INTEGRATION USING THE TRAPEZIUM RULE

The following definitions, copied from the hot bar of lesson 5, are again needed:

$$\text{ORIGIN} := 1 \qquad i := 1..6 \qquad t_{m_i} := (i-1)\cdot 0.5\text{min} \qquad zC := 0\,°C$$

$$T_x(t) := zC + \exp\left(-\frac{t}{3\text{min}}\right)\cdot 28.7K$$

$$T_m := \text{round}\left[\left(T_x(t_m) - zC\right)\cdot K^{-1}\right]K + zC$$

To find the total radiation emitted by the bar while it is cooling down to the 0 degree Celsius environment, we must integrate $\sigma(T^4 - (0 \text{ deg C})^4)$ with respect to time. Here σ is the Stefan-Boltzmann constant:

$$\sigma := 5.670400\cdot 10^{-8}\cdot\frac{\text{watt}}{m^2\cdot K^4}$$

Our measured temperature values, and the corresponding integrand f to integrate are

$$\text{ORIGIN} := 0$$

$$T_m = \begin{pmatrix} 302.15 \\ 297.15 \\ 294.15 \\ 290.15 \\ 288.15 \\ 285.15 \end{pmatrix} K \qquad f := \sigma\cdot\left[T_m^{\,4} - (273.15K)^4\right] = \begin{pmatrix} 156.952 \\ 126.437 \\ 108.852 \\ 86.228 \\ 75.261 \\ 59.234 \end{pmatrix}\cdot\frac{\text{watt}}{m^2}$$

The trapezium rule approximates the integral in each interval from one integrand value to the next by the average function value times the length of the interval. We have 5 time intervals, so define a range variable as:

$$i := 0..\text{last}(t_m) - 1$$

Now find the integrals over the time intervals:

$$\text{Trap}_{f_i} := \frac{f_i + f_{i+1}}{2}\cdot\left(t_{m_{i+1}} - t_{m_i}\right) \qquad \text{Trap}_f = \begin{pmatrix} 4.251\times 10^3 \\ 3.529\times 10^3 \\ 2.926\times 10^3 \\ 2.422\times 10^3 \\ 2.017\times 10^3 \end{pmatrix}\cdot\frac{J}{m^2}$$

For example, the first coefficient of $Trap_f$ is the integral from t_{m_0} to t_{m_1}. To get the total time integral, you must sum the results for all intervals together:

$$\sum Trap_f = 1.515 \times 10^4 \cdot \frac{J}{m^2}$$

This used the summation from the vector toolbar. If you want to avoid vectors, you can use the sum from the menu View / Toolbars / Calculus toolbar:

$$\sum_{i=0}^{last(t_m)-1} \left[\frac{f_i + f_{i+1}}{2} \cdot \left(t_{m_{i+1}} - t_{m_i}\right)\right] = 1.515 \times 10^4 \cdot \frac{J}{m^2}$$

INTEGRATION OF FUNCTIONS, INCLUDING INTERPOLATING FUNCTIONS

As a possible more accurate way to integrate the radiation, you can have Mathcad integrate an approximating function of the temperature. The following data from lesson5 are again needed:

$$funcs_{qls}(t) := \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix}$$

$$C_{qls} := \mathrm{linfit}\left(t_m \cdot \mathrm{min}^{-1}, T_m \cdot K^{-1}, funcs_{qls}\right) = \begin{pmatrix} 301.936 \\ -9.129 \\ 1 \end{pmatrix}$$

$$T_{qls}(t) := \left[C_{qls_0} + C_{qls_1} \cdot t \cdot \mathrm{min}^{-1} + C_{qls_2} \cdot \left(t \cdot \mathrm{min}^{-1}\right)^2\right] \cdot K$$

$$\int_{t_{m_0}}^{t_{m_{last(t_m)}}} \sigma \cdot \left(T_{qls}(t)^4 - zC^4\right) dt = 1.51 \times 10^4 \cdot \frac{J}{m^2}$$

SYMBOLIC OPERATIONS

So far, we always had Mathcad find *numbers*. For example, we could integrate

$$12 + 3x - 4x^2$$

between the limits 0 and 2 to find the number:

$$\int_0^2 12 + 3x - 4x^2\,dx = 19.333$$

But sometimes you do not want numbers but a symbolic expression. For example, what if you need to find an indefinite integral. That cannot be a number, for one because it has an undetermined integation constant. We can however let Mathcad find a *symbolic* answer by using Ctrl+. instead of =:

$$\int 12 + 3x - 4x^2\,dx \rightarrow \frac{3\cdot x^2}{2} - \frac{4\cdot x^3}{3} + 12\cdot x$$

We can also let Mathcad find the roots of the integrand *exactly* if we want. Below, use Ctrl+= for the equation equals sign, and Ctrl+Shift+. for the symbolic evaluation. Or use the Symbolic toolbar for the latter.

$$12 + 3x - 4x^2 = 0 \text{ solve } \rightarrow \begin{pmatrix} \frac{\sqrt{201}}{8} + \frac{3}{8} \\ \frac{3}{8} - \frac{\sqrt{201}}{8} \end{pmatrix}$$

If we give 12.0 instead of 12, Mathcad assumes it is not an exact integer and reverts to numbers:

$$12.0 + 3x - 4x^2 = 0 \text{ solve } \rightarrow \begin{pmatrix} 2.1471808598447281504 \\ -1.3971808598447281504 \end{pmatrix}$$

We can also find derivatives symbolically:

$$\frac{d}{dx}\left(12 + 3x - 4x^2\right) \rightarrow 3 - 8\cdot x$$

Mathcad can solve some simple nonpolynomial equations:

$e^x + 1 \text{ solve } \rightarrow \pi\cdot i$ Mathcad assumes = 0 if not specified

Sometimes we must specify which variable to solve for:

$$5\cdot x\cdot y - 9\cdot x - 1 \text{ solve}, y \rightarrow \frac{9\cdot x + 1}{5\cdot x}$$

$$5\cdot x\cdot y - 9\cdot x - 1 \text{ solve}, x \rightarrow \frac{1}{5\cdot y - 9}$$

$$5\cdot x\cdot y - 9\cdot x - 1 \text{ collect}, x \rightarrow (5\cdot y - 9)\cdot x - 1$$

Find a Taylor series:

$$e^{2x} \text{ series} \rightarrow 1 + 2\cdot x + 2\cdot x^2 + \frac{4\cdot x^3}{3} + \frac{2\cdot x^4}{3} + \frac{4\cdot x^5}{15}$$

$$e^{2x} \text{ series}, 10 \rightarrow 1 + 2\cdot x + 2\cdot x^2 + \frac{4\cdot x^3}{3} + \frac{2\cdot x^4}{3} + \frac{4\cdot x^5}{15} + \frac{4\cdot x^6}{45} + \frac{8\cdot x^7}{315} + \frac{2\cdot x^8}{315} + \frac{4\cdot x^9}{2835}$$

Very useful: find partial fractions:

$$\frac{2x^2 - 3x + 1}{x^3 + 2x^2 - 9x - 18} \text{ parfrac} \rightarrow \frac{1}{3\cdot(x-3)} - \frac{3}{x+2} + \frac{14}{3\cdot(x+3)}$$

$$\frac{1}{3\cdot(x-3)} - \frac{3}{x+2} + \frac{14}{3\cdot(x+3)} \text{ simplify} \rightarrow \frac{(x-1)\cdot(2\cdot x-1)}{(x+2)\cdot(x-3)\cdot(x+3)}$$

$$\frac{(x-1)\cdot(2\cdot x-1)}{(x+2)\cdot(x-3)\cdot(x+3)} \text{ expand} \rightarrow -\frac{2\cdot x^2 - 3\cdot x + 1}{9\cdot x - 2\cdot x^2 - x^3 + 18}$$

Used before in lesson 4:

$$\left[(x+1)^2 - 2\right]\left[(x-2)^2 + 5\right] \text{ expand} \rightarrow x^4 - 2\cdot x^3 + 22\cdot x - 9$$

$$x^4 - 2\cdot x^3 + 22\cdot x - 9 \text{ solve} \rightarrow \begin{pmatrix} \sqrt{2} - 1 \\ -\sqrt{2} - 1 \\ 2 + \sqrt{5}\cdot i \\ 2 - \sqrt{5}\cdot i \end{pmatrix}$$

Note that 5th order equations and higher do not have a general analytic solution.