u_f + x u_{fg} = 1004.76\ (6, 4) + 0.5 \cdot 1599.34\ (8.5) = 1804.43\ kJ/kg$$

$$s_1 = s_f + x s_{fg} = 2.6456 + 0.5 \cdot 3.5412\ (02) = 4.4162\ kJ/kg\text{-}K$$

$$m = \frac{V_1}{v_1} = \frac{20L \frac{1m^3}{1000L}}{0.033946 \frac{m^3}{kg}} = .58917\ kg$$



$U_2 = {}_1Q_2 - {}_1W_2 + mu_1 = 1539.11 \qquad u_2 = U_2/m = 2612.34$ (1st law O.K.)

Table B.1.2 (A-5) @ 1.2 MPa: $u_g = 2580.82$ (7.8) $< u_2$ so ② is superheated vapor

Table B.1.3 (A-6) @ 1.2 MPa and interpolated to $u_2 = 2612.34$:

$s_2 = 6.5898$ (909) (ignore difference between 2612.34 and 2612.74 (9))

Entropy generated

$$_1S_{2gen} = \Delta s_{net} = m(s_2 - s_1) - \frac{_1Q_2}{T_{surr}}$$

$$= .58917\ kg\ (6.5898 - 4.4162) \frac{kJ}{kg\text{-}K} - \frac{600\ kJ}{300+273\ K}$$

$$= 0.233 \frac{kJ}{\cancel{kg}\text{-}K}$$

Maybe O.K. ~~However,~~ $T_1 = 233.85°C < 300°C$

$T_2 = 200°C$

Show old exam 3/final formula sheet

AT END

I.G. $u, h, c_v, c_p = u, h, c_v, c_p(T)$

$T\,ds = du + p\,dv$ Gibbs

use v and T as indep. vars.

$du = \frac{\partial u}{\partial T} dT + \frac{\partial u}{\partial v} dv$

$$ds = \underbrace{\frac{1}{T}\frac{\partial u}{\partial T}}_{\frac{\partial s}{\partial T}} dT + \underbrace{\left(\frac{1}{T}\frac{\partial u}{\partial v} + \frac{p}{T}\right)}_{\frac{\partial s}{\partial v}} dv$$

Now $\frac{\partial}{\partial v}\frac{\partial s}{\partial T} = \frac{\partial}{\partial T}\frac{\partial s}{\partial v}$

$$\cancel{\frac{1}{T}\frac{\partial^2 u}{\partial v \partial T}} = \cancel{\frac{1}{T}\frac{\partial^2 u}{\partial v \partial T}} - \frac{1}{T^2}\frac{\partial u}{\partial v} + \frac{1}{T}\frac{\partial p}{\partial T} - \frac{p}{T^2}$$

$$\boxed{\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p}$$

Ideal gas $p = \frac{RT}{v}$ $\rightarrow T\frac{\partial p}{\partial T} - p = T\frac{R}{v} - \frac{RT}{v} = 0$

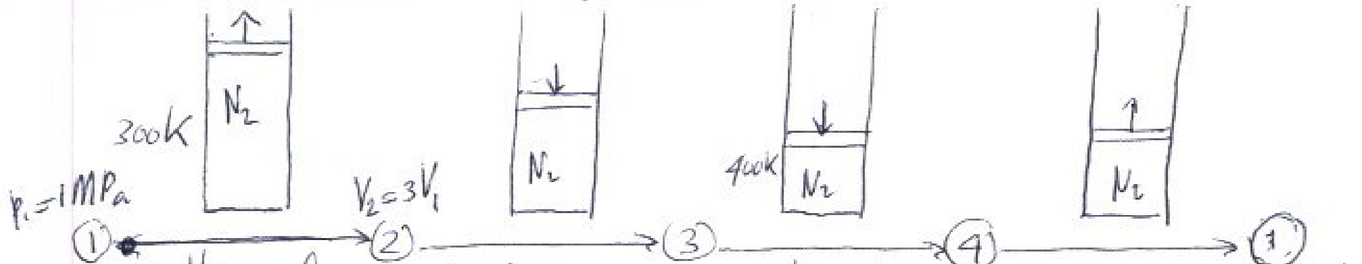
$\rightarrow \frac{\partial u}{\partial v} = 0 \rightarrow u = u(T)$

$\rightarrow h = u + pv = u + RT = h(T)$

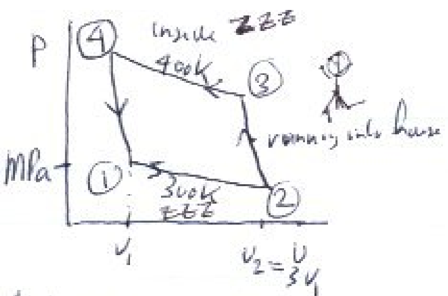
$\rightarrow c_v = \left(\frac{du}{dT}\right)_v = c_v(T)$ $c_p = \left(\frac{dh}{dT}\right)_p = c_p(T)$

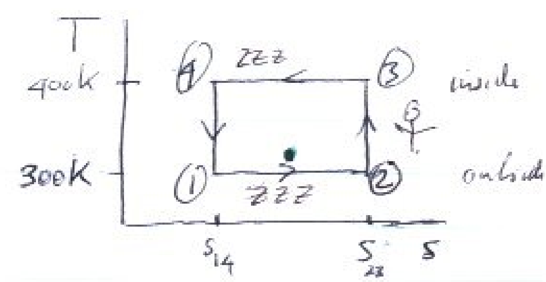
P8.89 Carnot cycle heat pump using 1 kg of N_2, piston-cylinder

~~outside~~ → time



1–2	2–3	3–4	4–1
isothermal heat addition at 300K (slow)	isentropic compression to 400K (fast)	isothermal heat removal at 400K (slow)	isentropic expansion to 300K (fast)





Asked: p, v, T at 1, 2, 3, 4; ${}_1Q_2, {}_2Q_3, {}_3Q_4, {}_4Q_1, {}_1W_2, {}_2W_3, {}_3W_4, {}_4W_1$.

END>

a)

	1	2	3	4	
p	1	.3333	.9123	2.7371	(MPa)
T	300	300	400	400	(K)
v	.08904	.2612	.1302	.04337	(m³/kg)

$v_1 = \frac{mRT_1}{p_1}$ $\quad v_2 = 3v_1$

$p_2 v_2 = p_1 v_1 \rightarrow p_2 = \frac{1}{3} p_1$

$\frac{p_3}{p_2} = \left(\frac{T_3}{T_2}\right)^{\frac{k}{k-1}} \left[\frac{p_4}{p_1} = \left(\frac{T_4}{T_1}\right)^{\frac{k}{k-1}}\right]$

$k = 1.4$ (table A.5)

A2(a)

b)

	1–2	2–3	3–4	4–1	
Q	97.82	0	−130.43	0	kJ
W	97.82	−74.48	−130.43	74.48	kJ

Since ${}_1Q_2 = U_2 - U_1 + {}_1W_2$, but $U_2 = U_1$ since I.G. and $T_1 = T_2$, ${}_1Q_2 = {}_1W_2$

${}_1W_2 = p_1 v_1 \ln \frac{v_2}{v_1}$ $\quad {}_1Q_2 = T_{12}(s_2 - s_1) = -T_{12} R \ln \frac{p_2}{p_1}$ (give same answer)

3–4 same as 1–2

Since ${}_2Q_3 = U_2 - U_1 + {}_1W_2$ but ${}_2Q_3 = 0$ by definition, ${}_1W_2 = U_2 - U_1 = mc_{v_0}(T_2 - T_1)$

or ${}_2W_3 = \frac{p_3 v_3 - p_2 v_2}{1-n}$ with $n = k = 1.4$

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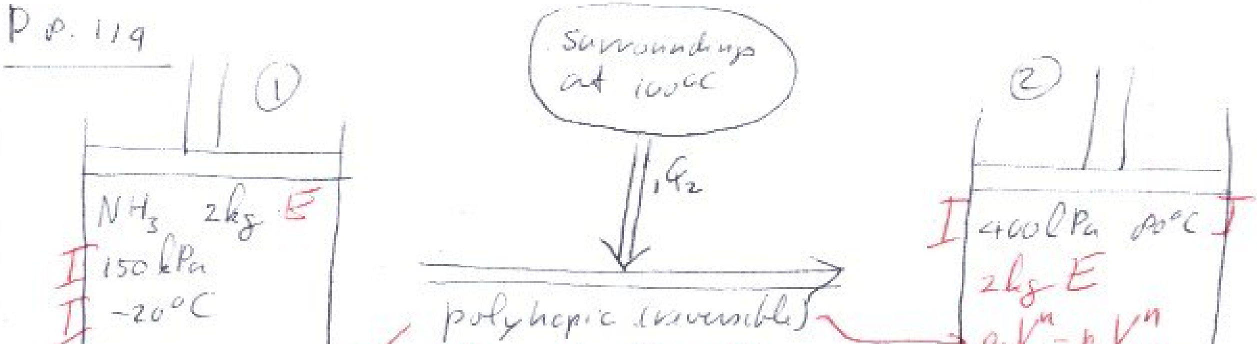


$${}_1Q_2 = U_2 - U_1 + {}_1W_2$$

Since p_1V_1 already known, gives n

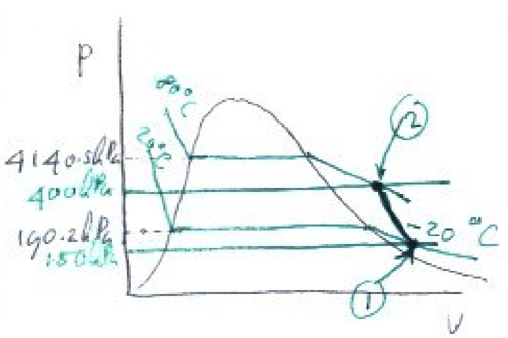
Asked: n
${}_1W_2$
${}_1Q_2$
${}_1S_{2,gen}$

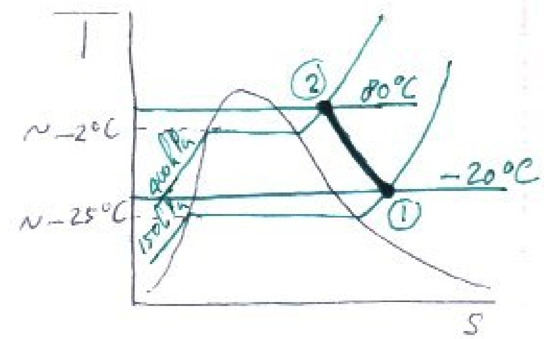
$${}_1W_2 = \frac{p_2V_2 - p_1V_1}{1-n} \quad (n \neq 1 \text{ assumed})$$

$${}_1S_{2gen} = S_2 - S_1 - \frac{{}_1Q_2}{T_{sur}}$$

Answer:

Find phase of states 1 and 2 in pv and Ts, Will use Broken line first in both.





Conclusion: both are superheated vapor.

Table B.2.2 @ 150 kPa, −20°C : $u_1 = 1303.3$ kJ/kg

$v_1 = .7977$ m³/kg

$s_1 = 5.7465$ kJ/kg-K

@ 400 kPa, 80°C : $u_2 = 1468.0$ kJ/kg

$v_2 = .42160$ m³/kg

$s_2 = 5.9907$ kJ/kg-K

Find n: $p_1 v_1^n = p_2 v_2^n$ or $\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n$

Trick: use ln: $\ln\frac{p_1}{p_2} = \ln\left(\frac{v_2}{v_1}\right)^n = n\ln\left(\frac{v_2}{v_1}\right)$

$$n = \ln\frac{p_1}{p_2} / \ln\frac{v_2}{v_1} = \ln\left(\frac{150}{400}\right) / \ln\left(\frac{.92160}{.7977}\right)$$

$$= \underline{\underline{1.5301}}$$

$${}_1W_2 = \frac{p_2 v_2 - p_1 v_1}{1-n} m = \frac{(400 \cdot .92160 - 150 \cdot .7977)}{1 - 1.5301} \, kPa \cdot \frac{m^3}{kg} \cdot 2\,kg = \underline{\underline{-182.1\ kJ}}$$

$${}_1Q_2 = U_2 - U_1 + {}_1W_2 = m(u_2 - u_1) + {}_1W_2 = 2\,kg\,(1460.0 - 1303.3)\frac{kJ}{kg} - 182.1\,kJ$$

$$= \underline{\underline{147.3\ kJ}}$$

$${}_1S_{2gen} = S_2 - S_1 - \frac{{}_1Q_2}{T_{surr}} = m(s_2 - s_1) - \frac{{}_1Q_2}{T_{surr}} = 2\,kg\,(5.4507 - 5.7465)\frac{kJ}{kg\,K} - \frac{147.3\,kJ}{(100+273)\,K}$$

$$= \underline{\underline{0.093\ \frac{kJ}{K}}}$$

3/30/06 end →