EML 5060 | **Analysis in Mechanical Engineering** | **11/17/17**
Closed book | *Van Dommelen* | *12:30-1:20 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. *Show all reasoning.* Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question. Answer what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

You *must* use the systematic procedures described in class, not mess around randomly until you get some answer. Echelon form is defined as in the lecture notes, not as in the book. Eigenvalues must be found using minors only. Eigenvectors must be found by identifying the basis vectors of the appropriate null space. Eigenvectors of symmetric matrices must be orthonormal. If there is a quick way to do something, you must use it.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** In finite volume fluid computations, the flow field may be divided into simple volumes, and you then need to find expressions for the surfaces of these volumes and the volumes.

   Question: Assume that point A has Cartesian coordinates (1,2,3), B (4,4,4), C (3,3,5), and D (2,3,5). Find the area of the parallelogram that has AB and AC as sides. Also find a vector of unit length normal to that parallelogram. Also find the volume of the parallelepiped with sides AB, AC, and AD.

2. **Background:** The eigenvalues of the 4×4 Dirac matrix

$$\begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & p & 0 \\ 0 & p & -1 & 0 \\ p & 0 & 0 & -1 \end{pmatrix}$$

   describe the possible energies of an electron moving in free space, where p is the scaled linear momentum of the electron. (Here p is real, not complex.)

   Question: Find the four eigenvalues of the matrix above. Do not use any Gaussian elimination steps. Note: the quartic is easy to factor, and in any case it can be written as a quadratic in terms of λ^2. Without solving for the eigenvectors, what can you say about the diagonalizability of the Dirac matrix and why?

3. **Background:** Many problems in physics require analysis of quadratic forms. Linear algebra is the general way to do so. Here we will restrict ourselves to a two-dimensional form, though in real life, the number of dimensions might be enormous.

   Question: Analyze the quadratic form

$$x^2 - 12xy + 17y^2 = -2$$

   using class procedures. Based on that, very neatly draw the solution curve(s) in the x, y-plane. In particular, show the principal axes, the unit vectors along the principal axes, principal axes intercepts, if any, asymptotes, if any, and all angles involved as accurately as reasonably possible, listing their exact and approximate values.