1 Linear interpolation

1.1 Description

Linear interpolation is a way to fill in the "holes" in tables. As an example, if you want to find the saturated pressure of water at a temperature of 40°C you can look in Table B.1.1, (p.674), for 40°C in the first column. The corresponding desired pressure is then in the next column; in this case, 7.384 kPa. But what if you want to find the saturated pressure at 38°C instead of 40°C?

A temperature of 38°C is not in the table. You could of course just ignore the difference between 38°C and 40°C, and still take the saturated pressure to be 7.384 kPa. But that is not acceptable in this class; it is too inaccurate. To get an accurate value, you must use linear interpolation. (Though taking the closest value, 40°C, is of course better than nothing in case you forgot how to do linear interpolation during an exam.)

Let's introduce a few symbols. Let g be your given value, 38°C in this example. Let g_1 and g_2 be the two closest *approximations* to g in the table. A look at Table B.1.1 shows that the two closest values you can find in the table are 35°C and 40°C, so in our example $g_1 = 35$°C and $g_2 = 40$°C. (The desired value is in between those two, hence the "in" in "interpolation.")

Also, let d be our desired value, in our example the saturated pressure. Let d_1 and d_2 be the approximate desired values corresponding to g_1 and g_2. In our example, Table B.1.1 gives the saturated pressure at g_1 =35°C to be d_1 =5.628 kPa and the saturated pressure at g_2 =40°C to be d_2 =7.384 kPa. Both d_1 and d_2 are approximations to our desired pressure, but neither is accurate enough.

The formula for linear interpolation is:

$$\boxed{d = d_1 + \frac{g - g_1}{g_2 - g_1}(d_2 - d_1)}$$

So, in our example, the desired saturated pressure d at 38°C is:

$$d = 5.628 \text{ kPa} + \frac{38°\text{C} - 35°\text{C}}{40°\text{C} - 35°\text{C}}(7.384 - 5.628) \text{ kPa} = 6.682 \text{ kPa}$$

1.2 A nonsaturated example

You need two variables to read off the compressed liquid or superheated vapor tables. In the next example, we will find the specific volume of steam at a given temperature of 100°C and a given pressure of 20 kPa.

Steam (superheated water vapor) is found in Table B.1.3. We have no difficulty finding the given 100°C in that table, but we cannot find the given pressure of 20 kPa. The closest pressures in the table are 10 kPa and 50 Kpa.

So in the linear interpolation formula from the previous section,

$$\boxed{d = d_1 + \frac{g - g_1}{g_2 - g_1}(d_2 - d_1)}$$

we set the given value g equal to 20 kPa, and the closest table values g_1 and g_2 to 10 kPa and 50 kPa.

The desired quantity d is now the specific volume at 100°C and 20 kPa. We set the value d_1 to the specific volume at g_1 =10 kPa (and 100°C,) so d_1 = 17.19561 m^3/kg according to the table, and d_2 to the specific volume at g_2 =50 kPa (and 100°C,) so d_2 = 3.41833 m^3/kg.

Our formula then gives the specific volume at 20 kPa and 100°C as:

$$d = 17.19561 \text{ m}^3/\text{kg} + \frac{20 \text{ kPa} - 10 \text{ kPa}}{50 \text{ kPa} - 10 \text{ kPa}}(3.41833 - 17.19561) \text{ m}^3/\text{kg} = 13.75129 \text{ m}^3/\text{kg}$$

1.3 Other problems

You might ask what happens to the last example if neither the given pressure nor the given temperature is in the table. For example, to find the specific volume at 20 kPa and 110°C, neither 20 kpa nor 110°C are in Table B.1.4. I do not think we would do this to you during the exam. But the answer would be to do three linear interpolations: first interpolate a specific volume at 110°C and 10 kPa (fill in the 110°C "hole" in the 10 kPa data), next interpolate a specific volume at 110°C and 50 kPa (fill in the 110°C "hole" in the 50 kPa data), and finally linear interpolate those 110°C values in the same way as we did for 100°C in the previous section.

Another problem arises if you try to interpolate the specific volume of steam at 11 kPa and 50°C. You can use the B.1.3 entry for 50°C and g_1 =10 kPa, giving d_1 = 14.86920 m^3/kg. But unfortunately, the 50 kPa data start at 81.33°C; no 50°C steam at 50 kPa exists. The key to find a second table entry, to give you g_2 and d_2, is to recognize that superheated steam ends at saturation, which is in table B.1.1. You can find the desired second table entry there; in particular, B.1.1 at 50°C gives a second pressure g_2 =12.350 and specific volume d_2 =12.0318. Which means that the formula

$$\boxed{d = d_1 + \frac{g - g_1}{g_2 - g_1}(d_2 - d_1)}$$

gives the specific volume of steam at 11 kPa and 50°C as:

$$d = 14.86920 \text{ m}^3/\text{kg} + \frac{11 \text{ kPa} - 10 \text{ kPa}}{12.350 \text{ kPa} - 10 \text{ kPa}}(12.0318 - 14.86920) \text{ m}^3/\text{kg} = 13.6618 \text{ m}^3/\text{kg}$$