EML 4930/5061	**Analysis in M.E. II**	**03/04/15**
Closed book	*Van Dommelen*	*12:30-1:20 pm*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. A surface is given by
$$x^2 + y^2 + 8z^2 = 4$$
   - What sort of surface is this?
   - Show that the point P with position vector $(1, 1, \frac{1}{2})$ is on the surface.
   - Find the equation for the plane that is tangential to the surface at P.
   - Find the vectorial description for the line through P normal to the surface at P, in terms of a parameter.
   - A ping-pong ball with velocity $2\hat{\imath}$ hits the surface at P and bounces off according to an elastic (Snell) reflection. What is the velocity of the bounced ball?

   Vector calculus only, absolutely no trig.

2. For the same surface as the previous question
   - Write $\vec{n}\,\mathrm{d}S$ in terms of Cartesian coordinates.
   - If a velocity field is given as
   $$\vec{v} = \hat{\imath}xz + \hat{\jmath}z^3 + \hat{k}x$$
   write the volumetric flow rate $\int \vec{v} \cdot \vec{n}\,\mathrm{d}S$ through the part of the surface with $z > 0$ using your obtained expression.
   - Convert to polar and integrate.

3. Consider the PDE
$$x^2 u_{xx} - 2xyu_{xy} + 2y^2 u_{yy} + xu_x + 2yu_y = \ln(x)$$
   What is the equation type? Using class procedures, convert this equation to 2D canonical coordinates. Take the straighforward choice for these coordinates, do not try to be smart. You can assume that x and y are positive. You should get a very simple equation for the transformed PDE. Find a particular solution u_p this way. In terms of the original coordinates, of course. What equation does the remainder u_h of u satisfy? What is the name of that simple equation?