Non-Newtonian Fluids

Non-Newtonian Flow

Goals

- Describe key differences between a Newtonian and non-Newtonian fluid
- Identify examples of Bingham plastics (BP) and power law (PL) fluids
- Write basic equations describing shear stress and velocities of non-Newtonian fluids
- Calculate frictional losses in a non-Newtonian flow system

Non-Newtonian Fluids

Newtonian Fluid

$$\tau_{rz} = -\mu \frac{du_z}{dr}$$

Non-Newtonian Fluid

$$\tau_{rz} = -\eta \frac{du_z}{dr}$$

η is the apparent viscosity and is not constant for non-Newtonian fluids.

η - Apparent Viscosity

The shear rate dependence of η categorizes non-Newtonian fluids into several types.

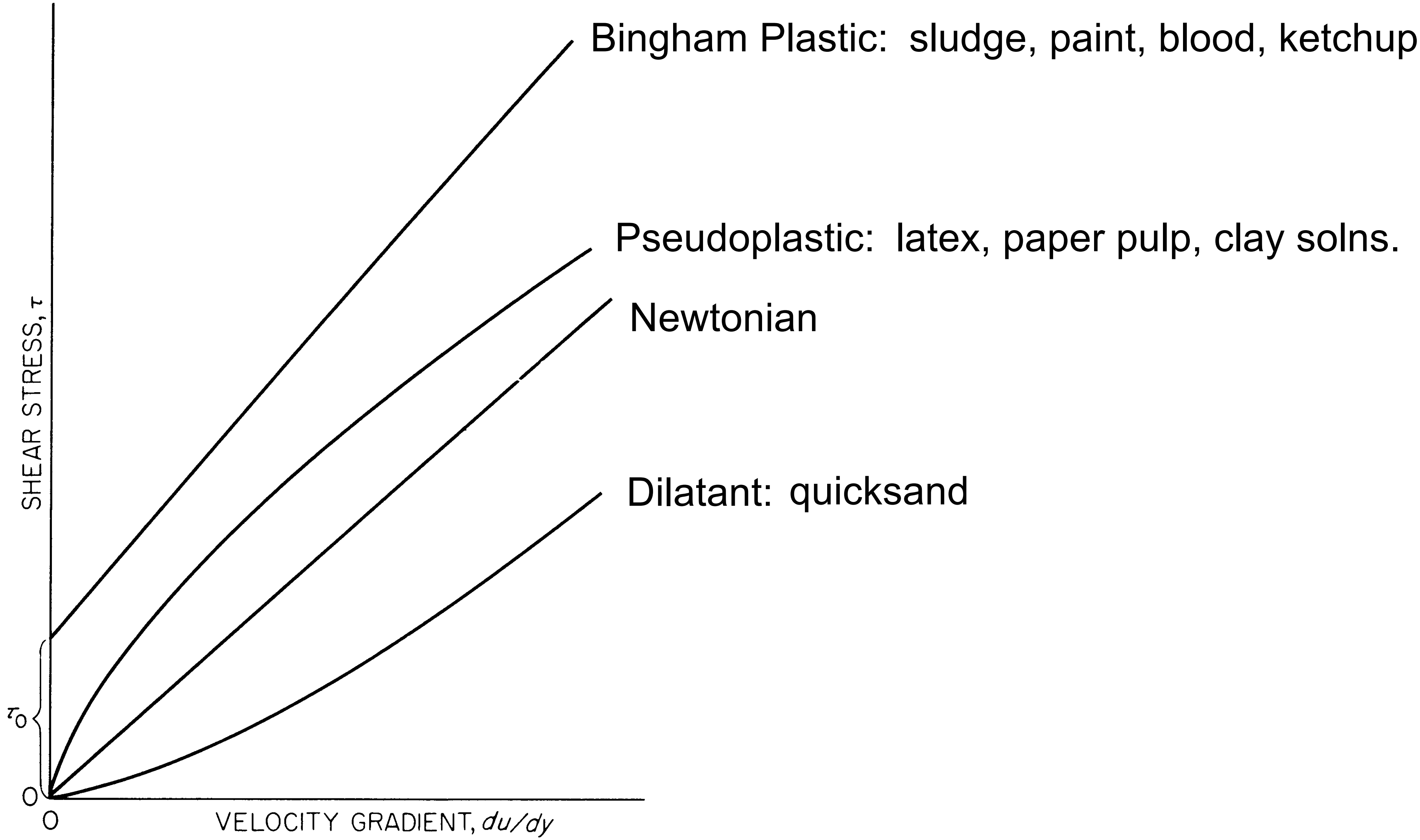
Power Law Fluids:

- Pseudoplastic – η (viscosity) decreases as shear rate increases (shear rate thinning)
- Dilatant – η (viscosity) increases as shear rate increases (shear rate thickening)

Bingham Plastics:

- η depends on a critical shear stress (τ_0) and then becomes constant

Non-Newtonian Fluids



Modeling Power Law Fluids

Oswald - de Waele

$$\tau_{rz} = K\left(-\frac{du_z}{dr}\right)^n = \underbrace{\boxed{\left[K\left(\frac{du_z}{dr}\right)^{n-1}\right]}}_{\mu_{eff}}\left(-\frac{du_z}{dr}\right)$$

where:

K = flow consistency index

n = flow behavior index

Note: Most non-Newtonian fluids are pseudoplastic n<1.

Modeling Bingham Plastics

$$\left|\tau_{rz}\right| < \tau_0 \qquad \frac{du_z}{dr} = 0 \qquad \text{Rigid}$$

$$\left|\tau_{rz}\right| \geq \tau_0 \qquad \tau_{rz} = -\mu_\infty \frac{du_z}{dr} \pm \tau_0$$

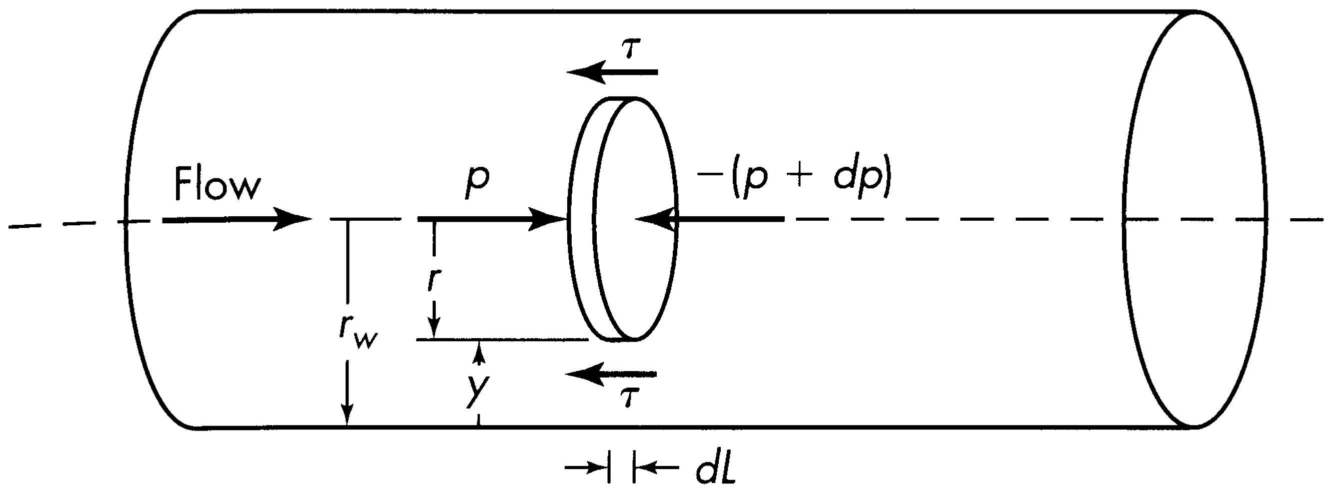
Frictional Losses
Non-Newtonian Fluids

Recall:

$$\boxed{h_f = 4f\frac{L}{D}\frac{\overline{V}^2}{2}}$$

Applies to any type of fluid under any flow conditions

Laminar Flow



Mechanical Energy Balance

$$\frac{\Delta p}{\rho}+\cancelto{0}{\frac{\Delta\alpha\overline{V}^{2}}{2}}+\cancelto{0}{g\Delta z}+h_f=\cancelto{0}{\hat{W}}$$

MEB (contd)

Combining:

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\frac{2}{\overline{V}^2}\left(-\frac{\Delta p}{\rho}\right)$$

Momentum Balance

$$\cancelto{0}{\dot{m}\left(\beta_2\overline{V}_2-\beta_1\overline{V}_1\right)}=p_1S_1-p_2S_2-F_w-\cancelto{0}{F_g}$$

$$2\pi rL\tau_{rz}=\pi r^2\left(-\Delta p\right)$$

$$-\Delta p=2\frac{L}{r}\tau_{rz}$$

Power Law Fluid

$$\tau_{rz} = K\left(-\frac{du_z}{dr}\right)^n$$

$$\frac{du_z}{dr} = -\left(-\frac{1}{2}\frac{\Delta p}{KL}\right)^{1/n} r^{1/n}$$

Boundary Condition

$$r = R \qquad u_z = 0$$

Velocity Profile of Power Law Fluid Circular Conduit

Upon Integration and Applying BC

$$u_z = \left(-\frac{1}{2}\frac{\Delta p}{KL} \right)^{1/n} \left(\frac{n}{n+1} \right) \left[R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}} \right]$$

Power Law (contd)

Need bulk average velocity

$$\bar{V} = \frac{1}{S}\int_S u\,dS = \frac{1}{\pi R^2}\int (2\pi r u_z)\,dr$$

$$\boxed{\bar{V} = \left[-\frac{1}{2}\frac{\Delta p}{KL}\right]^{1/n}\left(\frac{n}{3n+1}\right)R^{\frac{n+1}{n}}}$$

Power Law Results (Laminar Flow)

$$\Delta p = -\frac{2^{n+2}\left(\frac{3n+1}{n}\right)^{n} LK\overline{V}^{\,n}}{D^{n+1}}$$

↑ Hagen-Poiseuille (laminar Flow) for Power Law Fluid ↑

Recall

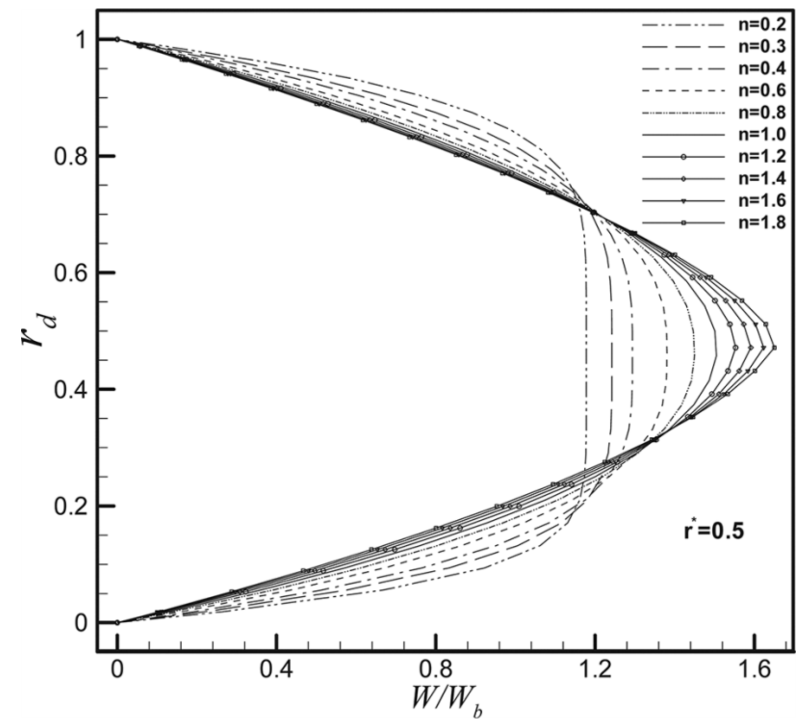
$$f = -\frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{2}{\overline{V}^{2}}\right)\frac{\Delta p}{\rho}$$

Power Law Fluid Behavior

Power Law Reynolds Number and Kinetic Energy Correction

$$Re_{PL} = 2^{3-n}\left(\frac{n}{3n+1}\right)^{n}\frac{\overline{V}^{\,2-n}D^{n}\rho}{K}$$

$$\mathrm{Re}_{PL,critical} = 2100\,\frac{(4n+2)(5n+3)}{3(3n+1)^{2}}$$



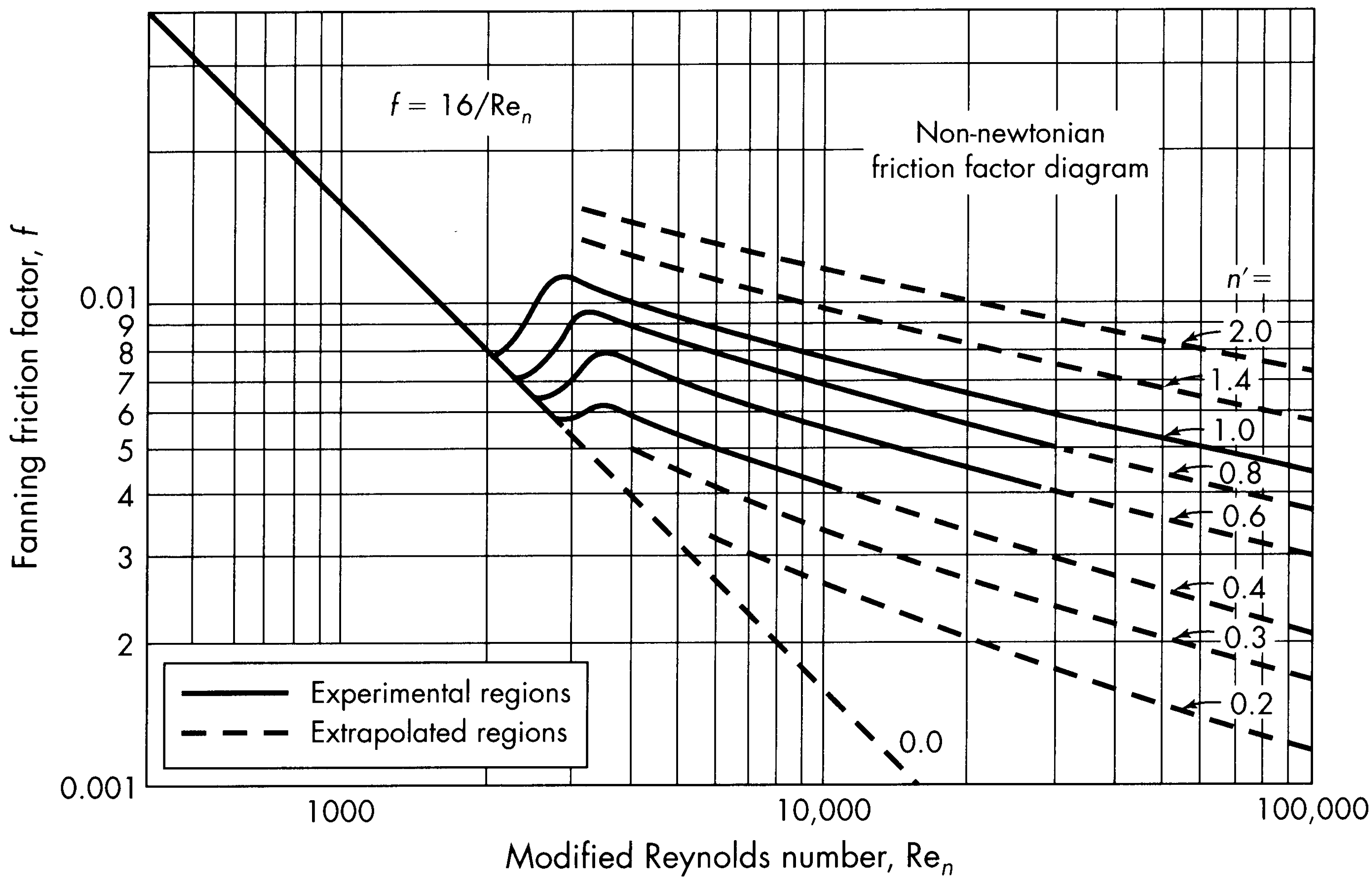
$$\alpha = \frac{3(3n+1)^{2}}{(2n+1)(5n+3)}$$

Laminar Flow Friction Factor
Power Law Fluid

$$f = \frac{2^{n+1}\left(\frac{3n+1}{n}\right)^{n} K}{\overline{V}^{\,2-n} D^{n} \rho}$$

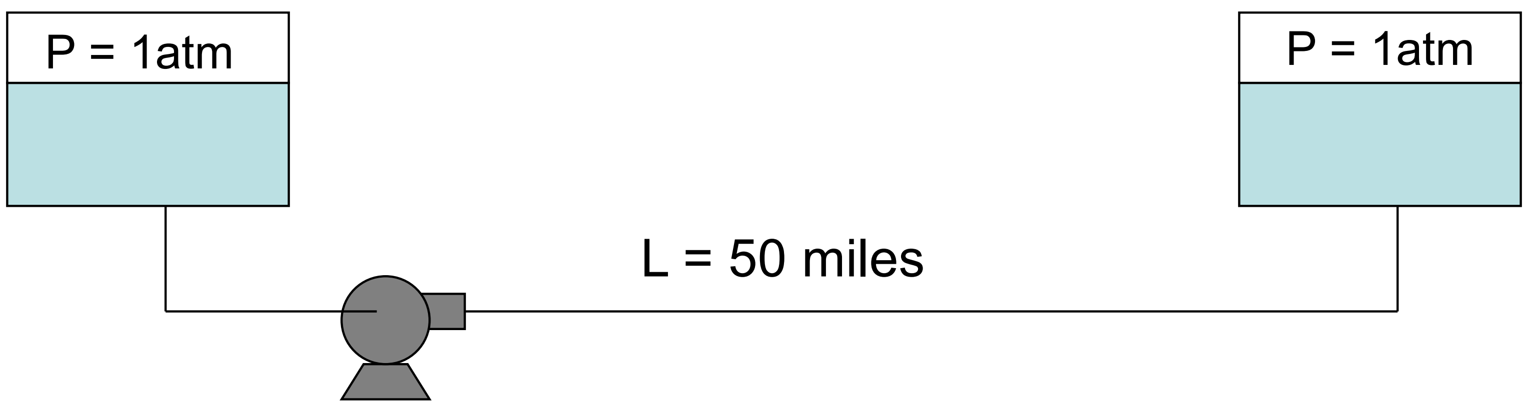
$$f = \frac{16}{Re_{PL}}$$

Turbulent Flow Friction Factor Power Law Fluid (Smooth Pipe)



Power Law Fluid Example

A coal slurry is to be transported by horizontal pipeline. It has been determined that the slurry may be described by the power law model with a flow index of 0.4, an apparent viscosity of 50 cP at a shear rate of 100 /s, and a density of 90 lb/ft3. What horsepower would be required to pump the slurry at a rate of 900 GPM through an 8 in. Schedule 40 pipe that is 50 miles long ?



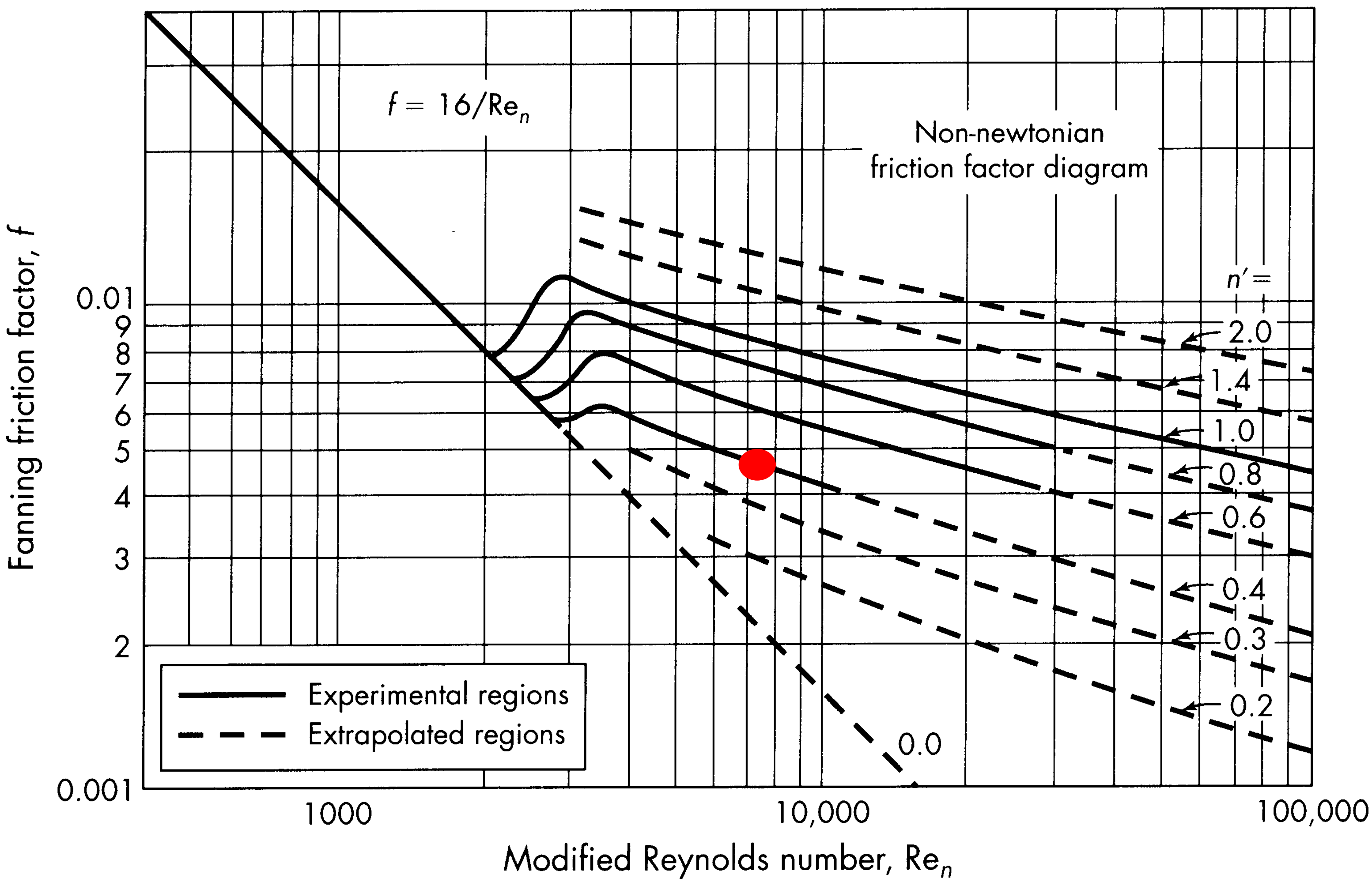
$$\tau_{rz} = K\left(\frac{\partial V}{\partial r}\right)^n = \mu_{eff}\left(\frac{\partial V}{\partial r}\right)$$

$$K = 50cP\left(\frac{100}{s}\right)^{1-0.4} = 0.792\frac{kg}{m\ s^{1.6}}$$

$$\tilde{V} = \left(\frac{900gal}{\min}\right)*\left(\frac{1ft^3}{7.48gal}\right)*\left(\frac{1\min}{60s}\right)*\left(\frac{1}{0.3474ft^2}\right)*\left(\frac{m}{3.281ft}\right) = 1.759\frac{m}{s}$$

$$RE_N = 2^{(3-0.4)}\left(\frac{0.4}{3*(0.4)+1}\right)^{0.4}\left[\frac{(0.202\ m)^{0.4}\left(1442\frac{kg}{m^3}\right)\left(1.759\frac{m}{s}\right)^{1.6}}{0.792\frac{kg}{m\ s^{1.6}}}\right] = 7273$$

Friction Factor (Power Law Fluid)



$$W_p = \frac{\Delta P}{\rho} + \frac{\Delta \alpha V^2}{2g_c} + \frac{g\Delta Z}{g_c} + h_f$$

$$W_p = h_f = 4f\left(\frac{L}{D}\right)\frac{V^2}{2}$$

$$f = 0.0048 \quad Fig\ 5.11$$

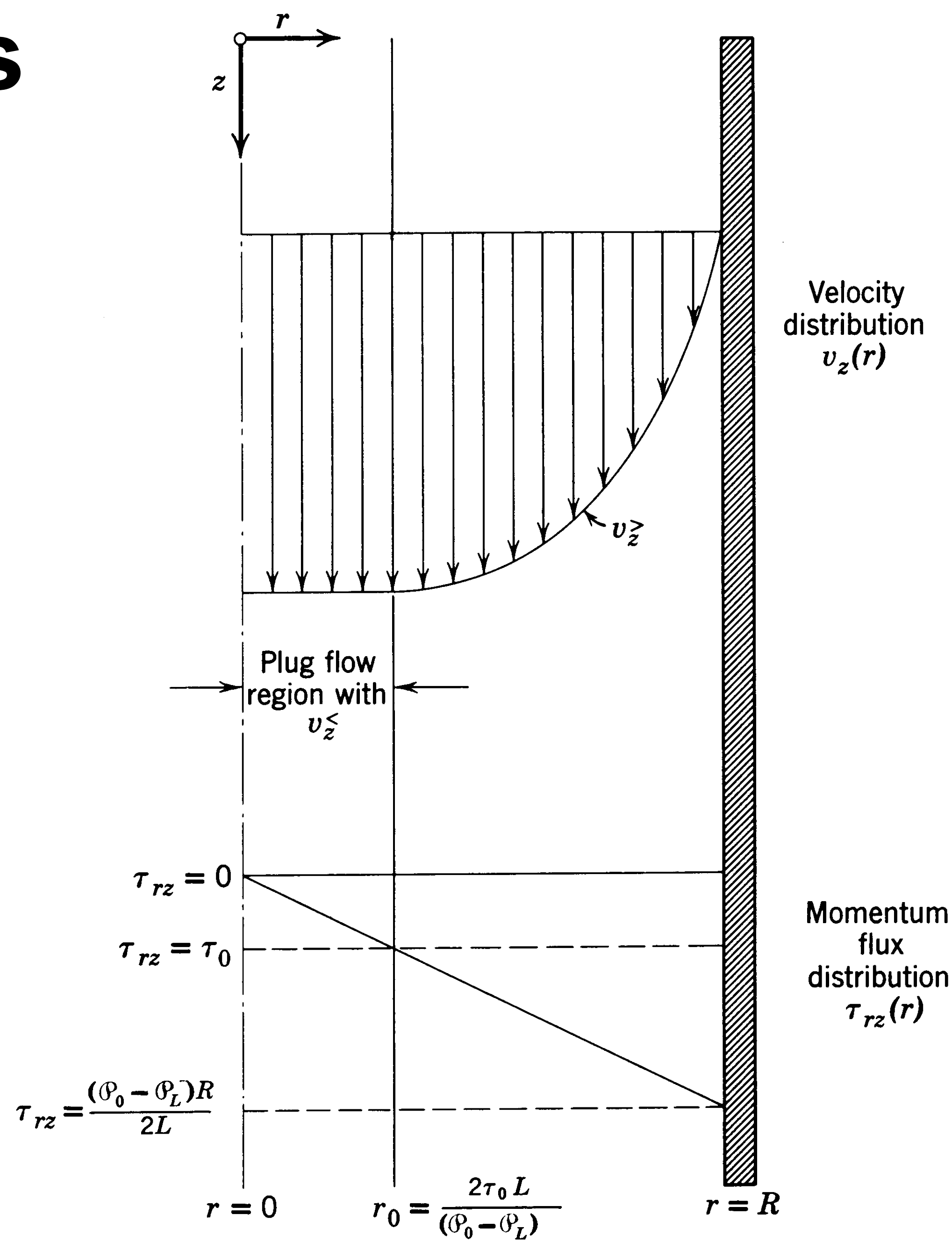
$$W_p = h_f = 4(0.0048)\left(\frac{80460m}{0.202m}\right)\frac{\left(1.760\frac{m}{s}\right)^2}{2} = 11{,}845\frac{m^2}{s^2}$$

$$\dot{m} = 1.759\frac{m}{s} * \left(0.0323\ m^2\right) * \left(1442\frac{kg}{m^3}\right) = 81.9\frac{kg}{s}$$

$$Power = \frac{81.9\frac{kg}{s}\left(11{,}845\frac{m^2}{s^2}\right)}{1000} = 970.1\ kW = 1300\ Hp$$

Bingham Plastics

Bingham plastics exhibit Newtonian behavior after the shear stress exceeds τ_o. For flow in circular conduits Bingham plastics behave in an interesting fashion.



Bingham Plastics

Unsheared Core

$$r \le r_c \qquad u_z = u_c = \frac{\tau_0}{2\mu_\infty r_c}\left(R - r_c\right)^2$$

Sheared Annular Region

$$r > r_c \qquad u_z = \frac{\left(R - r\right)}{\mu_\infty}\left[\frac{\tau_{rz}}{2}\left(1 + \frac{r}{R}\right) - \tau_0\right]$$

Laminar Bingham Plastic Flow

$$f = \frac{16}{\mathrm{Re}_{BP}}\left[1 + \frac{He}{6\,\mathrm{Re}_{BP}} - \frac{He^4}{3f^3\left(\mathrm{Re}_{BP}\right)^7}\right]$$ (Non-linear)

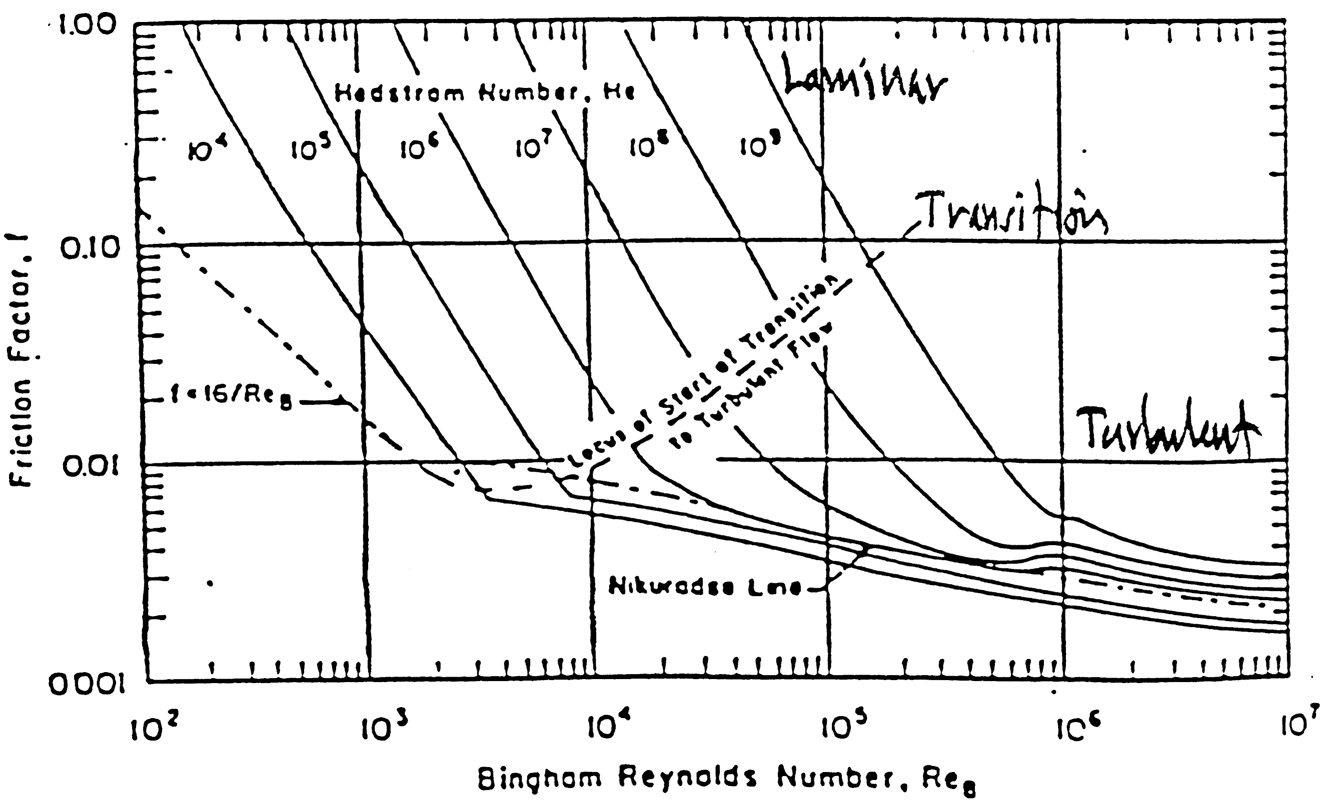
$$He = \frac{D^2\rho\tau_0}{\mu_\infty^2}$$ Hedstrom Number

$$\mathrm{Re}_{BP} = \frac{D\rho\overline{V}}{\mu_\infty}$$

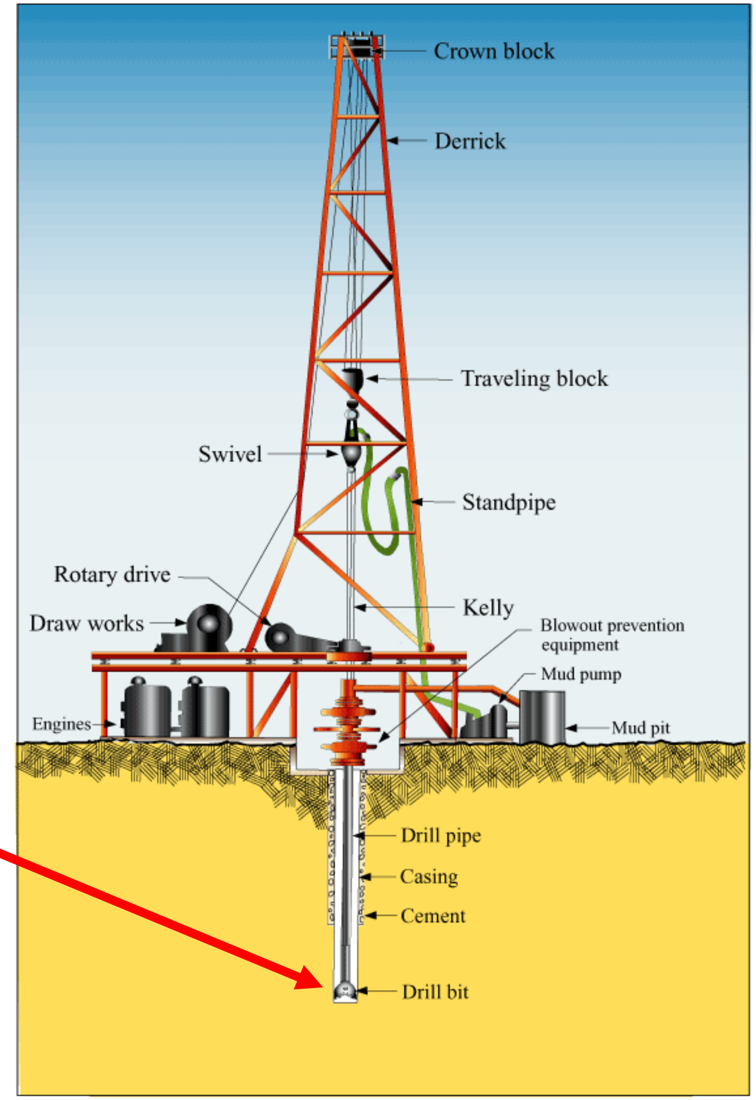
Turbulent Bingham Plastic Flow

$$f = 10^{a}\,\mathrm{Re}_{BP}^{-0.193}$$

$$a = -1.378\left(1 + 0.146e^{-2.9x10^{-5}\,He}\right)$$

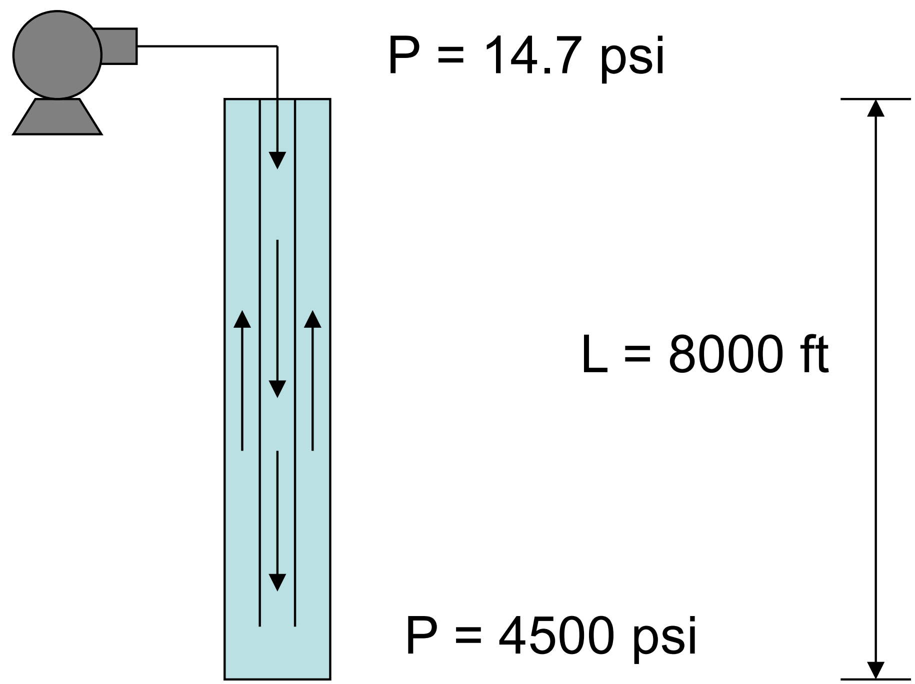


Drilling Rig Fundamentals



Bingham Plastic Example

Drilling mud has to be pumped down into an oil well that is 8000 ft deep. The mud is to be pumped at a rate of 50 GPM to the bottom of the well and back to the surface through a pipe having an effective diameter of 4 in. The pressure at the bottom of the well is 4500 psi. What pump head is required to pump the mud to the bottom of the drill string ? The drilling mud has the properties of a Bingham plastic with a yield stress of 100 dyn/cm^2, a limiting (plastic) viscosity of 35 cP, and a density of 1.2 g/cm3.



$$D = \frac{4}{12}\,ft = 0.3333\ ft \qquad Area = 0.0873\ ft^2$$

$$V = 50\,\frac{gal}{\min} * \left(\frac{\min}{60s}\right) * \left(\frac{ft^3}{7.48\,gal}\right) * \left(\frac{1}{0.0873\,ft^2}\right) = 1.276\,\frac{ft}{s}$$

$$\rho = 1.2 * 62.4\,\frac{lb_m}{ft^3} = 74.88\,\frac{lb_m}{ft^3}$$

$$\mu = 35\ cP * \left(\frac{6.7197 \times 10 - 4\,\frac{lb_m}{ft\ s}}{cP}\right) = 0.0235\,\frac{lb_m}{ft\ s}$$

$$N_{RE} = \frac{0.3333\,ft * \left(1.276\,\frac{ft}{s}\right) * \left(74.88\,\frac{lb_m}{ft3}\right)}{0.0235\,\frac{lb_m}{ft\ s}} = 1355$$

$$\tau_o = 100\,\frac{dyn}{cm^2} = 100\,\frac{g}{s^2 cm}$$

$$N_{HE} = \frac{\left(4in\left(\frac{2.54\ cm}{in}\right)\right)^2 * \left(1.2\frac{g}{cm^3}\right) * \left(\frac{100g}{s^2cm}\right)}{\left(0.35\frac{g}{cms}\right)^2} = 1.01\times10^5$$

$$f = 0.14$$

$$W_p = \frac{\Delta P}{\rho} + \frac{\Delta\alpha V^2}{2g_c} + \frac{g\Delta Z}{g_c} + h_f$$

$$W_p = \frac{(4500-14.7)\frac{lb_f}{in^2}\left(\frac{144\ in^2}{ft^2}\right)}{74.88\frac{lb_m}{ft^3}} - 8000\frac{ft\ lb_f}{lb_m} + \frac{4*0.14*(8000\,ft)}{0.3333\,ft}\left(\frac{\left(1.276\frac{ft}{s}\right)^2}{2*\left(\frac{32.2\,ft\ lbm}{lb_f s^2}\right)}\right)$$

$$Wp = (8626-8000+339) = 965\frac{ft\ lb_f}{lb_m}$$

$$f = \frac{16}{\mathrm{Re}_{BP}}\left[1 + \frac{He}{6\,\mathrm{Re}_{BP}} - \frac{He^4}{3 f^3 (\mathrm{Re}_{BP})^7}\right] = 0.14$$