EML 5060 | **Analysis in Mechanical Engineering** | **9/25/15**
Closed book | *Van Dommelen* | *12:30-1:20 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. *Show all reasoning.* Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question. Answer exactly what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** Graphical depiction of a function is often an essential part to understand its properties.

   Question: Analyze and very neatly graph

   $$y = \sqrt{(x-1)(x+2)} \qquad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{-9}{4\left(\sqrt{(x-1)(x+2)}\right)^3}$$

   Discuss x and y intercepts and extents, asymptotic behavior for large positive x and large negative x (each separately!), horizontal, oblique and vertical asymptotes, symmetries, local and global maxima and minima, concavity, inflection points, kinks, cusps, horizontal and vertical slopes and other singularities. Draw the function very neatly, clearly showing all features.

2. **Background:** Not all functions can be integrated analytically. Approximation may be needed

   Question: Find

   $$\int_0^{1/2} \sqrt{1+x^3}\,\mathrm{d}x$$

   by writing a *five* term Taylor series, but without evaluating the actual numbers. Then sum this five term series term by term, (evaluate term, add to total, repeat), *stopping all computations as soon as you know that your last computed partial sum has error less than 0.00001.* Explain how you know that your result is accurate to an error less than 0.00001. *Show the value of every term and the partial sums you compute.*

3. **Background:** Areas of plates are important for such diverse purposes as weight, cost, resistance, etcetera.

   Question: Consider the region R

   $$\text{inside the parabola } y^2 = 4x \qquad \text{above the line } y = x - 3$$

   Draw this region neatly. Now write out the integral

   $$\int_R f(x,y)\,\mathrm{d}x\mathrm{d}y$$

   completely, including limits of integration, for both the case in which you integrate x first and the one in which you integrate y first. Discuss which of the two approaches is simplest, and why. Finally find the area of the region by setting $f = 1$ in your expression and integrating using the simpler approach.