EML 5060 | **Analysis in Mechanical Engineering** | **11/01/13**
Closed book | *Van Dommelen* | *11:50-12:40 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. Show all reasoning. Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question if there is any ambiguity. Answer exactly what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

You *must* use the systematic procedures described in class, not mess around randomly until you get some answer. Eigenvalues must be found using minors only. Eigenvectors must be found by identifying the basis vectors of the appropriate null space if there are multiple eigenvalues. Eigenvectors to symmetric matrices must be orthonormal. If there is a quick way to do something, you must do it.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** *Typically*, if you use too few bars in a truss, it will not be able to support the required applied forces. If you use too many bars, there will typically be "residual" tension forces in the bars even if no forces are applied. These can be big and lead to unexpected failure.

   Question: A truss was designed to support the three externally applied force components F_{2x}, F_{2y}, and F_{3x}. The truss consists of four bars, under tension forces T_1, T_2, T_3, and T_4 respectively. These tension forces relate to the applied forces as

$$-\frac{1}{\sqrt{2}}T_2 - \frac{1}{\sqrt{2}}T_4 = F_{2x} \tag{1}$$

$$T_1 + \frac{1}{\sqrt{2}}T_2 + T_3 + \frac{1}{\sqrt{2}}T_4 = F_{2y} \tag{2}$$

$$\frac{1}{\sqrt{2}}T_2 + \frac{1}{\sqrt{2}}T_4 = F_{3x} \tag{3}$$

   What makes you suspect immediately that the designer may not come from a very good Engineering school? Write the system in a suitable matrix form. Solve it using class procedures and also find the null space. Based on the results, address what of the two potential problems mentioned above, if any, might affect this truss. What do you learn in more detail from the structure of the null space?

2. **Background:** For purposes such as analyzing natural frequencies, or in quantum mechanics, sometimes an analytical expression of a determinant is needed.

   Question: Find, without *any* row (or column) operations

$$\begin{vmatrix} 0 & 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 \\ 12 & 0 & 13 & 0 & 14 \\ 0 & 0 & 16 & 0 & 17 \end{vmatrix}$$

   At every stage, choose the approach that requires the smallest possible number of terms. If you do it correctly, it will be quick.

3. **Background:** If an appropriate streamfunction exists for a flow, then the streamlines in the vicinity of stagnation points can be found as lines on which a quadratic form is constant.

   Question: Using the class procedures for quadratic forms, find and very accurately and neatly draw the lines on which $2x^2 + 2\sqrt{3}xy = 3$. List all relevant angles in the picture to fully define it. Describe the points that come closest to the origin, and their distance from the origin.