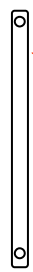
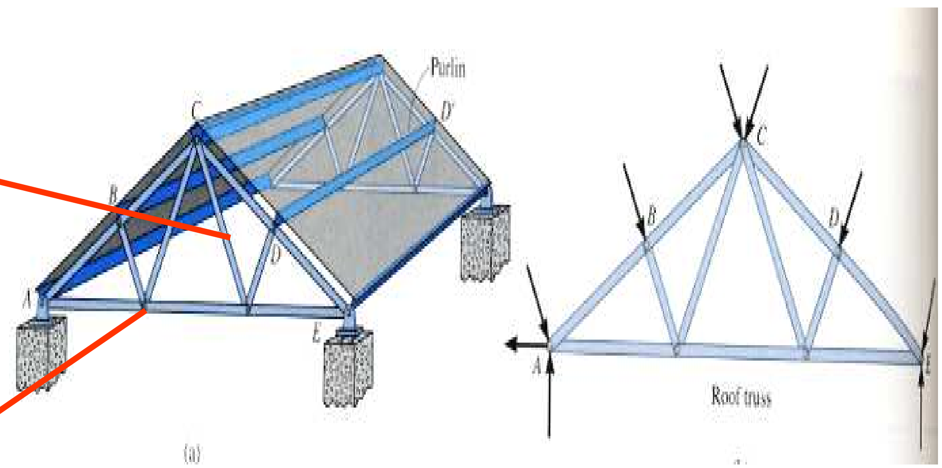
Mechanics & Materials I

Chapter 5
Structural Analysis

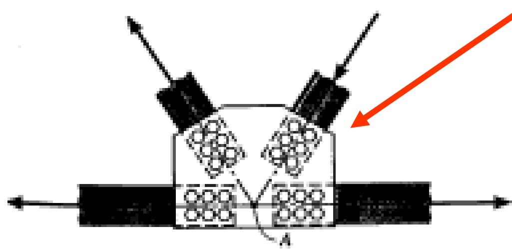
FAMU-FSU College of Engineering
Department of Mechanical Engineering

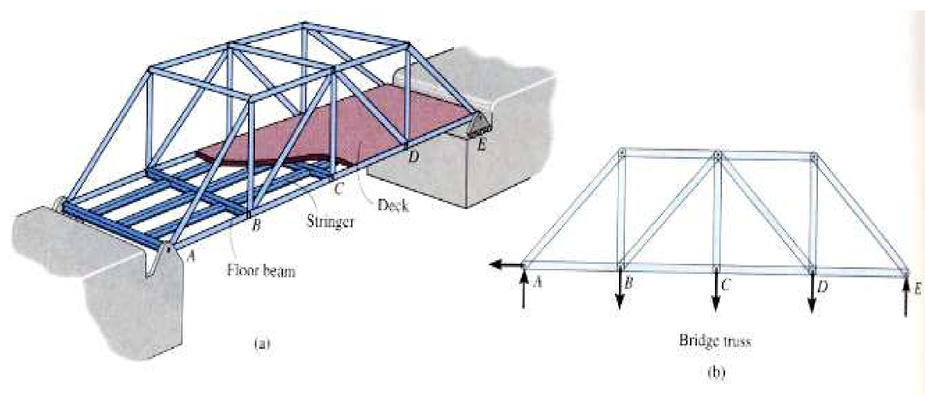
- Trusses
  - Slender members





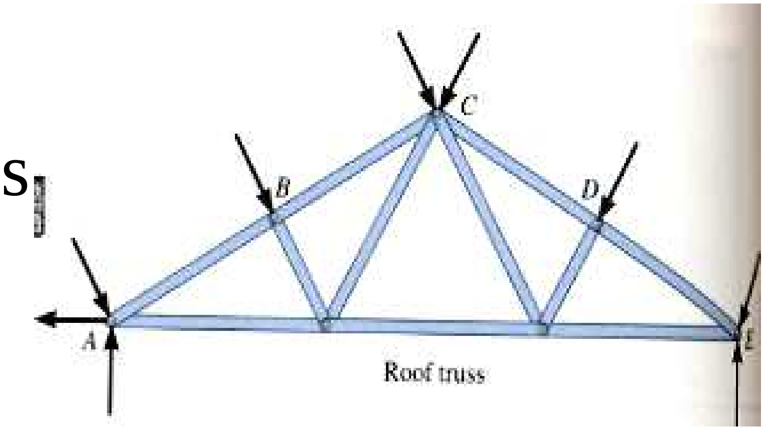
  - Joined at end points



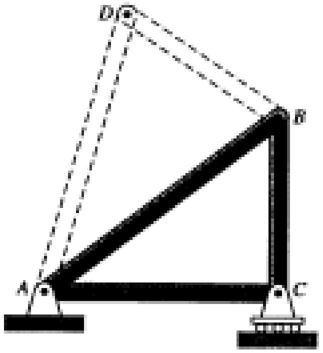


Trusses

- Planar Trusses
  - Single plane
  - Supports roofs and bridges

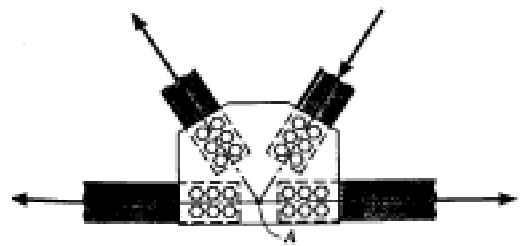


- Simple Truss
  - Basic Triangle element

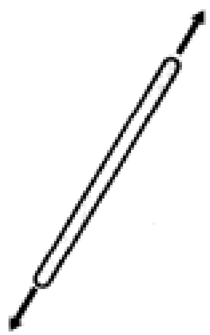


Trusses

- Assumptions for design
  - All loadings are applied at the joints.
    - neglect weights of the members
  - The member are joined together by smooth pins.
    - *concurrent* joining members

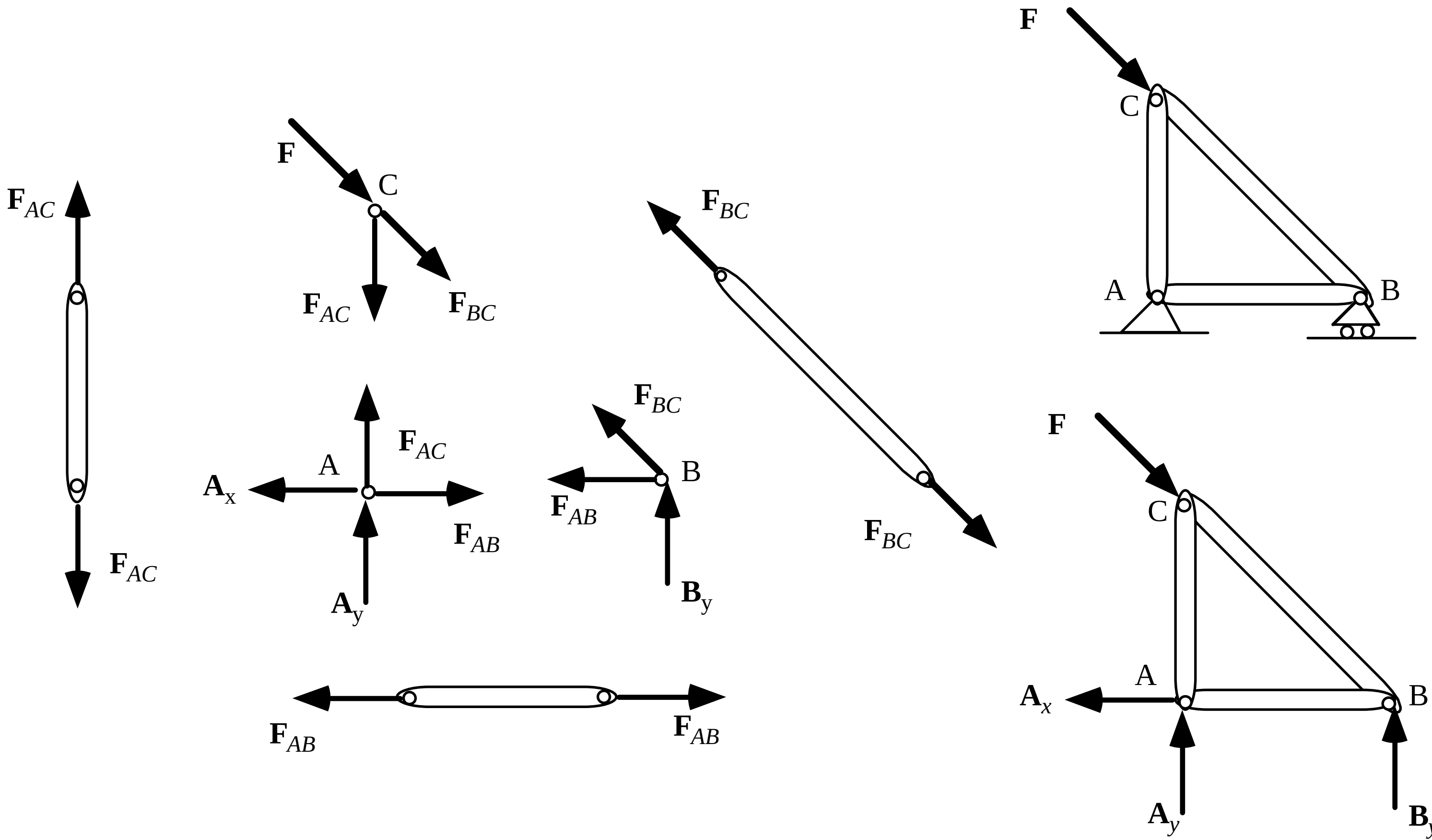


  - Two force members



Trusses

- Example -1: Free-body diagram



Trusses

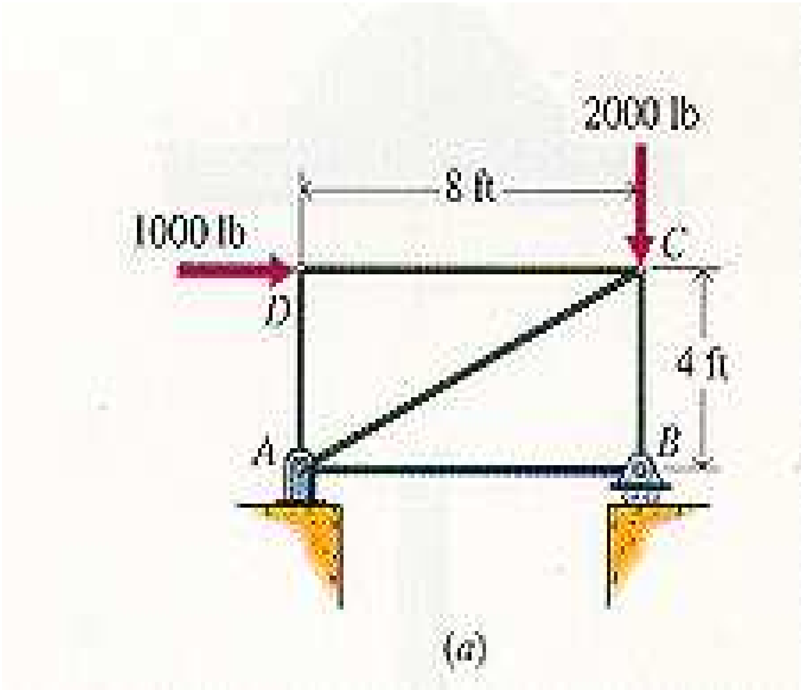
- Correct sense of an unknown member force
  - Always assume that the unknown member forces are in *tension*.
    - A positive answer indicates tension (T) and a negative answer indicates compression (C).
  - The correct sense of an unknown member force can in many cases be determined "by inspection."
    - A positive answer indicates that the sense is correct, whereas a negative answer indicates that the sense is wrong and must be reversed.

Method of Joints

- Draw the free-body diagram of the truss.
- Find the external reactions (if necessary).
- (Try to) Start with a joint having at least one known force and at most two unknown forces (why 2?).
- Draw the joints free-body diagram and find the unknown forces.
- Continue to analyze each of the other joints.

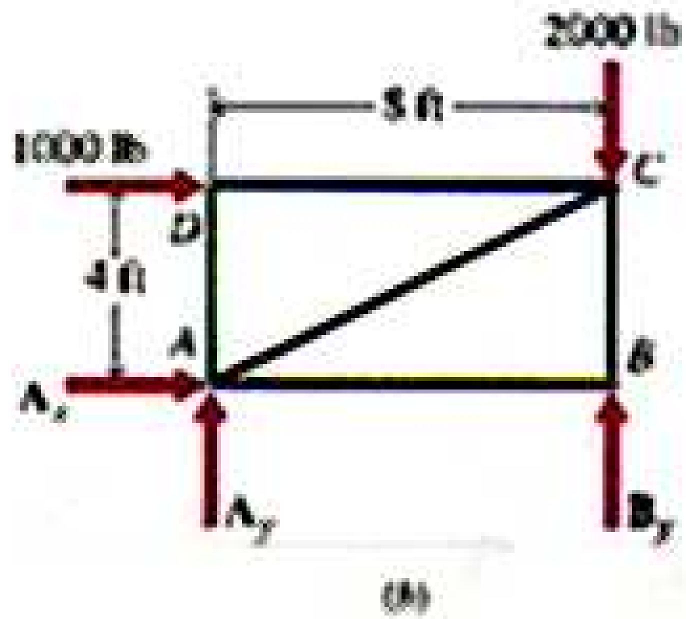
Example-1: Method of Joints

- Problem:
- Use the method of joints to find the force in each member of the truss at the shown figure.



Solution of Example-1

- First draw FBD of the entire truss and find the reactions from the equilibrium equations



$$\overset{+}{\rightarrow}\sum \overline{F}_x = 1000 + A_x = 0 \rightarrow A_x = -1000\ lb$$

$$+\uparrow \sum \overline{F}_y = -2000 + A_y + B_y = 0$$

$$\sum M_A = -(4)(1000) - 8(2000) + 8B_y = 0$$

$$B_y = -2500\ \text{lb}$$

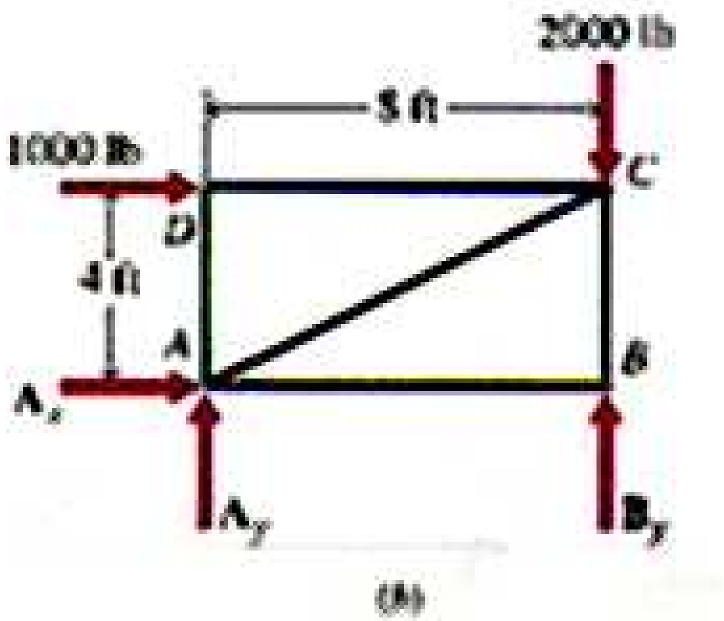
$$\text{A}_\text{y} = -500\ \text{lb}$$

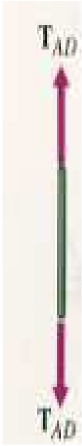
Solution Cont.,..

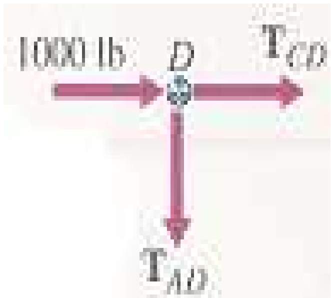
- FBD of pin D

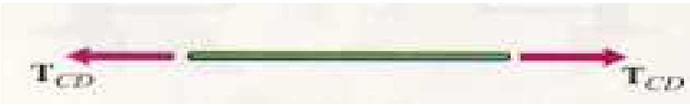
$$+\rightarrow \sum F = 1000 + T_{CD}$$

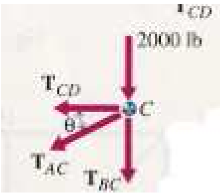
$$+\uparrow \sum F = -T_{AD} = 0$$











Solution cont.

- FBD of pin C

$$+\rightarrow \sum F = -T_{CD} - T_{AC}\cos\theta = 0$$

$$+\uparrow \sum F = -2000 - T_{CB} - T_{AC}\sin\theta = 0$$

where

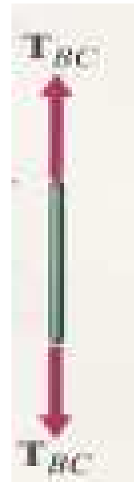
$$\sin\theta = \frac{AD}{AC} = \frac{8}{\sqrt{4^2+8^2}}$$

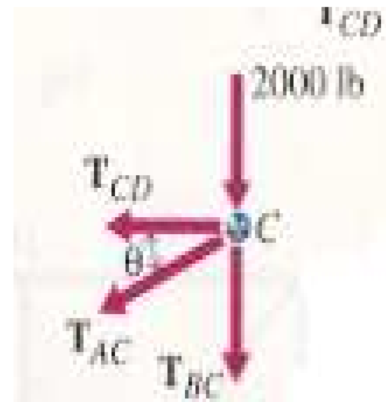
$$\cos\theta = \frac{CD}{AC} = \frac{4}{\sqrt{4^2+8^2}}$$

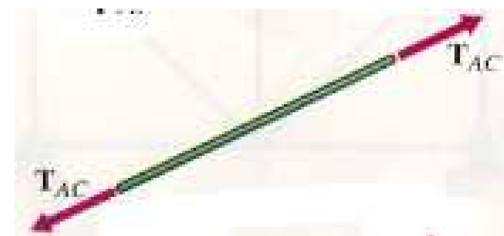
$$\text{knowing that } \mathrm{T_{cd}} = -1000 \quad \text{lb}$$

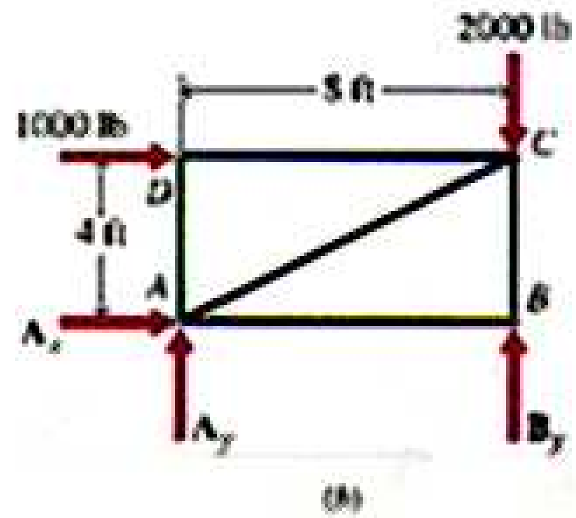
$$\mathrm{T_{AC}} = \frac{-T_{CD}}{\cos\theta} = 1118 \ \text{ lb}$$

$$\mathrm{T_{BC}} == -2000 - (1118)\cos\theta = -2500 \ \text{ lb}$$



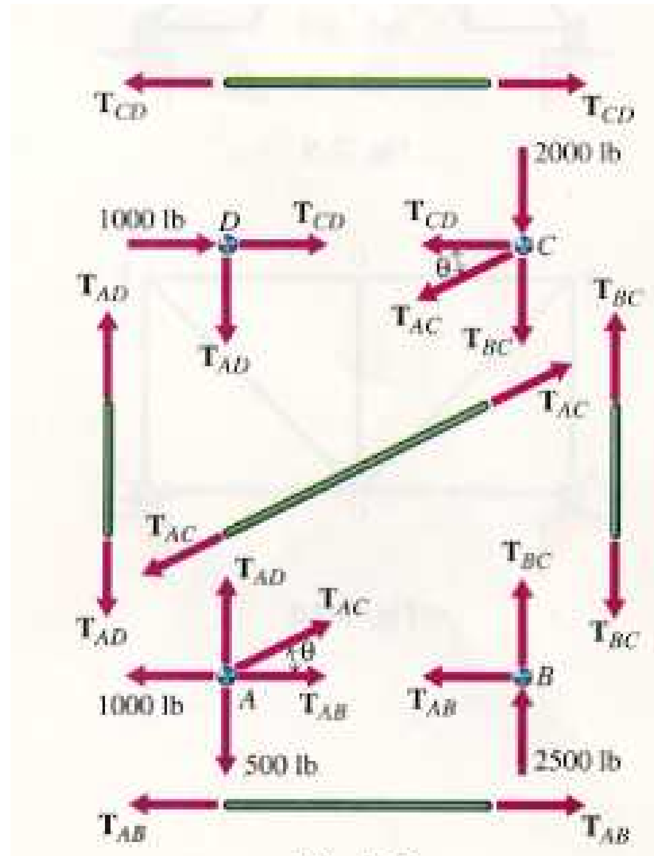






Solution cont.

- Following same analysis for pins B and A we get the following



$T_{AD} = T_{AB} = 0 \ \text{lb} \ (\text{Zero Force Member})$

$T_{AC} = 1118 \ \text{lb} \ (\text{Tension})$

$T_{BC} = 2500 \ \text{lb} \ (\text{Compression})$

$T_{CD} = 1000 \ \text{lb} \ (\text{Compression})$

Self-Practice !!

- Problem:
- The truss shown in the figure supports one side of a bridge,; an identical truss supports the other side. Floor beams carry vehicle loads to the truss joints. A 2000 kg car is stopped on the bridge. Calculate the forces in each member of the truss using the method of joints.

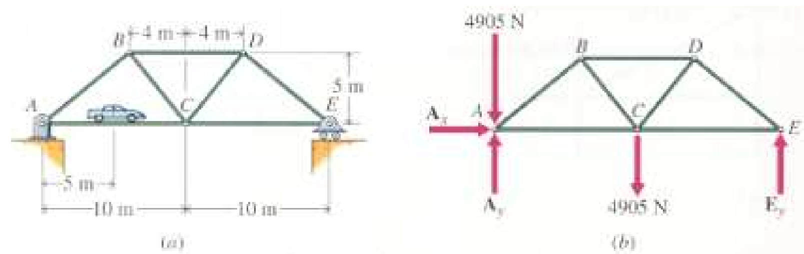
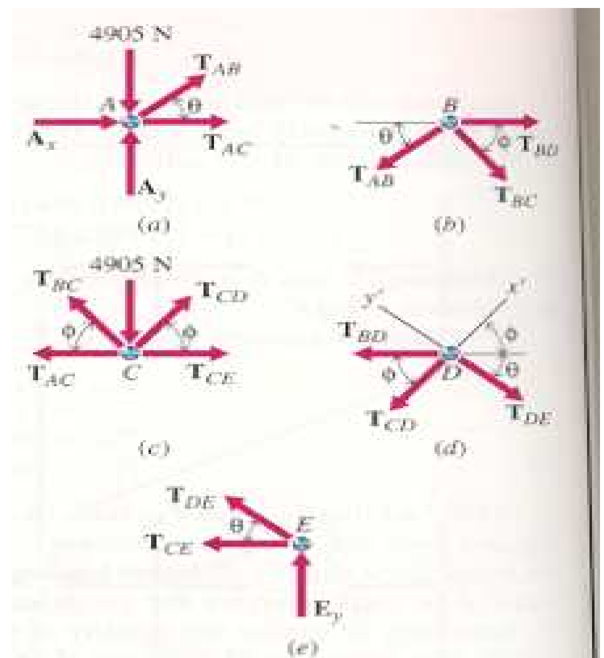


Fig. 7-13



- Answers:

$$T_{AB} = T_{DE} = 3830 \text{ N (Compression)}$$
$$T_{AC} = T_{CE} = 2942 \text{ N (Tension)}$$
$$T_{BC} = T_{CD} = 3140 \text{ N (Tension)}$$
$$T_{BD} = 4904 \text{ N (Compression)}$$

Zero-Force Members

- The zero force members are used to increase the stability of the truss during construction and to provide support if the applied loading is changed.
- There are two geometries that lead to zero force members – learn to look for and identify them readily!

Zero Force Members

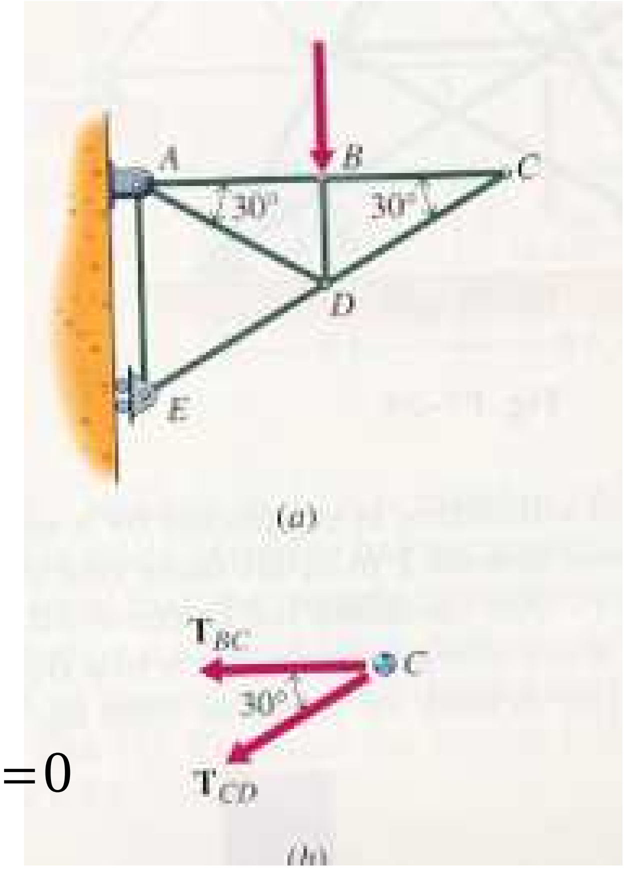
- Case-1:
- If only two members form a truss joint and no external load or support reaction is applied to the joint, the members must be zero-force members.

Example: Two Members

- Member BC and CD are two non-collinear members, they share joint C.
- There are no external forces or reactions applied to joint C
- Hence, members BC and CD are zero force members.
- Proof:

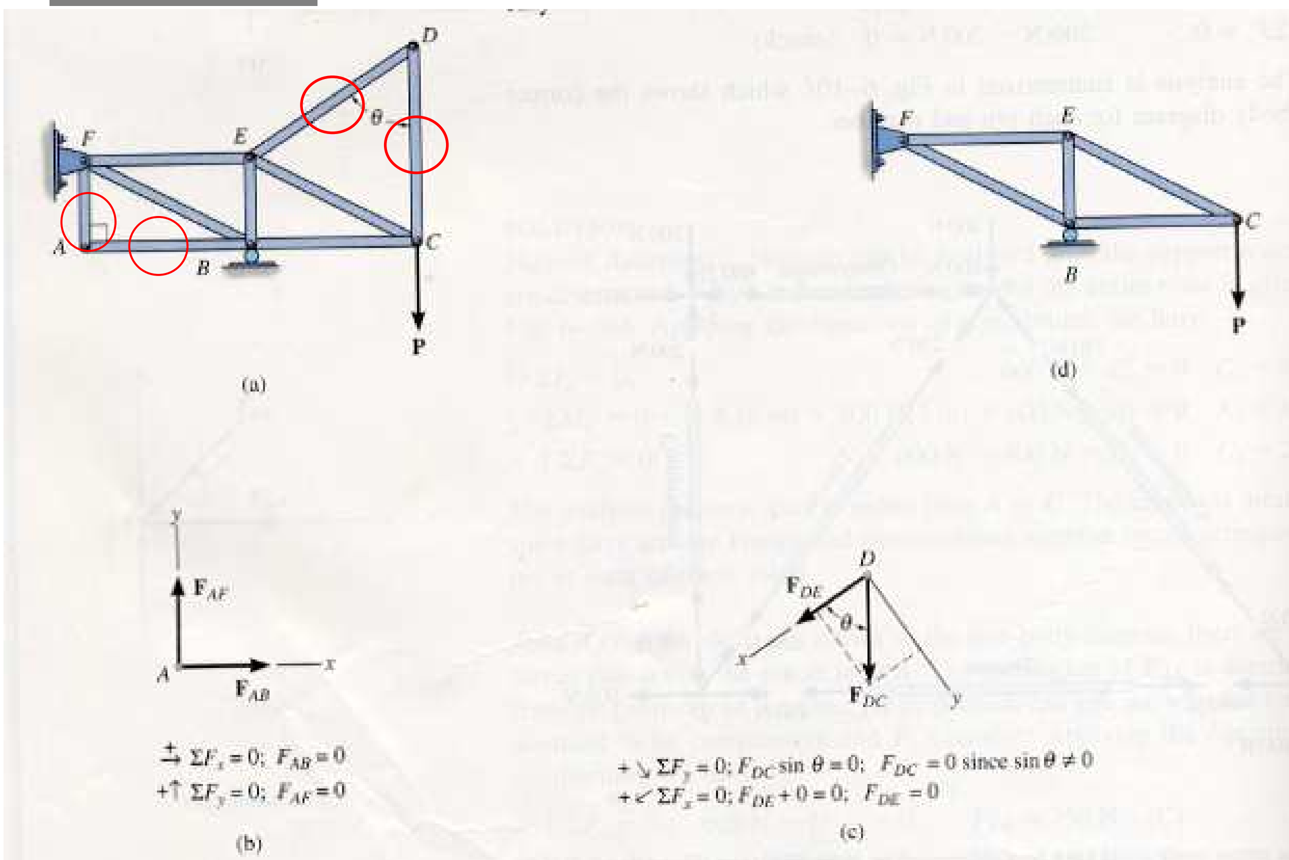
$$+\rightarrow \sum F = -T_{BC} - T_{CD}\cos 30^{o} = 0$$

$$\left.+\uparrow \sum F = -T_{CD}\sin 30^{o} = 0 \right\} \Rightarrow T_{CD} = 0 \text{ and } T_{BC} = 0$$



Initial Truss

After Eliminating the Zero Force Members



Zero-Force Members

- Case 2: If three members form a truss joint for which two of the members are collinear, and the third forms an angle with the first two , then the non-collinear member is a zero force member provided no external load or support reaction is applied to the joint. The two collinear members carry equal loads(either both in tension or both compression)

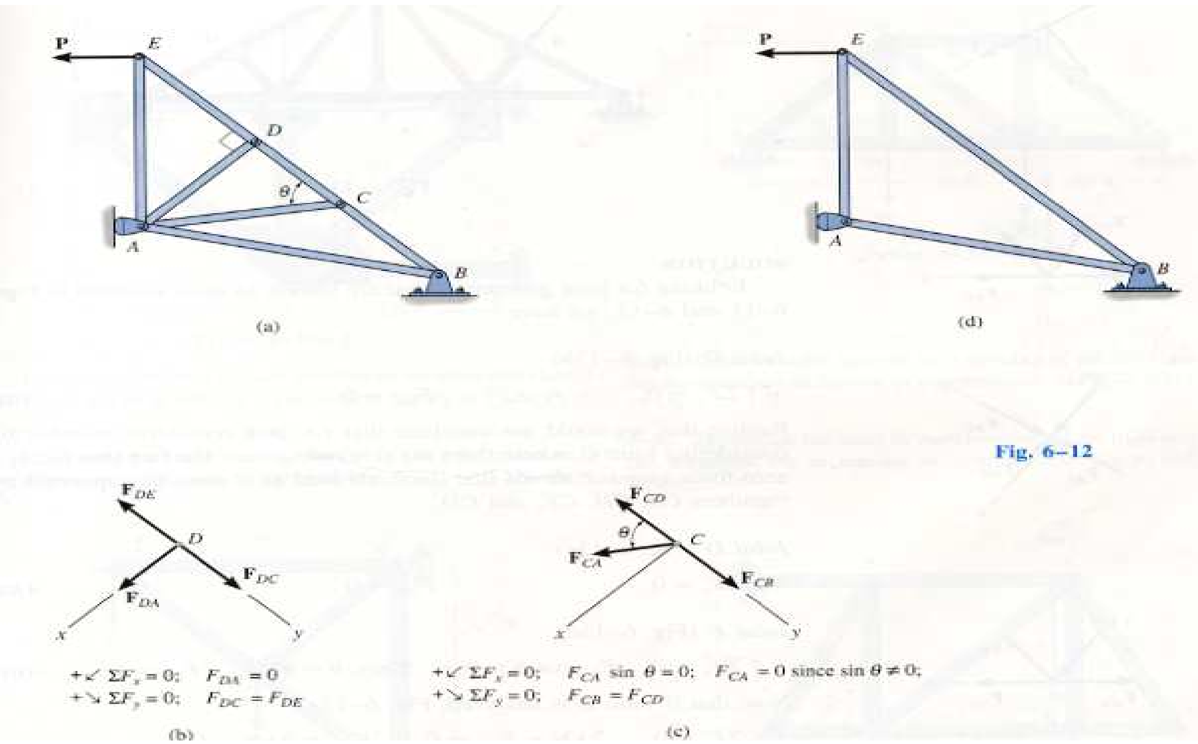


Fig. 6–12

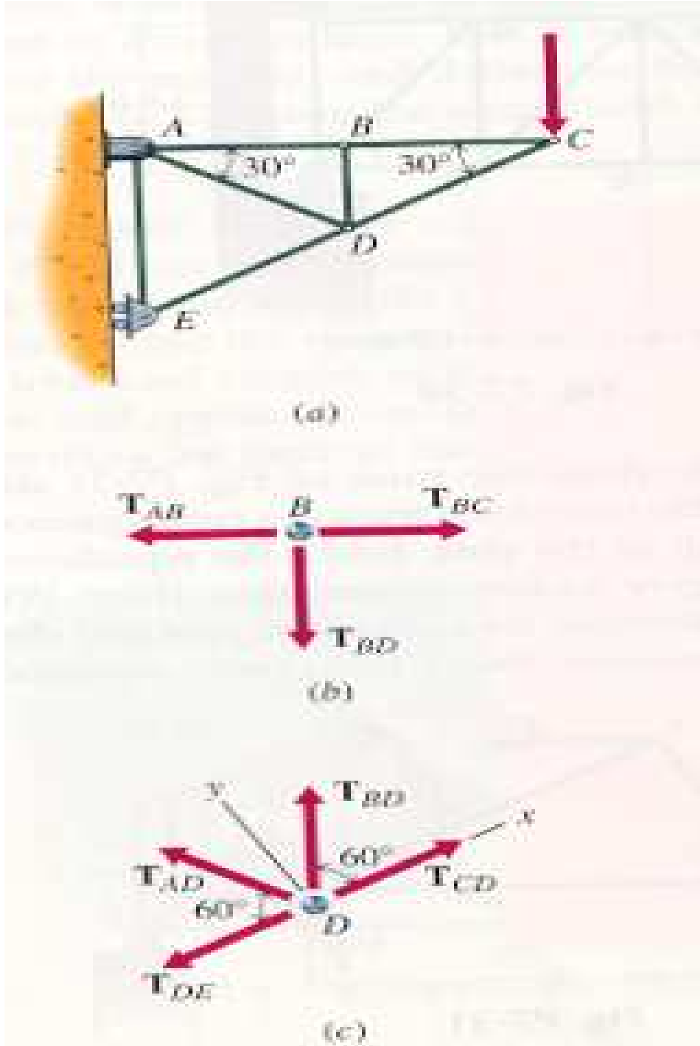
Example: 3 Members

- Joint B holds three members , and there is no external loads or reactions at the joint.
- Members AB and BC are collinear, while member BD makes an angle with the two collinear members. Hence,member BD is zero force member.
- Proof:

$$+ \rightarrow \sum F_x = -T_{AB} + T_{BC} = 0$$

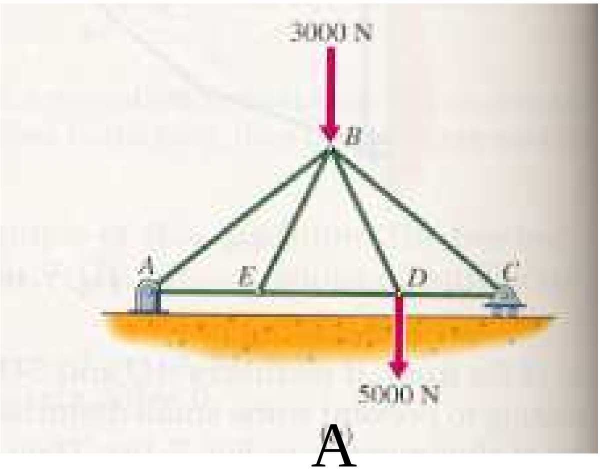
$$+ \uparrow \sum F_y = -T_{BD} = 0$$

- Question is AD a zero force member?

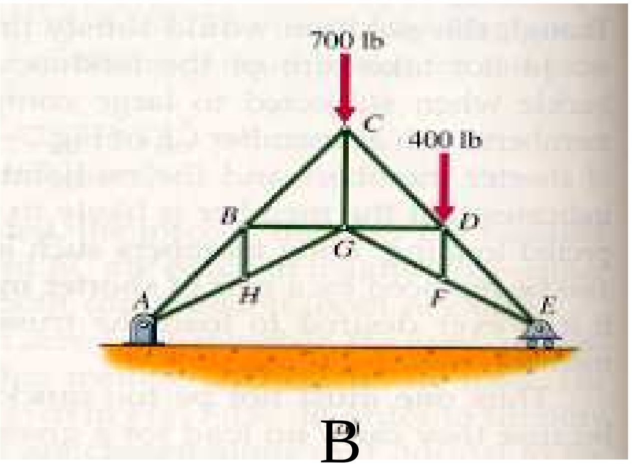


Practice !!

- Following the same analysis we used, find all the zero force members for each one of the following trusses.



A

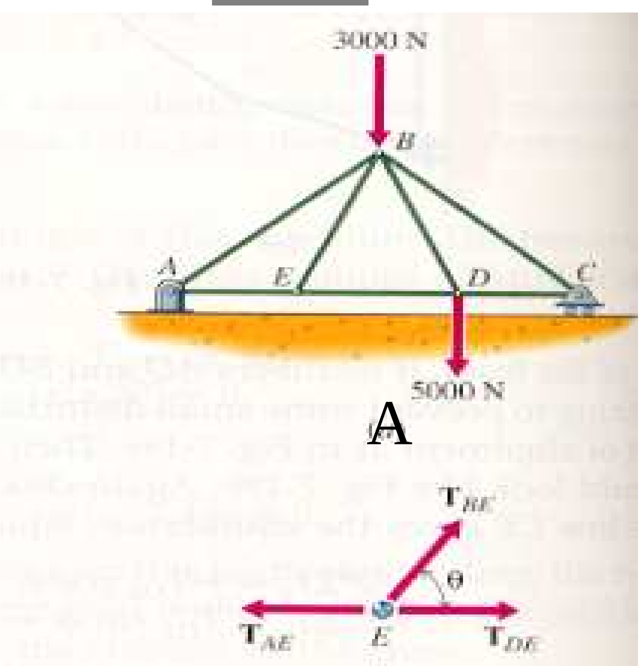


B

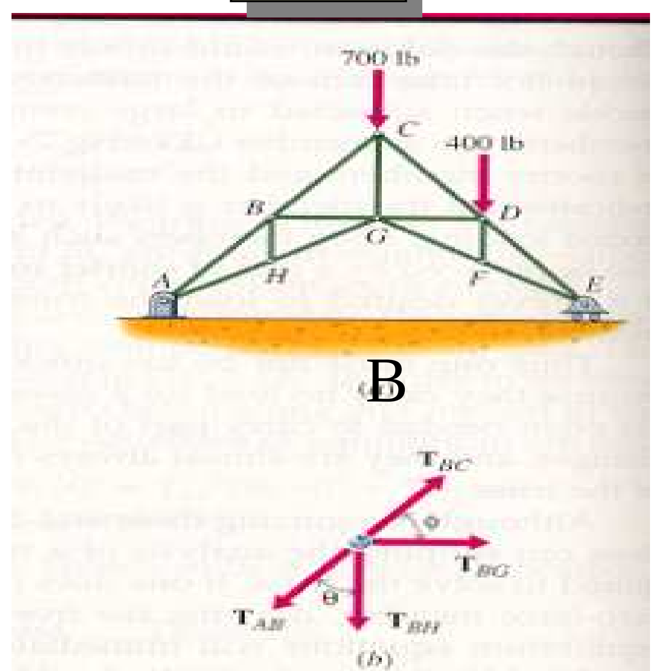
Answer to Practice

- (a) BE is a zero force member
- (b) BG and BH are zero force members

(A)

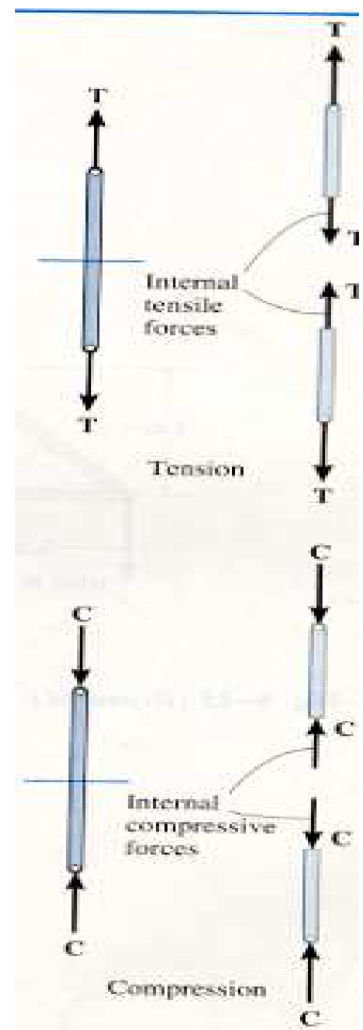


(B)



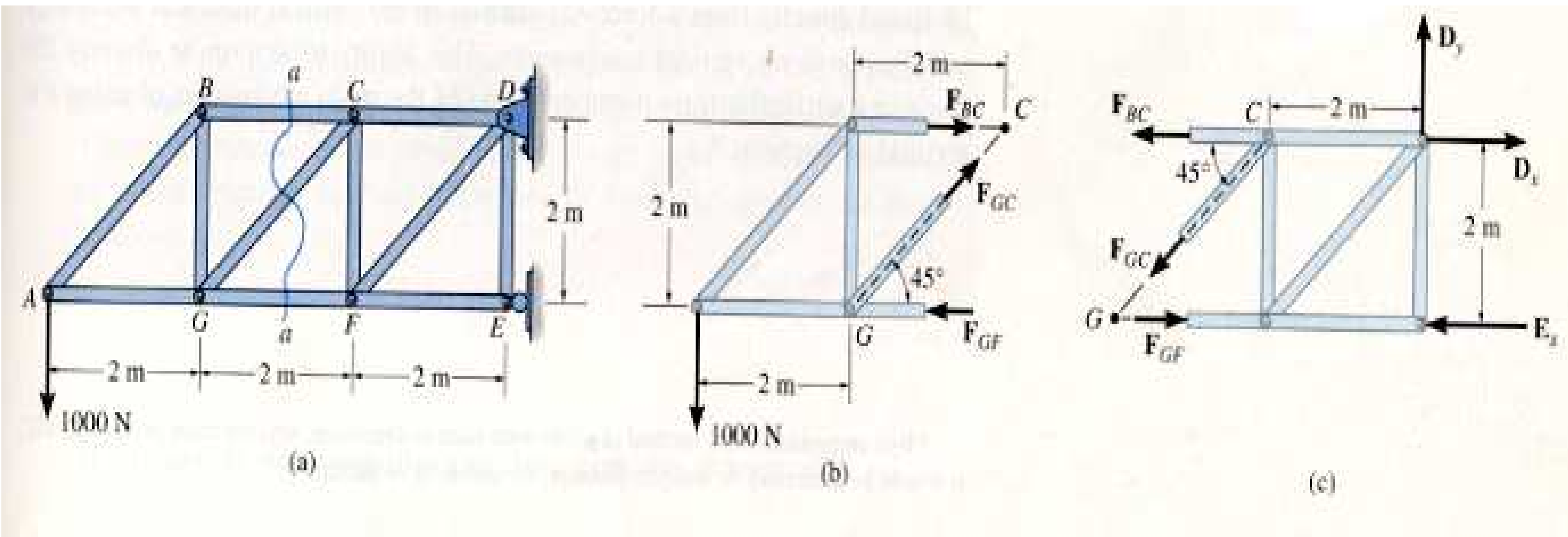
Method of Sections

- Passing an imaginary section through a member results in internal loading.
- The loads acting at the section must be included on the free-body diagram.
- Any "cut" must be replaced by a load … same conceptually as support reactions!



Method of Sections

- If a body is in equilibrium, then any part of the body is also in equilibrium.



Procedure for Analysis

- In most cases, it is first necessary to determine the truss's external reactions.
- Decide how to "cut" or section the truss through the members where forces to be determined (try to select a section that, in general, passes through not more than three members – why 3?).

Procedure for Analysis (cont..)

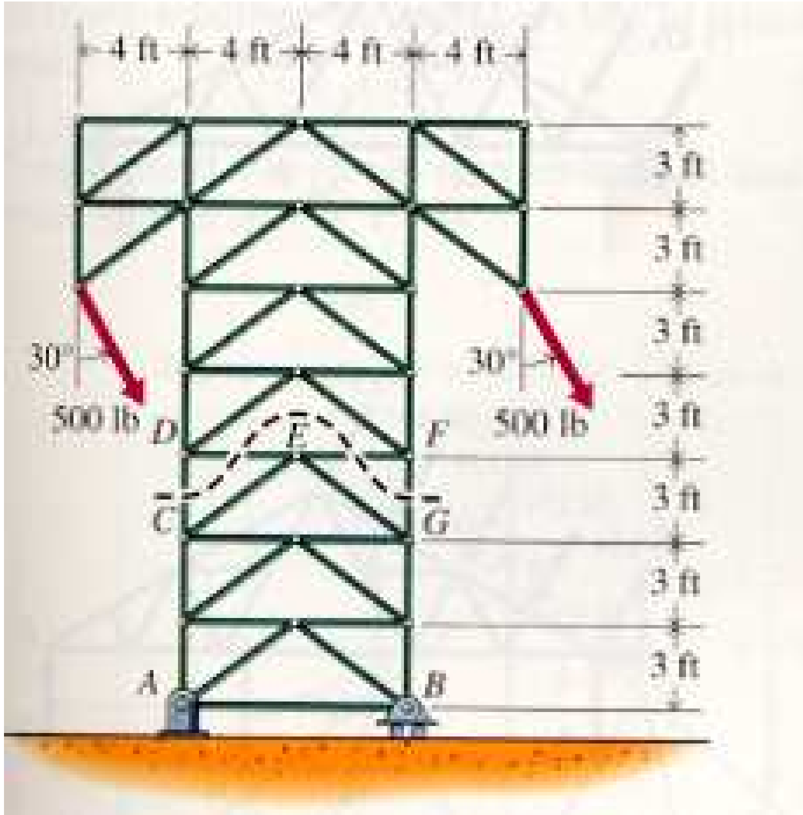
– Draw the free-body diagram of that part of the sectioned truss which has the least number of forces acting on it.

– Use one of the two methods (described previously) for the direction (sense) of an unknown member force.

– Apply the three equations of equilibrium to find the unknown forces.

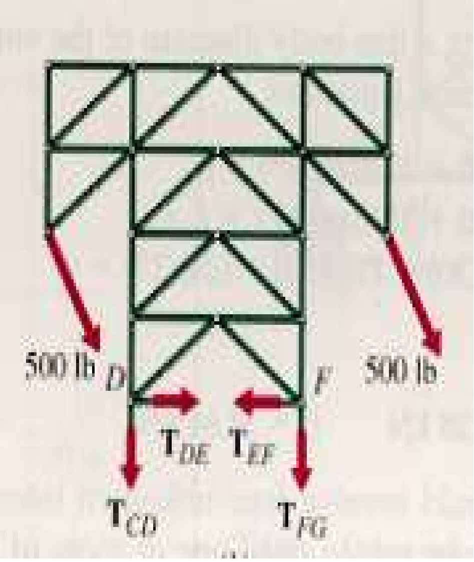
Example: Method of Sections

- Use the method of sections to find the forces in members CD and FG of the truss shown in the figure.



Solution

- Cut a section through members CD,DE,EF, and FG as shown
- Draw a FBD of the upper part of the truss.
- Apply equilibrium equations



$$\sum M_D = 4(500\cos 30^o) - (6)(500\sin 30^o) - 8T_{FG}$$

$$-(12)(500\cos 30^o) - (6)(500\sin 30^o) = 0$$

$$\Rightarrow T_{FG} = -808 \text{ lb (Compression)}$$

$$\sum M_F = (12)(500\cos 30^o) - (6)(500\sin 30^o) + 8T_{CD}$$

$$-(4)(500\cos 30^o) - (6)(500\sin 30^o) = 0$$

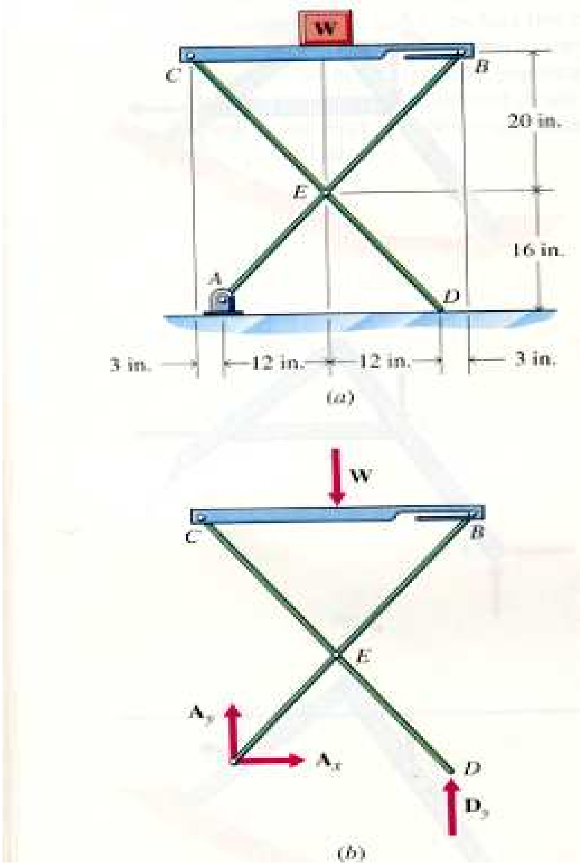
$$\Rightarrow T_{CD} = -58.01 \text{ lb (compression)}$$

Frames and Machines

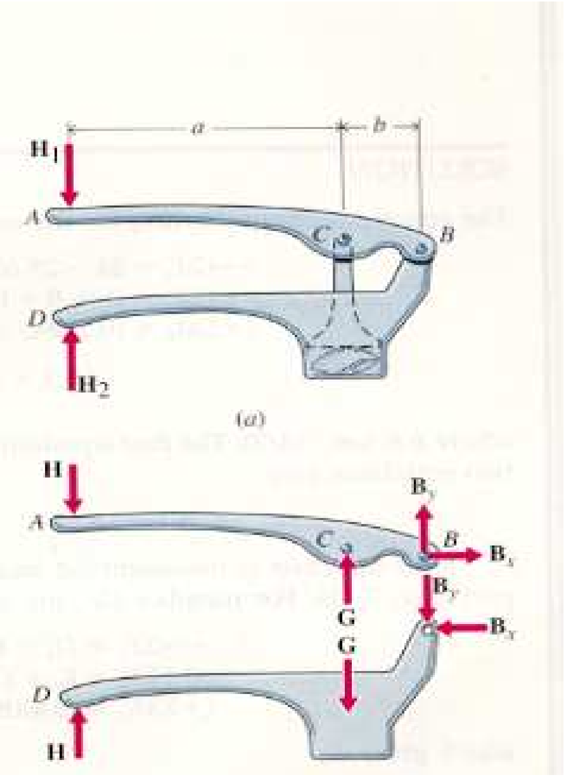
- Frames and machines are structures that composed of pin-connected multi-force members; members that are subjected to more than two forces.
- Main difference between frames and machines: frames are rigid structures whereas machines are not.
- Rigid: doesn't depend on its supports to maintain its shape.

Frames and Machines

Frame



Machine

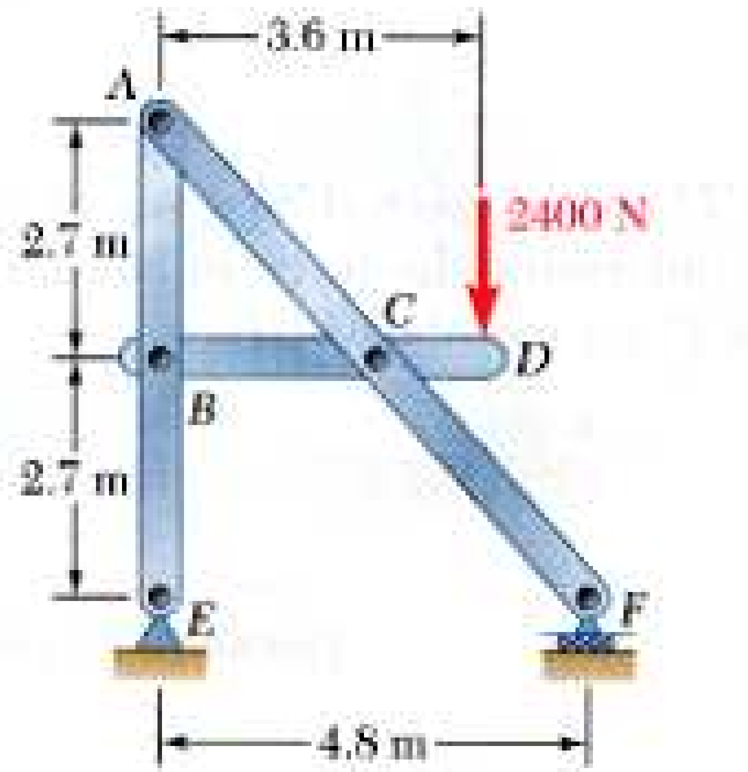


Frames and Machines

- Procedure for Analysis
  - Isolate each part by drawing its outlined shape. Then show all the forces and/or couple moments that act on the part.
  - Identify all the two-force members in the structure.
  - Assume a sense for unknown forces.
  - Use equilibrium equations to calculate the unknown forces.

Example

- Determine the components of the forces acting on each member of the frame shown



Solution cont.

- FBD : Entire Body

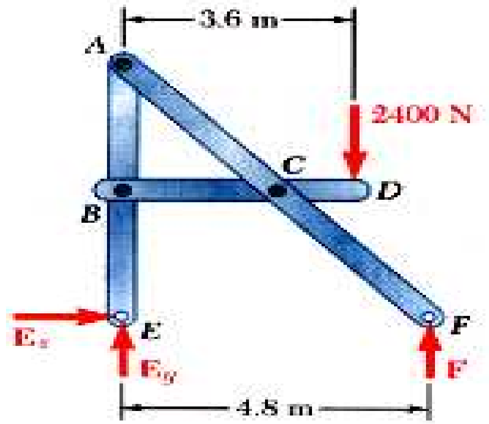
$+\circlearrowleft \Sigma M_E = 0$: $-(2400\ \text{N})(3.6\ \text{m}) + F(4.8\ \text{m}) = 0$

$F = +1800\ \text{N}$ $\qquad$ $\mathbf{F} = 1800\ \text{N}\uparrow$

$+\uparrow \Sigma F_y = 0$: $-2400\ \text{N} + 1800\ \text{N} + E_y = 0$

$E_y = +600\ \text{N}$ $\qquad$ $\mathbf{E}_y = 600\ \text{N}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0$: $\qquad$ $\mathbf{E}_x = 0$

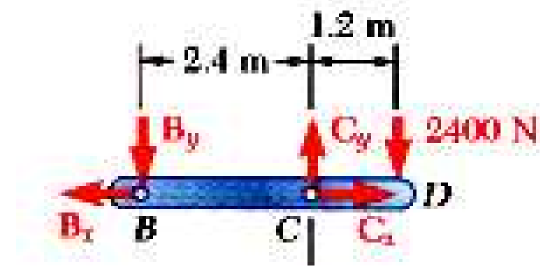


- FBD : Member BCD

$+\circlearrowleft \Sigma M_B = 0$: $-(2400\ \text{N})(3.6\ \text{m}) + C_y(2.4\ \text{m}) = 0$ $\qquad$ $C_y = +3600\ \text{N}$

$+\circlearrowleft \Sigma M_C = 0$: $-(2400\ \text{N})(1.2\ \text{m}) + B_y(2.4\ \text{m}) = 0$ $\qquad$ $B_y = +1200\ \text{N}$

$\xrightarrow{+} \Sigma F_x = 0$: $-B_x + C_x = 0$



Solution cont.

- FBD: Member ABE

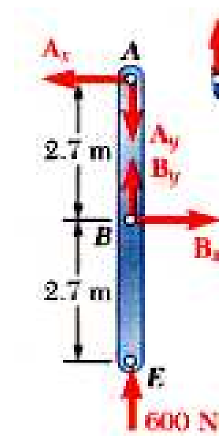
Free Body: Member ABE

$+\circlearrowleft \Sigma M_A = 0: \quad B_x(2.7\text{ m}) = 0 \qquad B_x = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad +B_x - A_x = 0 \qquad A_x = 0$

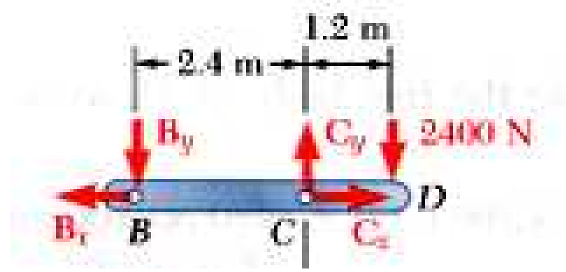
$+\uparrow \Sigma F_y = 0: \quad -A_y + B_y + 600\text{ N} = 0$

$-A_y + 1200\text{ N} + 600\text{ N} = 0 \qquad A_y = +1800\text{ N}$



- FBD: Member BCD

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -B_x + C_x = 0 \qquad 0 + C_x = 0 \qquad C_x = 0$



Solution, Cont.

- FBD: Member ACF

$$+\circlearrowleft \Sigma M_C = (1800\text{ N})(2.4\text{ m}) - A_y(2.4\text{ m}) - A_x(2.7\text{ m})$$
$$= (1800\text{ N})(2.4\text{ m}) - (1800\text{ N})(2.4\text{ m}) - 0 = 0 \quad \text{(checks)}$$