hi 3002

**Common approximation for solids and liquids**

$$ {}_1Q_2 \approx m\, C_{(p)} (T_2 - T_1) $$

$C = C_p =$ specific heat at constant P

Example:

Engine $5\,^{o}C$
100kg cast iron
20kg aluminum
20kg steel
5kg oil
6kg antifreeze
= glycerine

$$\xrightarrow[7000\,kJ\ =\ {}_1Q_2]{\text{heat added}}$$

$T_2$ ?

$${}_1Q_2 = m_{iron}\, C_{p\,iron} (T_2 - T_1) + m_{al}\, C_{p\,al} (T_2 - T_1) + \dots$$

$${}_1Q_2 = 7000\ kJ$$

$$= [100\ kg\ 0.42 + 20\ 0.9 + 20\ 0.46 + 5\ 1.9 + 6\ 2.42] \frac{kJ}{kg\ K} (T_2 - T_1)$$

$$T_2 = T_1 + \underbrace{\frac{7000\ kJ}{[\ldots\ldots] \frac{kJ}{kg\ C}}}_{75^\circ C} = 80^\circ C$$

Ideal gasses:

Specific heat at constant pressure $C_p$

Specific heat at constant volume $C_v$

$\delta Q = m C_p dT$ if P is constant

${}_1Q_2 = \int_1^2 m C_p dT \rightarrow$

$\delta Q = m C_v dT$ if v is constant

${}_1Q_2 = \int_1^2 m C_v dT$

But if v is constant $\to$ v constant

$$U_2 - U_1 = {}_1Q_2 - \cancel{{}_1W_2}$$

$$U_2 - U_1 = \int_1^2 m\, C_v\, dT$$

$$U_2 - U_1 = m\, C_v\, (T_2 - T_1) \text{ if } C_v \text{ constant}$$

true for any process

$$H_2 - H_1 = {}_1Q_2 \text{ if isobaric}$$

$$H_2 - H_1 = m\, C_p\, (T_2 - T_1) \text{ if } C_p \text{ constant}$$

$$H = U + PV$$

Specific quantities

$$\boxed{\begin{aligned} u_2 - u_1 &= c_v (T_2 - T_1) \\ h_2 - h_1 &= c_p (T_2 - T_1) \end{aligned}} \quad \boxed{\begin{aligned} \left(\frac{\partial u}{\partial T}\right)_v &= c_v \\ \left(\frac{\partial h}{\partial T}\right)_p &= c_p \end{aligned}}$$

For ideal gases

$$\boxed{c_p - c_v = R} \qquad \bar{c}_p - \bar{c}_v = \bar{R}$$

Also $u = u(T)$ $h = h(T)$ $c_v = c_v(T)$

$c_p = c_p(T)$

$u, h$ in tables A·7·1 and A8

If not in these tables use $u_2 - u_1 = \ldots$

$$\left.\begin{aligned} u_2 - u_1 &= c_v (T_2 - T_1) \\ h_2 - h_1 &= c_p (T_2 - T_1) \end{aligned}\right\}$$

accurate for noble gasses,

For other ideal gasses:

$$u_2 - u_1 = \int_1^2 c_v(T)\, dT$$

$$h_2 - h_1 = \int_1^2 c_p(T)\, dT$$

Approximate

$(T_2+T_1)/2$

$$u_2 - u_1 = c_v(T_{ave})(T_2 - T_1)$$

$$h_2 - h_1 = c_p(T_{ave})(T_2 - T_1)$$

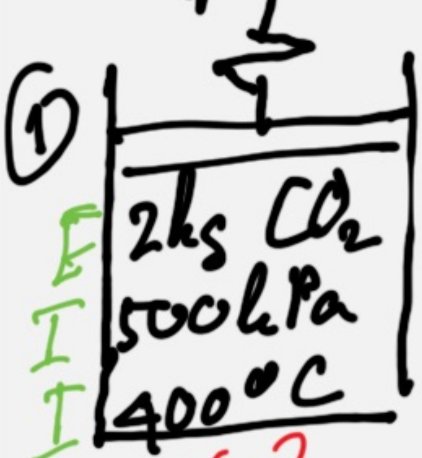
$25°C$

Or even worse

$$u_2 - u_1 = c_{v_{A\cdot5}}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p_{A\cdot5}}(T_2 - T_1)$$

Example



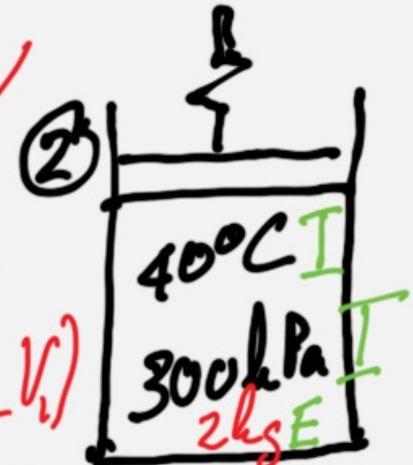

$V_1 = ?$

P linear in V

cool

$${}_1W_2 = \frac{P_2+P_1}{2}(V_2-V_1)$$

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2$$




$V_2 = ?$

Asked: ${}_1Q_2$

Most accurate only! Nope!

Solution

→ A.8 → must use

$P_1V_1 = mRT_1$

$R$ → A.5 0.1889 kJ/kgK

$T_1$ → 673.15 K

$m$ → 2 kg

$P_1$ → 500 kPa

→ $V_1 = 0.5087\ m^3$

Same way $V_2 = 0.3942$

$$ {}_1W_2 = \frac{P_1 + P_2}{2}(V_2 - V_1) = \frac{500 + 300}{2}\ kPa\ (0.3942 - 0.5087)\ m^3 $$

$$ = -45.72\ kJ $$

400°C

$$ {}_1Q_2 = U_2 - U_1 + {}_1W_2 $$

$$ = m(u_2 - u_1) - 45.72\ kJ $$

2kg 40°C

For $u_1$ $\quad g = T = 673.15\ K \quad g_1 = 650 \quad g_2 = 700$

$d = u \quad d_1 = 457.71 \quad d_2 = 483.94$

$$ d = u_1 = d_1 + \frac{g - g_1}{g_2 - g_1}(d_2 - d_1) $$

$u_1 = 459 \text{ kJ/kg}$ same way $u_2 = 166.6 \text{ kJ/kg}$

$\xrightarrow{} 2\,kg$

${}_1Q_2 = m(u_2 - u_1) - 45.72 \text{ kJ} = \underline{\underline{-630.5 \text{ kJ}}}$

Suppose $CO_2$ was not in A.8: $\overline{u_2 - u_1 = c_v(T_2 - T_1)}$

worst : use $c_v$ from table A.5 ($25^\circ C$).

${}_1Q_2 = \cancel{\;}\,0.2\,kg \; 0.653 \frac{kJ}{kg\,K} (40 - 400) K - 45.72 kJ$

${}_1Q_2 = -515.9 \text{ kJ}$ Bad

Better: use $u_2 - u_1 = c_v(T_{ave})(T_2 - T_1)$

$c_v(T_{ave}) = c_{v_{ave}}$ $\quad T_{ave} = \frac{T_1 + T_2}{2} =$

$T_{ave} = \frac{40 + 400}{2} = 220^\circ C = 493.15 K$

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$c_v = c_p - R \rightarrow 0.1889 \frac{kJ}{kg K}$

Table A:6

$\theta = \frac{T_{ave}}{1000 K} = \frac{493.15 K}{1000 K}$

$= 0.493$

$c_{p0} = c_p = 0.45 + 1.67 \, 0.493$

$- 1.27 (0.493)^2 + 0.39 (0.493)^3$

$C_0$, $C_1$, $C_2$, $C_3$

$= 1.01136 \, kJ/kgK$

$c_v = c_p - R = (1.01136 - 0.1889) \, kJ/kgK = 0.8225 \, kJ/kgK \rightarrow \therefore Q = -6327.4 \, kJ$