Problem Sets –

1. **Transport lab – Wild Card Experiment –**

Problem Statement – Classify the following samples as Netwonian, pseusoplastic, dilatant and Bingham plastics using a viscometer. Treating the samples as power law fluids determine the power law coefficient and argument.

(a) Calibration Oil
(b) Salad dressing
(c) Corn starch in water

2. Problem Statement – Evaluation of creams (curve fitting, yield stress, shear thinning)

The flow behavior of systems can be characterized by understanding how the shear stress (τ) in the system changes as a function of applied shear rate ($\dot{\gamma}$). A generalized model, known as the Herschel Bulkley model is often used to relate these parameters and evaluate important constants such as the yield stress (τ_0), consistency index (k) and the power law index (n). The model is given as –

$$\tau = \tau_0 + k\dot{\gamma}^n$$

The data given in Table 1 represents the shear stress vs shear rate data for 5 samples of moisturizer.

(a) Fit the data sets to the Herschel Bulkley model and evaluate τ_0, k and n for each sample
(b) Which of the samples are suited for the application if it is required that the yield stress remain between 200-500 Pa?
(c) What does the power law exponent tell you about the flow behavior of the different samples? What values of n are suited for this application? Why?

Shear Rate ($\dot{\gamma}$)					
0.1	148.0	325.8	207.2	800.6	484.0
0.5	158.4	338.2	216.0	804.4	490.6
0.7	162.8	343.6	220.4	806.5	492.9
1.0	169.0	351.0	227.0	810.0	496.0
5.0	234.6	429.7	315.0	869.0	522.0
7.0	262.6	463.3	359.0	903.3	531.4
10.0	301.7	510.3	425.0	958.5	543.7
20.0	418.6	650.6	645.0	1164.1	576.5
50.0	715.6	1007.0	1305.0	1893.4	647.3
70.0	892.2	1218.8	1745.0	2437.3	684.7
100.0	1139.3	1515.3	2405.0	3311.9	733.6
120.0	1295.6	1702.9	2845.0	3926.2	762.9
150.0	1520.6	1972.9	3505.0	4886.1	803.4
170.0	1665.6	2146.9	3945.0	5548.3	828.6
200.0	1876.9	2400.4	4605.0	6570.8	864.4
220.0	2014.2	2565.2	5045.0	7270.0	887.0
250.0	2215.5	2806.8	5705.0	8342.7	919.4
270.0	2347.1	2964.7	6145.0	9072.5	940.2
300.0	2540.8	3197.2	6805.0	10187.4	970.2
320.0	2667.8	3349.6	7245.0	10943.3	989.6
350.0	2855.4	3574.7	7905.0	12094.9	1017.7
370.0	2978.7	3722.6	8345.0	12873.7	1036.0

400.0	3161.1	3941.5	9005.0	14057.8	1062.6
420.0	3281.2	4085.6	9445.0	14857.2	1079.9
450.0	3459.2	4299.2	10105.0	16070.6	1105.2
470.0	3576.6	4440.1	10545.0	16888.6	1121.8
500.0	3750.7	4649.1	11205.0	18128.6	1146.0

3. Flow of a Bingham Fluid in a Pipe (Transport Lab or Transport 1)

In a food processing unit manufacturing Ketchup, you are required to design a process to transport the fluid over 5ft. You have a 316 L Stainless Steel Pipe (2.469" ID) and a pump which can operate between 20 – 200 psi.

(a) Assuming steady state, unidirectional Laminar flow in the system, model the shear stress and velocity profile in the pipe. Ketchup behaves as a Bingham Fluid with viscosity given by the following relation –

$$\mu = \mu_0 + \frac{\tau_0}{\dot{\gamma}} \; at \; \tau > \tau_0$$

$$\mu = \infty \; at \; \tau < \tau_0$$

Here, τ_0 is the yield stress and $\dot{\gamma}$ is the applied shear rate. (Hint: $\tau = \mu\dot{\gamma}$ and $\dot{\gamma} = \frac{dv_r}{dz}$)
Derive an expression for the average velocity and the volumetric flow rate.

(b) Carry out a rotational viscometry experiment for Ketchup over a suitable range of shear rates to determine μ_0 and τ_0.

(c) Using the values of μ_0 and τ_0 from (b), calculate the pressure drop required to achieve a volumetric flow rate of 1 L/min.

4. A coal slurry is to be transported by horizontal pipeline. It has been determined that the slurry may be described by the power law model with a flow index of 0.4, an apparent viscosity of 50 cP at a shear rate of 100 /s, and a density of 90 lb/ft3. What horsepower would be required to pump the slurry at a rate of 900 GPM through an 8 in. Schedule 40 pipe that is 50 miles long?

5. Drilling mud has to be pumped down into an oil well that is 8000 ft deep. The mud is to be pumped at a rate of 50 GPM to the bottom of the well and back to the surface through a pipe having an effective diameter of 4 in. The pressure at the bottom of the well is 4500 psi. What pump head is required to pump the mud to the bottom of the drill string ? The drilling mud has the properties of a Bingham plastic with a yield stress of 100 dyn/cm^2, a limiting (plastic) viscosity of 35 cP, and a density of 1.2 g/cm3.

6. Consider the fully developed steady laminar circular pipe flow of an incompressible non-Newtonian fluid due to a constant pressure gradient. Gravitational effects maybe neglected. The normal stress in this fluid in the z-direction, i.e. σ_{zz}, is equal to – p where p is the pressure. The shear stress σ_{rz} is related to the velocity gradient by

$$\sigma_{rz} = C\left(-\frac{du_z}{dr}\right)^2$$

where C is a known constant

Find

1. The velocity profile $u_z(r)$ and

2. The friction factor, f, (i.e. the wall shear stress made dimensionless using the dynamic pressure based on the average velocity in the pipe) for this pipe flow in terms of C, ρ (fluid density), dp/dz, r and R (pipe radius) or a subset of these parameters.