EML 4930/5061	**Analysis in M.E. II**	**10/30/15**
Closed book	*Van Dommelen*	*9:35-10:25 pm*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. The height of the ground above sea level is given as

$$h(x,y) = \sin(x)\sin(2y)$$

where x and y are the horizontal coordinates.

- For an arbitrary point (x,y), find the two-dimensional unit vector $\vec{n}_\mathrm{h}$ in the direction in which height increases at the highest rate.
- What is the rate of increase of h with horizontal distance in that direction?
- Also find a unit vector $\vec{n}_\mathrm{c}$ in the direction in which h does not change at the point.
- Returning to the direction of highest increase, find the ratio $\mathrm{d}y/\mathrm{d}x$ for which h increases at the highest rate.
- Integrate the ordinary differential equation of the previous question to find an expression for the path along which h is always increasing at the highest rate.

2. A three-dimensional force field is given by

$$\vec{F} = \begin{pmatrix} 2x\sin(x^2+y) + e^{x^2+z^2} + y - z \\ \sin(x^2+y) + e^{y^2+z^2} + z - x \\ x - y \end{pmatrix}$$

Find the work $\oint \vec{F}\cdot \mathrm{d}\vec{r}$ done by this force over the closed contour in the x,y-plane given by:

- From the origin (0,0,0) to the point (2,2,0) along the quarter circle with radius 2 and center (0,2,0).
- From the point (2,2,0) to the point (0,4,0) along a straight line.
- From the point (0,4,0) back to the origin (0,0,0) along the y-axis.

Hint: convert the integral.

3. Consider the PDE

$$e^y u_{xx} + e^x u_{xy} = (e^x + e^y) u_x$$

Using characteristics coordinates, reduce this equation to something like $u_{\xi\eta} = u_\xi$ depending on details. Use that equation to find the general solution of the PDE, in terms of the original coordinates x and y.