EML 4930/5061	**Analysis in M.E. II**	**10/26/16**
Closed book	*Van Dommelen*	*9:35-10:25 am*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. Given the three vectors

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad \vec{c} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$$

find

(a) The area of the parallelogram with sides $\vec{a}$ and $\vec{b}$.

(b) The volume of the parallelepiped with sides $\vec{a}$, $\vec{b}$, and $\vec{c}$.

(c) The angle that the diagonal from the origin to the opposite corner of the parallelepiped makes with side $\vec{a}$.

2. Given the electric field

$$\vec{E} = \left[x + 2y + 3z + zx^2 + z^3 + xz\sin(xyz)\right]\hat{\imath} + \left[2x + 2y + 2z - yz\sin(xyz)\right]\hat{\jmath} + \left[5x + 2y + z - x^3 - xz^2\right]\hat{k}$$

find the flux $\int \vec{E}\cdot\vec{n}\,\mathrm{d}S$ through the surface whose top half is given by the conical surface fragment

$$x^2 + y^2 = (2-z)^2 \quad \text{for} \quad 0 \le z \le 2$$

and whose bottom half is given by the half spherical surface

$$x^2 + y^2 + z^2 = 4 \quad \text{for} \quad -2 \le z \le 0$$

3. Consider the PDE

$$3u_{xx} + 3u_{yy} - u_{zz} - 2u_{xz} + 4u_{yz} + u_z + 2u + 3 = 0$$

Classify this PDE. Get rid of the inhomogeneous term(s) by defining a new variable. Then reduce it to n-dimensional canonical form by rotating the coordinate system to a new coordinate system (x', y', z'). Give the axial unit vectors $\hat{\imath}'$, $\hat{\jmath}'$, and $\hat{k}'$ of the new coordinate system. List the transformations to go from original coordinates and variable to final coordinates and variable and vice-versa. Make sure the PDE is fully in terms of the new coordinates and variable.