

# Ideal gases

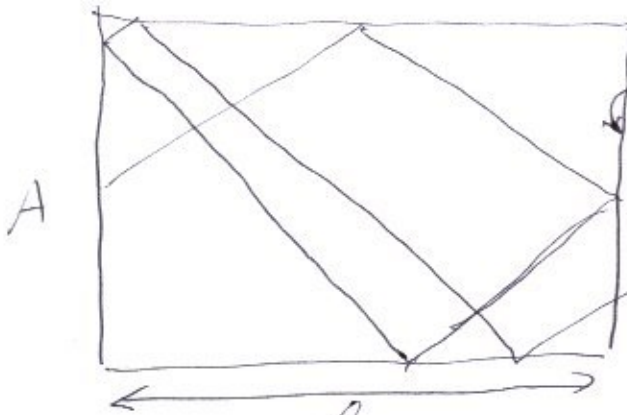
3.18 p2

Good for normal air.

Must use most accurate data available unless told otherwise

→ Tables for water vapor, other  
A4, 5, 6

## General idea



Force  $\rightarrow$   $PA \Delta t = 2m_p v_x \frac{v_x \Delta t}{2l_x} N$  (momentum change)

$PV = N m_p v_x^2 = N \frac{1}{3} m v^2$  (momentum change)

$\rightarrow \frac{2}{3} N \frac{1}{2} m v^2 = \frac{2}{3} N k_B T$  (number of collisions, number of molecules)

$P = \frac{1}{3} n m v^2$  (number of molecules)

kinetic energy of molecules  
→  $\bar{RT}$

Formulae  $\bar{v} = \frac{V}{n} l_x$   $v = \frac{V}{m}$

~~$p\bar{v} = RT$~~

$pV = nRT$      $p\bar{v} = RT$

$pV = mRT$      $p v = RT$

$R = \frac{R}{M}$

$\bar{R} = 8.314.5 \text{ J/kmol-K}$

## Alternate

$PA \Delta t = \underbrace{2m_p v_x}_{\text{momentum change / collision}} \underbrace{\frac{v_x \Delta t}{2l_x}}_{\text{\# collisions}} \underbrace{N}_{\text{number of molecules}} \rightarrow \frac{pV}{A l_x} = N m_p v_x^2 = N k_B T = n \frac{1}{3} k_B T$

### 3.6 Ideal gases

Good for normal air, other gases, water vapor under certain conditions.

Must use most accurate data available unless told otherwise

Superheated tables for water vapor more accurate than ideal gas ~~equation~~.

Ideal gas equation  $v = \frac{V}{m}$   $\bar{v} = \frac{V}{n}$  ← number of moles

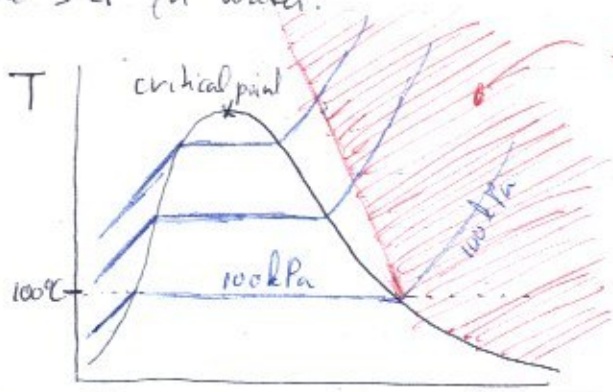
|                               |                 |                 |  |
|-------------------------------|-----------------|-----------------|--|
| intensive                     | $p\bar{v} = RT$ | $p\bar{v} = RT$ | bar left ↔ bar right<br>(if intensive) |
| extensive                     | $pV = mRT$      | $pV = nRT$      |  |
| WARNING: ABSOLUTE TEMPERATURE |                 |                 |  |

|                                    |                                       |
|------------------------------------|---------------------------------------|
| R gas constant for specific gas    | $R = \bar{R}/M$                       |
| $\bar{R}$ : universal gas constant | $\bar{R} = 8314.5 \text{ J/(kmol K)}$ |

Values for R are in table A5 (or F4) A1 or A-2

$n = \frac{m}{M}$       M: molecular mass in kg/kmole

Figure 3-21 for water:

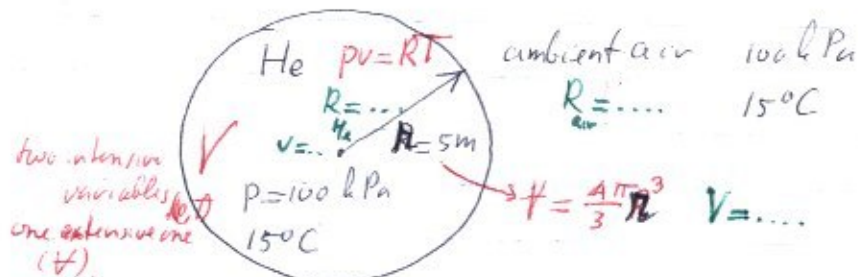


error in  $pv = RT$  is less than 1.5% in the red region.

a vapor is a good ideal gas when ~~1)  $T \gg T_{crit}$~~  or 2)  ~~$p \ll p_{crit}$~~  and it is vapor of course, not liquid.

- 1)  $T \gg T_{crit}$  (reasonably speaking)
- or 2)  $p \ll p_{crit}$  and it is vapor.

3.65 Given: Balloon, spherical



Asked: how much can the helium lift?

Answer: Lifting capability according to Archimedes:

(weight of the helium was <sup>ambient</sup> air) - (weight of helium)

↓  
 $m_{\text{air}} g$

↓  
 $m_{\text{He}} g$

Need  $m_{\text{air}}$ ,  $m_{\text{He}}$ . Ideal gas will give  $v$ , which is  $V/m$

$$v_{\text{He}} = \frac{V}{m_{\text{He}}}$$

$$v_{\text{air}} = \frac{V}{m_{\text{air}}}$$

$$V = \frac{4\pi}{3} (5 \text{ m})^3 \dots$$

Table A.5 A-2

| Gas    | $R$ [ $\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ ] |
|--------|--|
| air    | 0.287  |
| Helium | 2.0771   |

$$p v_{\text{He}} = R_{\text{He}} T \quad 100 \text{ kPa} \quad v_{\text{He}} = 2.0771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (15 + 273) \text{ K}$$

$$v_{\text{He}} = \frac{2.0771 (15 + 273)}{100} \frac{\text{kJ}}{\text{kPa} \cdot \text{kg}} = 5.902 \frac{\text{kJ} \cdot \text{m}}{\text{kPa} \cdot \text{kg}} = 5.902 \frac{\text{m}^3}{\text{kg}}$$

$$v_{\text{He}} = \frac{V}{m_{\text{He}}} \quad 5.902 \frac{\text{m}^3}{\text{kg}} = \frac{\frac{4\pi}{3} (5 \text{ m})^3}{m_{\text{He}}} \quad m_{\text{He}} = \frac{\frac{4\pi}{3} (5 \text{ m})^3}{5.902} = 0.75 \text{ kg}$$

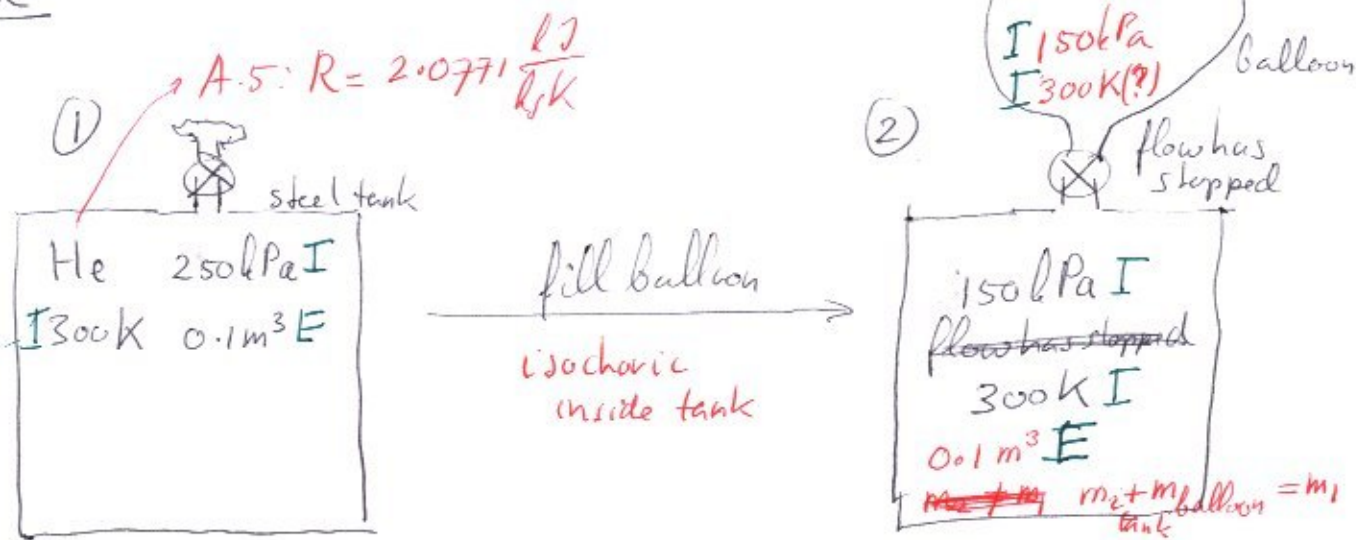
$$p_{\text{air}} = R_{\text{air}} T \quad 100 \text{ kPa} \quad v_{\text{air}} = 0.207 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (15 + 273) \text{K}$$

$$\rightarrow v_{\text{air}} = 0.0266 \frac{\text{m}^3}{\text{kg}}$$

$$v_{\text{air}} = \frac{V}{m_{\text{air}}} \quad 0.0266 \frac{\text{m}^3}{\text{kg}} = \frac{\frac{4\pi}{3} 5^3 \text{m}^3}{m_{\text{air}}} \quad m_{\text{air}} = 633 \text{ kg}$$

$$\text{Lifting capability} \quad m_{\text{air}} g - m_{\text{He}} g = 633 \text{ kg} \cdot 9.81 - 0.7 \text{ kg} \cdot 9.81 = 5.36 \text{ kN}$$

3.52 SC



Asked How big is the balloon?

Solution: full data for 1 → can compute  $m_1$   
full data for tank at 2 → can compute  $m_{2 \text{ tank}}$  } ⇒  $m_{\text{balloon}}$

$$P_1 V_1 = m_1 R T_1 \rightarrow 250 \text{ kPa } 0.1 \text{ m}^3 = m_1 \cdot 2.0771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \cdot 300 \text{ K}$$

$$m_1 = 0.04012 \text{ kg}$$

$$P_2 V_{2t} = m_{2t} R T_2 \rightarrow 150 \text{ kPa } 0.1 \text{ m}^3 = m_{2t} \cdot 2.0771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \cdot 300 \text{ K}$$

$$m_{2t} = 0.02407 \text{ kg}$$

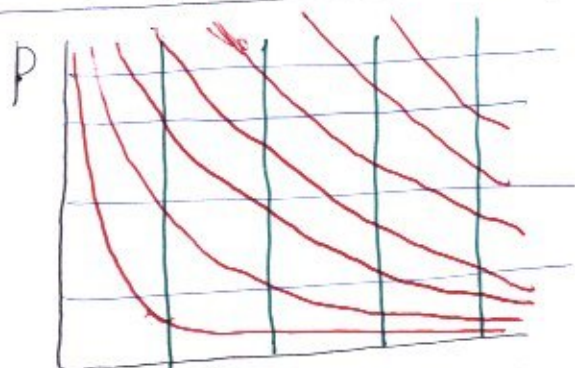
$$\rightarrow m_{2b} = m_1 - m_{2t} = 0.016048 \text{ kg}$$

$$P_2 V_{2b} = m_{2b} R T_2 \quad 150 \text{ kPa } V_{2b} = 0.016048 \text{ kg} \cdot 2.0771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \cdot 300 \text{ K}$$

$$V_{2b} = \frac{0.016048 \text{ kg} \cdot 2.0771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \cdot 300 \text{ K}}{150 \text{ kPa}} = 0.0666 \frac{\text{kJ}}{\text{kPa}} = 0.0666 \text{ m}^3$$

Appendix A: 1 J = 1 Nm  
1 Pa = 1 N/m²

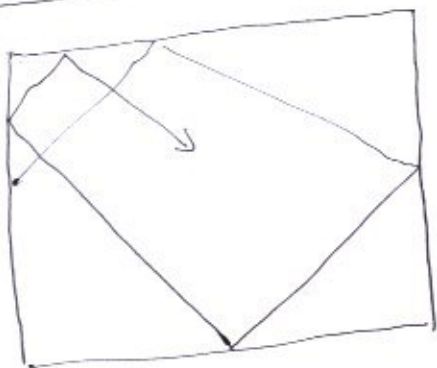
# The P-v diagram of an ideal gas



— isobars  
— isochores  
— isotherms

Yes, isotherms go down, but where is the bubble??

## Van der Waals



v: must subtract volume b unavailable due to molecule sizes

$$Pv = RT \Rightarrow \frac{1}{2}P(v-b) = RT$$

P: must subtract "pull back" by other molecules when molecules approach the wall

$$P \rightarrow P + \frac{1}{v} \frac{1}{v} a$$

↑ amount of molecules pulling back      ↓ amount of molecules being pulled back

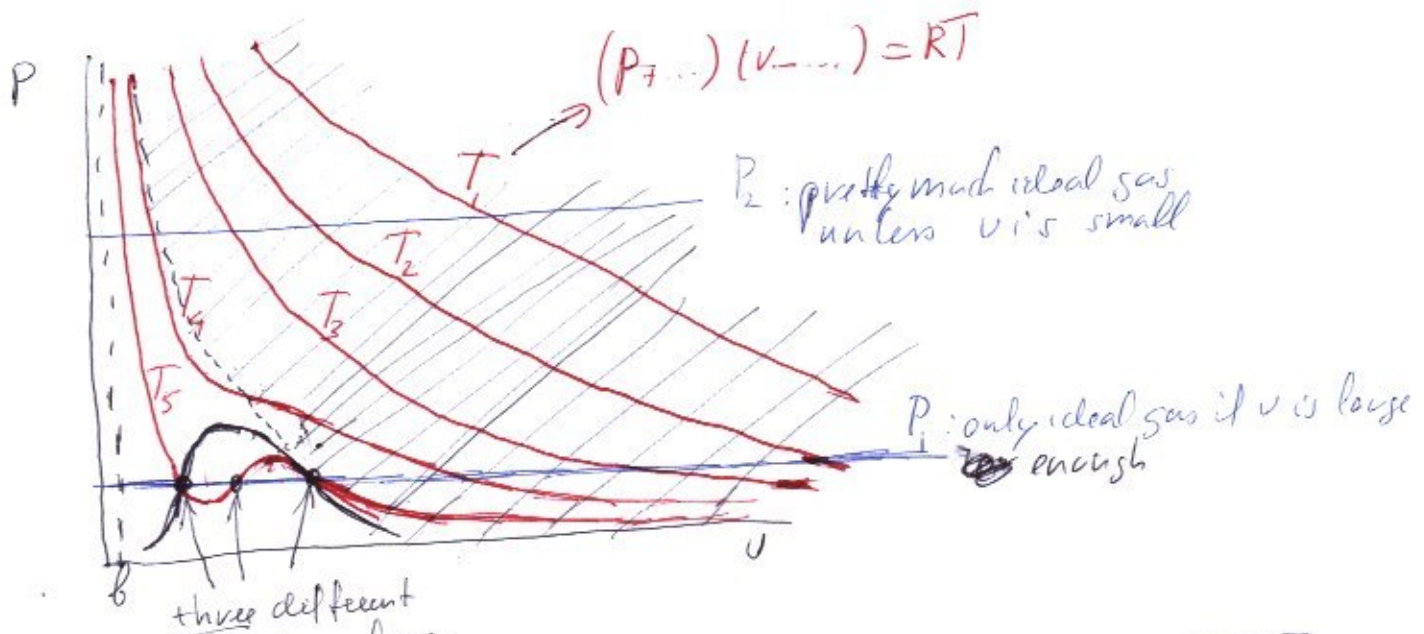
→ same constant, measure!

$$\left(P + \frac{a}{v^2}\right)(v-b) = RT$$

Van der Waals gas

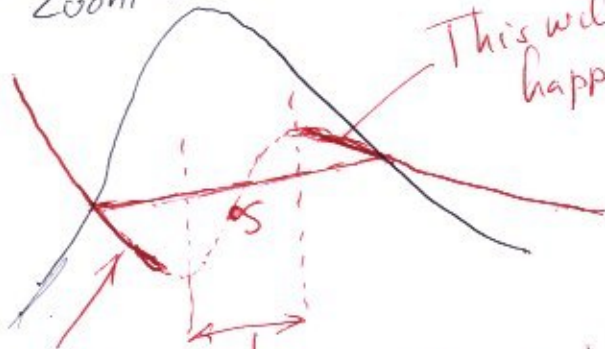
If v is large compared to b, can ignore -b.

If P is large and/or v is large, can ignore  $\frac{a}{v^2}$



Two phase bubble of ideal gas is "hidden in the corner"

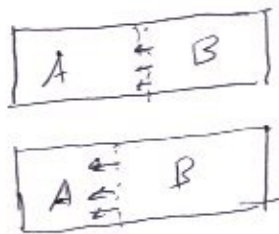
Zoom in on bubble



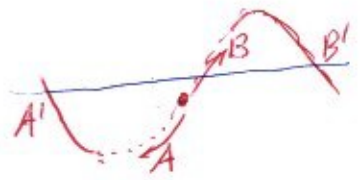
This will actually happen: "super saturation"

This will actually happen: "Subsaturation"

This will never happen: violently unstable:  
 Suppose the substance is in state S and the sound of my voice compresses the left half substance just a little bit



Pressure of A half becomes lower  
 Pressure of B half becomes higher  
 → B pushes A even further in  
 → Pressure of A becomes still lower  
 B higher  
 → worse!



~~Process~~ Collapse of A cannot stop until A has reached A' (sat liq) and B B' (sat vap) and the pressures are the same again

Compressibility factor Z

Z gives the error in the ideal gas law:

Ideal gas approximation:  $pv = RT$   
 True:  $pv = ZRT$     so  $\frac{R}{R} \rightarrow ZR$   
 or  $\frac{R}{R} \rightarrow ZR$

When  $Z \approx 1$ , ideal gas is good

How to find Z:

- a) Look up critical pressure  $p_c$  and critical temperature  $T_c$  for your gas in Table A.2 p 65-6
- b) Compute the values

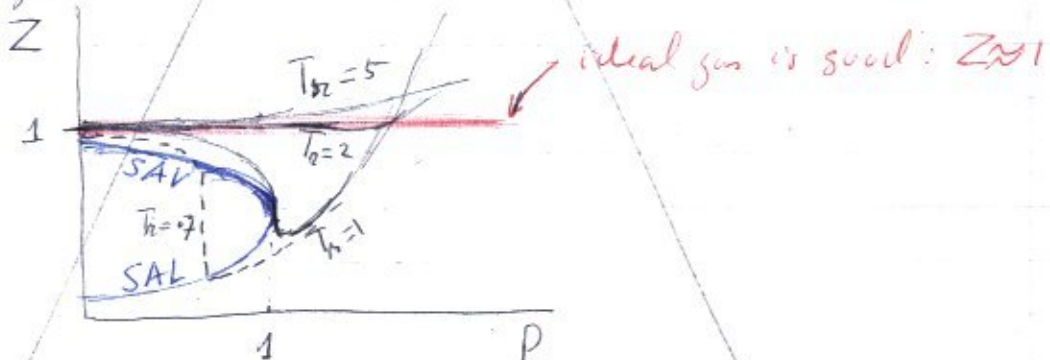
$$P_r = \frac{P}{P_c} \quad \text{and} \quad T_r = \frac{T}{T_c}$$

for your  $p$  and  $T$

- c) Look in figure D.1, p 728 to find Z

(Alternatively, if you have tables like for superheated water vapor, use  $Z = pv/RT$  with  $v$  from the table)

Figure D.1



Note that ideal gas is good if it's vapor and  $P_r$  is small, i.e.  $p < p_c$



SKIP

3.22

3.01 Given:  $\text{NH}_3$   $40^\circ\text{C}$   $500 \text{ kPa}$

Asked: % error in  $v$  if computed using the ideal gas law, and using the compressibility factor

Answer:

Exact value: ~~from table B.2.1~~

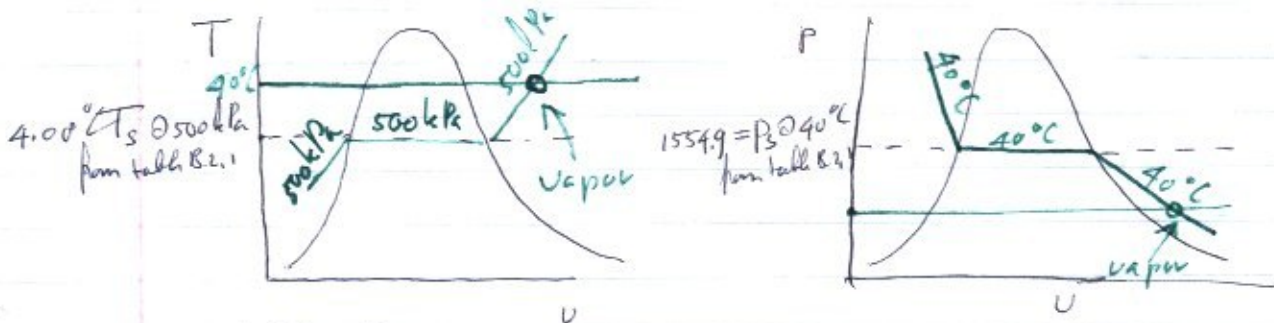


Table B.2.1 saturated ammonia

| Temp [ $^\circ\text{C}$ ] | Press [kPa] |
|---------------------------|-------------|
| 0                         | 429.6       |
| 5                         | 515.9       |
| 40                        | 1554.9      |

$T_s \propto \sqrt[0.4]{\frac{500 - 429.6 \text{ kPa}}{515.9 - 429.6 \text{ kPa}}} (5 - 0) = 4.00^\circ\text{C}$

Vapor state, so use table B.2.2 on page 695

| Temp [ $^\circ\text{C}$ ] | $v$ [ $\text{m}^3/\text{kg}$ ] |
|---------------------------|--------------------------------|
| 40                        | 0.29227                        |

500 kPa

$v_{\text{exact}} = \underline{0.29227}$  (Must use exact value unless instructed otherwise)

Ideal gas:  $pv = RT$

Table A.5

Gas - - - -  $R \left[ \frac{\text{kJ}}{\text{kg K}} \right]$

Ammonia

0.4402

$$\text{So } 500 \text{ kPa } v_{IG} = 0.4402 \frac{\text{kJ}}{\text{kg K}} (40 + 273) \text{ K}$$

$$v_{IG} = \frac{0.4402 (40 + 273)}{500} \frac{\text{kJ}}{\text{kPa kg}}$$

$$= 0.30561 \frac{\text{m}^3}{\text{kg}}$$

$$\text{Error: } \frac{v_{IG} - v_{\text{exact}}}{v_{\text{exact}}} \times 100\% = \frac{0.30561 - 0.29227}{0.29227} 100\% = \underline{\underline{4.6\% \text{ too high}}}$$

Compressibility factor:

Table A.2: critical constants (page 656)

Substance                      Temp [K]                      Press [MPa]

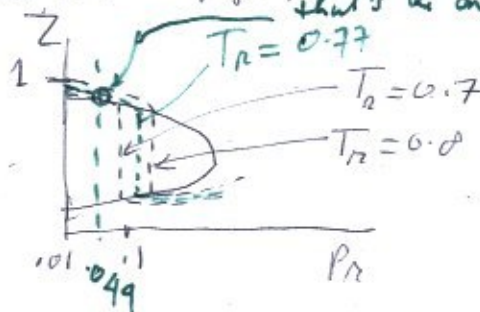
Ammonia

405.5

11.35

$$T_r = \frac{T}{T_c} = \frac{(40 + 273) \text{ K}}{405.5 \text{ K}} = 0.772 \quad P_r = \frac{P}{P_c} = \frac{500 \text{ kPa}}{11350 \text{ kPa}} = 0.044$$

Figure D.1 on page 730 <sup>what is the one</sup>:  $Z \approx 0.97$



$$Z \approx 0.97 \quad pv = ZRT \quad 500 \text{ kPa } v_2 = 0.97 \cdot 0.4402 \frac{\text{kJ}}{\text{kg K}} (40 + 273) \text{ K}$$