Matlab Homework 11c

The same requirements as for homework 3c apply.

1. You will need to make each of the next items a separate section (started with a double percent), as each produces a separate graph.

   (a) Use `ezplot` to plot both $J_0(\omega)$ and $\frac{1}{2}\omega J_1(\omega)$ in the same graph without actually defining either function or any plot points. The horizontal axis must again extend from 0 to 3.5π. Use title "Drum Frequency Functions" and legend "J_0(omega)" and "0.5 omega J_1(omega)". Put the legend in the empty space in the southwest corner of the graph.

   (b) Plot the "lemniscate of Bernoulli," given by the equation

   $$(x^2 + y^2)^2 - (x^2 - y^2) = 0$$

   Use title "Lemniscate of Bernoulli" and hidden axes with sizes from -1 to 1, respectively -0.5 to 0.5 with equal x and y scalings.

2. We have the following measured data

x:	12.9	16.6	21.5	27.8	35.9	46.4	59.9	77.4
y:	6.7	8.2	9.5	10.2	11.6	12.8	15.8	16.6

   See whether the relationship of x and y seems to follow some power law of the form $y = Cx^p$. If so, try to find approximate values for the constants C and p. Compare your found relationship to the measured data in a `loglog` plot. Plot the power relationship from $x = 10$ to 100. Make the vertical axis range from 5 to 20. Add appropriate title, axes labels, and legend in the southeast corner, Comment with `disp` on how well you think the agreement is.

3. A rectangular plate has width 2 and length 1.5. The bottom edge of the plate is in contact with boiling water, so the temperature T there is 100 degrees Centrigrade (in Tallahassee at least). The other three edges are in contact with ice water, so their temperature T is zero degrees Centrigrade. Your task is to find and plot the temperature, including in the interior of the plate.

   After defining a grid of n x-values and m y-values, the temperatures T at the grid points may be found using function `simplePoisson`. You should already have obtained that function in lecture 11. If not, you can find it at the link in "Course Library", "Matlab", under "Some notes".

   In using this function, you will need to define array `data`. In doing so note that in this problem the "forcing" is zero. (In this problem, forcing would correspond to heat radiated away from the plate surface, and that can be ignored at 100°C.) The boundary values are also all zero, except for the 100°C bottom boundary $i = 1$. There is a slight problem with grid points $(1, 1)$ and $(1, n)$, as these points are both on the 100°C boundary and on a 0°C boundary. To fix that, give those two special points the average temperature of 50°C.

   You will need to make each of the next items a separate section (started with a double percent), as each produces a separate graph.

(a) Take $n = 29$ and $m = 22$. Define and plot the grid points on the plate, as circles. Use appropriate title and labels, x-axis from 0 to 2 and y-axis from 0 to 1.5.

(b) Use `simplePoisson` to find the temperatures at the interior points for these m and n values. Plot the raw data as stems and so check that they seem generally sane. Use appropriate title and labels on all three axes, x-axis from 0 to 2, y-axis from 0 to 1.5, and z-axis from 0 to 100.

(c) Plot the temperature as a three-dimensional surface above the plate. Use appropriate title and labels on all three axes, x-axis from 0 to 2, y-axis from 0 to 1.5, and z-axis from 0 to 100.

(d) Plot the isotherms (lines of constant temperature) within the plate. Use appropriate title and labels on both axes, x-axis from 0 to 2 and y-axis from 0 to 1.5. Note that the top three boundaries form the 0°C isotherm and the bottom boundary the 100°C isotherm. The other isotherms are temperatures somewhere in between.

4. If you did the previous question correct, the temperatures are generally OK, except near the mentioned two special corner points where the temperature jumps from 0°C to 100°C. To make this less noticeable, you could use much bigger values of m and n, but that would make the computation very slow.

   "Mesh-stretching" to the rescue. In this question, define your grid point x and y values as follows:

```
stretchPar=0.875
angles=linspace(0,2*pi,n);
xValues=2*(angles-stretchPar*sin(angles))/(2*pi);
angles=linspace(0,2*pi,m);
yValues=1.5*(angles-stretchPar*sin(angles))/(2*pi);
```

   Repeat all four plots. Do not forget to recompute everything that needs recomputing. You should get much better results near the problem corners.

5. The "streamfunction" ψ for slow flow of superfluid liquid helium around a circular cylinder, (or of Hele-Shaw flow around a circular cylinder, for that matter), is given in polar coordinates r and θ by

$$\psi = U\left(r - \frac{a^2}{r}\right)\sin(\theta)$$

   where U is the incoming flow velocity far from the cylinder and a is the radius of the cylinder. Take both to be one. Now contour lines of the streamfunction are "streamlines" that show you in which direction the fluid flows. To plot them, define a polar grid where r takes 100 values from a to $5a$, and θ takes 100 values from 0 to 2π. Find the streamfunction at those points and use it to plot the streamlines in Cartesian coordinates. Note: to get the streamlines you really want to see, you must specify their values of ψ in function `contour`. To do so, use the following vector

```
[-5.001:0.25:-0.001 0.001:0.25:5.001]
```

   Use axis sizes from -3 to 3 in both directions, and make sure you use square axes so that the cylinder looks like a cylinder and not an ellipse. Add appropriate labels and title.