EML 4930/5930	**Analysis in M.E. II**	**04/24/07**
Closed book	*Van Dommelen*	*3:00-5:00 pm*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. Solve the first order partial differential equation and initial condition:

$$yu_x - xu_y = yu \qquad u(x,0) = \sin(x)$$

2. Find the numerical value of u(0.3,3) for acoustics in a pipe of length one with open ends. The partial differential equation is

$$u_{tt} = u_{xx}$$

with boundary conditions and initial conditions

$$u(0,t) = u(1,t) = 0 \qquad u(x,0) = x \quad u_t(x,0) = 0$$

3. Solve the following problem using the method of separation of variable. The partial differential equation is

$$u_{tt} = u_{xx}$$

with boundary conditions and initial conditions

$$u(0,t) = 2 \quad u(1,t) = 1 \qquad u(x,0) = 2 \quad u_t(x,0) = 0$$

4. Solve the following problem of a viscous flow next to a plate. The partial differential equation is:

$$u_t = u_{xx}$$

and the initial and boundary conditions are:

$$u(x,0) = 0 \qquad u(0,t) = \frac{1}{\sqrt{t}}$$

Table 6.3: Properties of the Laplace Transform

$f(t)$	$\hat{f}(s) = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$
1. $C_1 f_1(t) + C_2 f_2(t)$	$C_1\hat{f}_1(s) + C_2\hat{f}_2(s)$
2. $f(at)$	$a^{-1}\hat{f}(s/a) \qquad (a > 0)$
3. $f^{(n)}(t)$	$s^n\hat{f}(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0) \qquad (n = 1, 2, \ldots)$
4. $t^n f(t)$	$(-1)^n\hat{f}^{(n)}(s) \qquad (n = 1, 2, \ldots)$
5. $e^{ct}f(t)$	$\hat{f}(s-c) \qquad (s = \text{const.})$
6. $H(t-b)f(t-b)$, where $H(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$	$e^{-bs}\hat{f}(s) \qquad (b > 0)$
7. $f * g(t) \equiv \int_0^t f(t-\tau)g(\tau)\,\mathrm{d}\tau$	$\hat{f}(s)\hat{g}(s)$

Table 6.4: Laplace Transform Pairs

$f(t)$	$\hat{f}(s) = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$
1. 1	$\dfrac{1}{s}$
2. t^n	$\dfrac{n!}{s^{n+1}} \qquad (n = 1, 2, \ldots)$
3. e^{kt}	$\dfrac{1}{s-k}$
4. $\sin(at)$	$\dfrac{a}{s^2+a^2}$
5. $\cos(at)$	$\dfrac{s}{s^2+a^2}$
6. $\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}}$
7. $\dfrac{1}{\sqrt{\pi t}}e^{-k^2/4t}$	$\dfrac{1}{\sqrt{s}}e^{-k\sqrt{s}} \qquad (k > 0)$
8. $\dfrac{k}{\sqrt{4\pi t^3}}e^{-k^2/4t}$	$e^{-k\sqrt{s}} \qquad (k > 0)$
9. $\operatorname{erfc}\left(k/2\sqrt{t}\right)$, where $\operatorname{erfc}(z) \equiv \dfrac{2}{\sqrt{\pi}}\int_z^\infty e^{-u^2}\,\mathrm{d}u$	$\dfrac{1}{s}e^{-k\sqrt{s}} \qquad (k > 0)$