Mechanics & Materials I

Chapter 4

Equilibrium of Rigid Bodies

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Department of Mechanical Engineering

Equilibrium of Rigid Bodies

- Equilibrium of rigid bodies:
- The situation when the external forces and moments acting on a rigid body form a system equivalent to zero

$$\sum \overline{F} = 0$$

$$\sum M_o = \sum (\overline{r} \times \overline{F}) = 0$$

Equilibrium in Two Dimensions

- Here the structure and the forces applied to it are contained in the same plane.
- Example:

$$\overset{+}{\rightarrow} \sum \overline{F}_x = 0$$

$$\overset{+}{\uparrow} \sum \overline{F}_y = 0$$

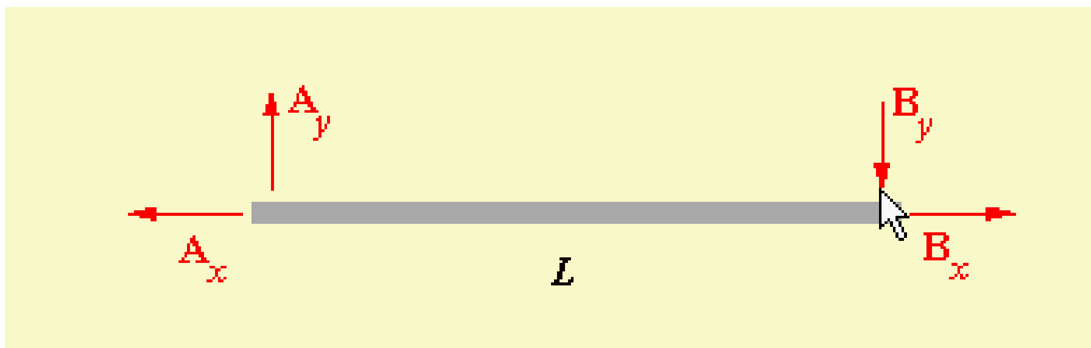
$$\overset{+}{\hookleftarrow} \sum M_0 = 0$$

Equilibrium

- Free-Body Diagram (FBD)
  - Step 1. Identify the body or combination of bodies
  - Step 2. Isolate this body and draw an outline
  - Step 3. Replace the surrounding with forces to keep the structure in equilibrium
  - Step 4. Choose a set of coordinate axes

Equilibrium

- Types of Forces:
  - Two force members (trusses, frames, machines,...)



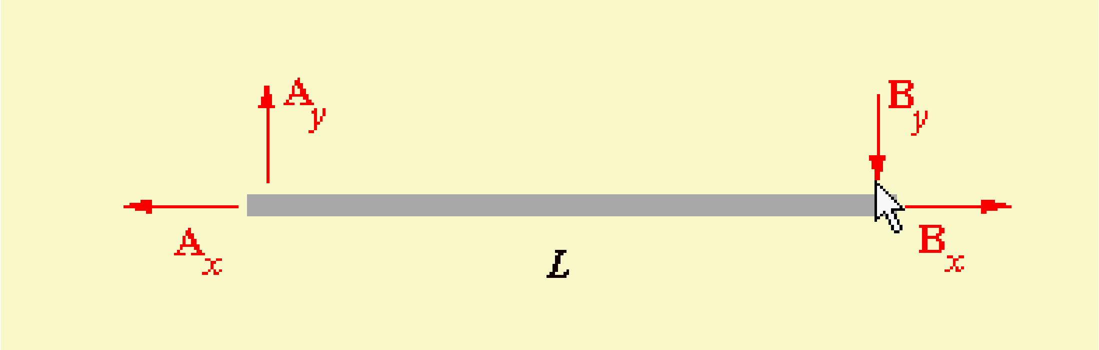
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - A_x = 0 \quad A_x = B_x$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - B_y = 0 \quad A_y = B_y$$

Equilibrium

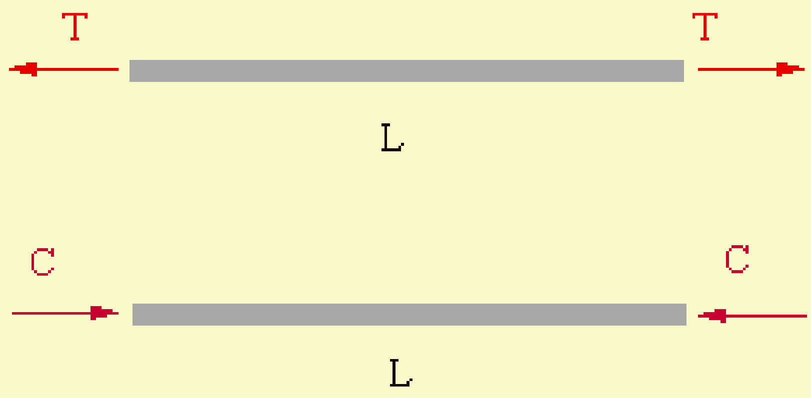
- Taking moment about A or B:

$+\circlearrowright \Sigma M_A = 0: \quad -B_y L = 0$ Or, $B_y = A_y = 0$



Equilibrium

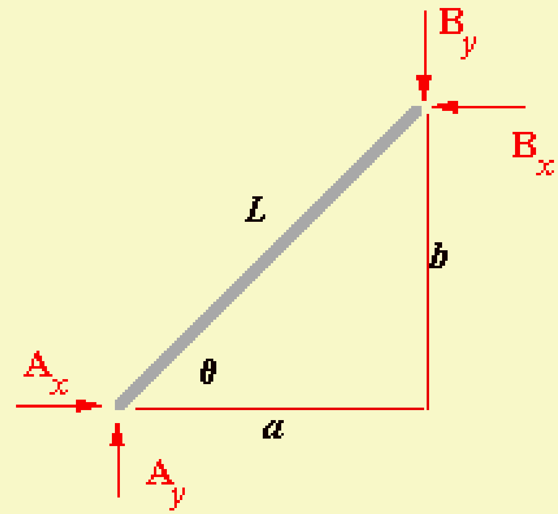
Therefore, the loads transmitted through the member must lie along the member, and can be either tensile (T) or compressive (C).



Equilibrium

The inclined two-force member shown can be modeled so that $\boldsymbol{\theta}$ is defined by an appropriate trigonometric relationship involving $\boldsymbol{a}$, $\boldsymbol{b}$, and (or) $\boldsymbol{L}$.

Apply the conditions of equilibrium of forces to the member.



$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x - B_x = 0 \qquad A_x = B_x$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - B_y = 0 \qquad A_y = B_y$$

Taking moments about point $\boldsymbol{A}$ **:**

$$+\circlearrowleft \Sigma M_A = 0: \quad bB_x - aB_y = 0 \qquad B_y = (b/a)B_x$$

B_x and B_y can be expressed as a resultant force:

$$B = [B_x^{\,2} + B_y^{\,2}]^{1/2} = [B_x^{\,2} + (bB_x/a)^2]^{1/2} = B_x\,[(1+b/a)^2]^{1/2}$$

$$= (B_x[(a+b)^2]^{1/2})/a$$

Using geometry, this expression can be reduced to the following:

$$B = B_x\,L/a = B_x/\cos\theta \qquad \text{and} \qquad B_x = B\cos\theta$$

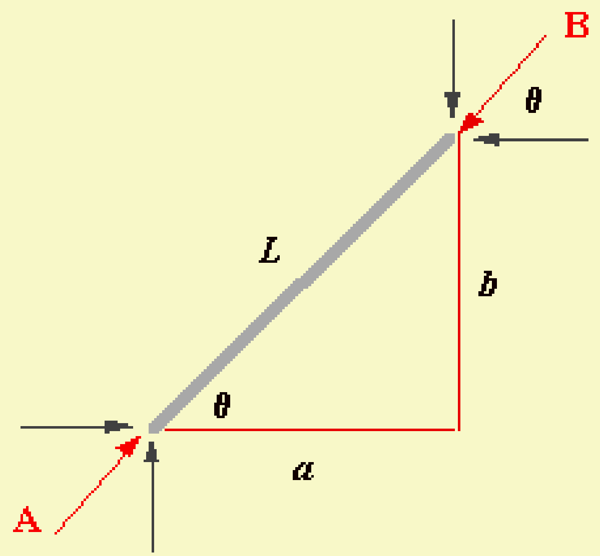
Equilibrium

Therefore, knowing that $A_x = B_x$ and $A_y = B_y$, the member loads at A and B must be identical, and must lie along the member.

The load can be either compressive (as shown) or tensile (in the opposite direction).

If the weight of the member had been considered to act at its center, or a load were applied at some location between A and B , this member would not be a 2-force member.

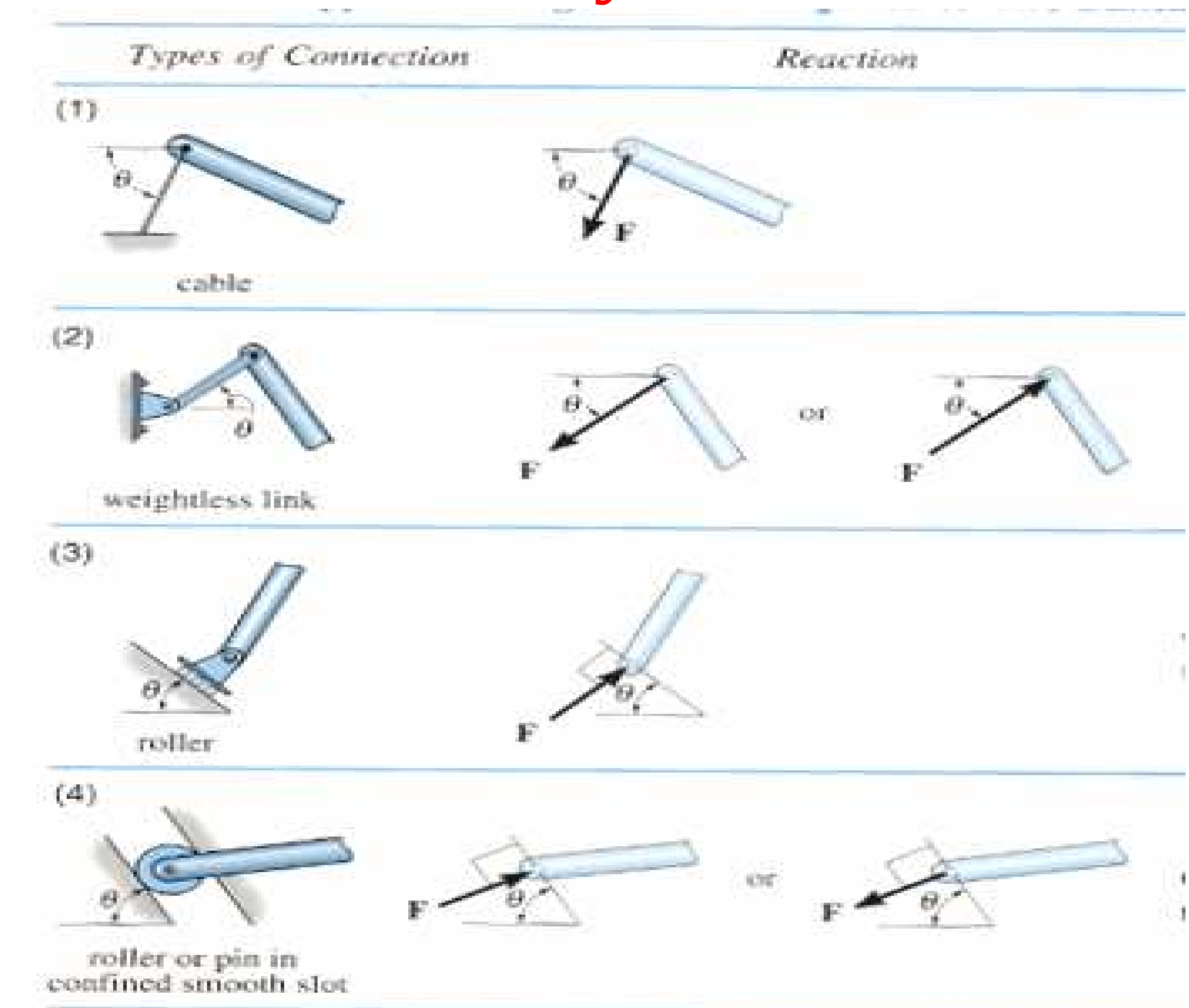
The line of action of the forces at A and B are along the member.



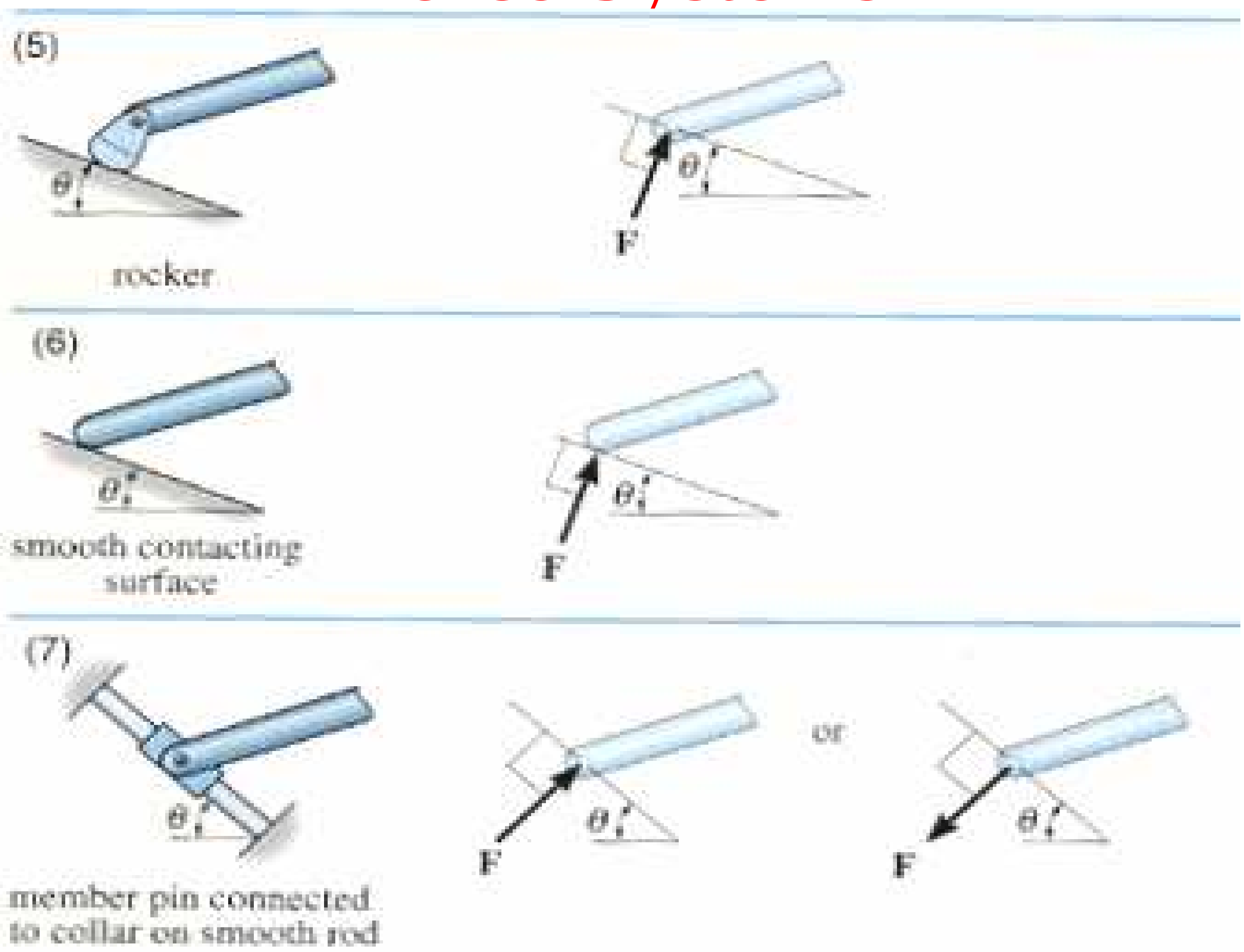
Equilibrium: support reactions

- Support reactions are required for static equilibrium of bodies with external forces
  - For example, the table top is subjected to gravity; the legs provide support!
  - The door is subjected to gravity, the hinges provide support
  - We need to represent the supports by the forces and moments that they exert on the body – these are called the “support reactions”
  - In general, ***a support reaction exists for each type of motion that is prevented.*** This is true in 1D, 2D, and 3D problems.

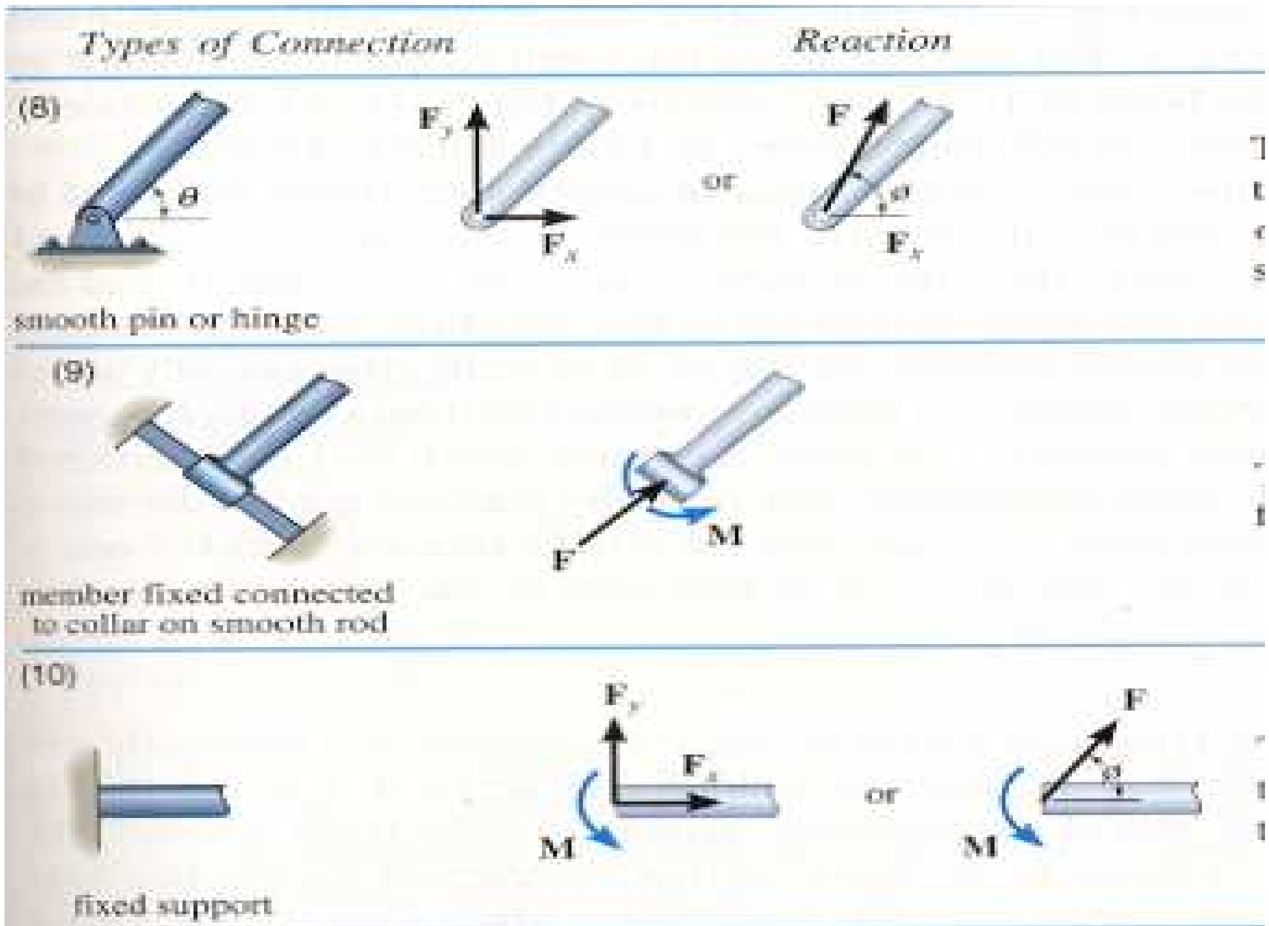
Supports for Rigid Bodies Subjected to 2-D Force Systems-1



Supports for Rigid Bodies Subjected to 2-D Force Systems-2

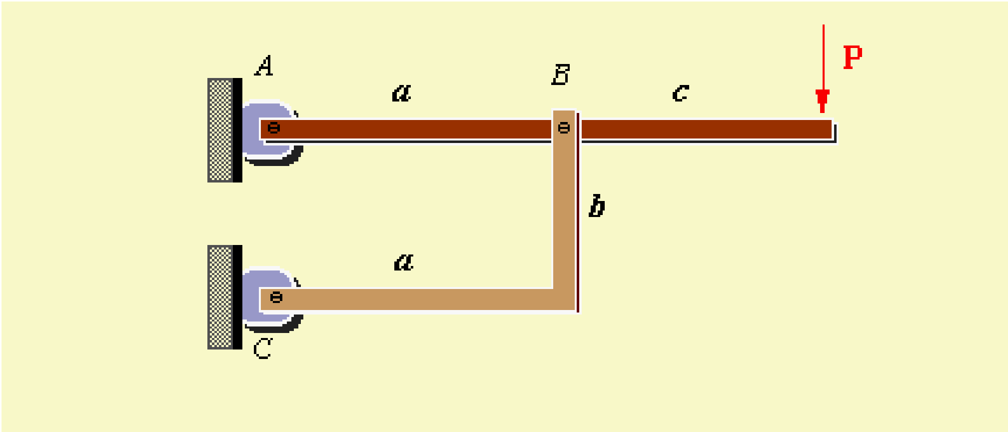


Supports for Rigid Bodies Subjected to 2-D Force Systems-3

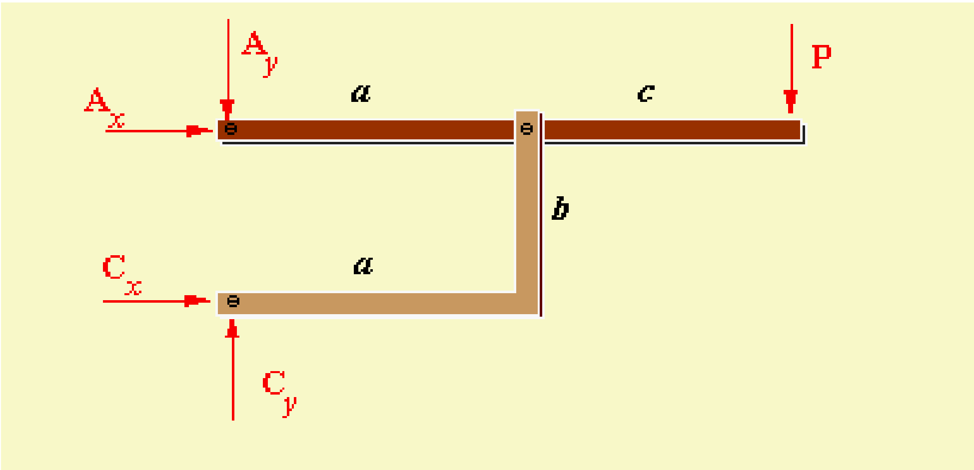


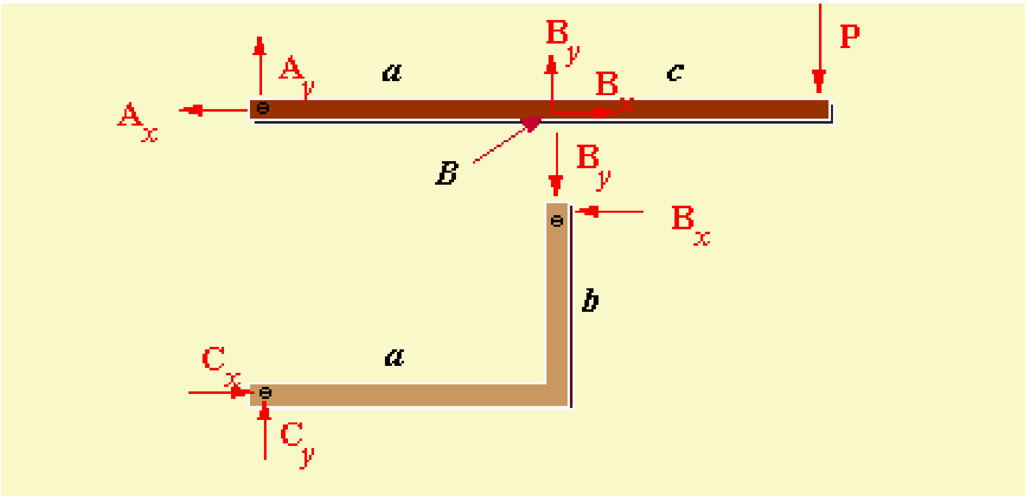
Equilibrium

Example:



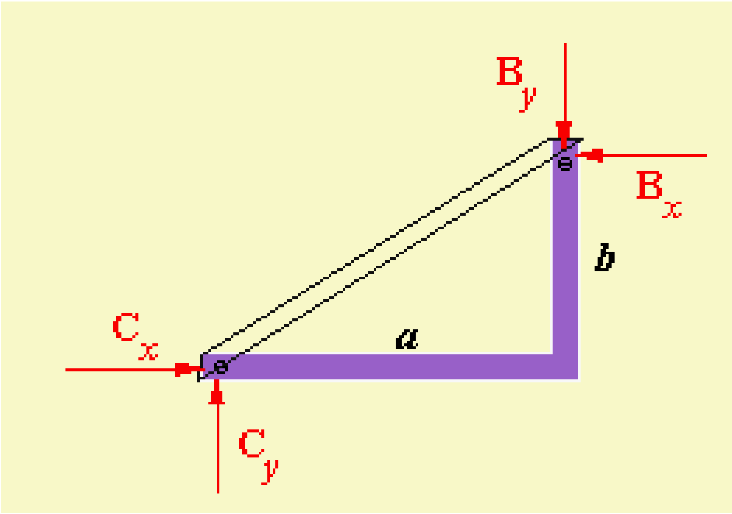
FBD:

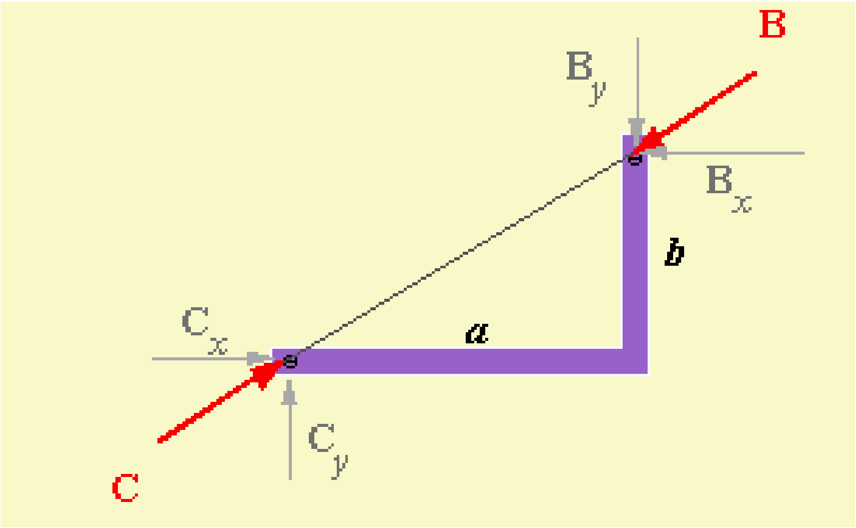




Equilibrium

- Another example:





Equilibrium in 3-D

For a rigid body in equilibrium:

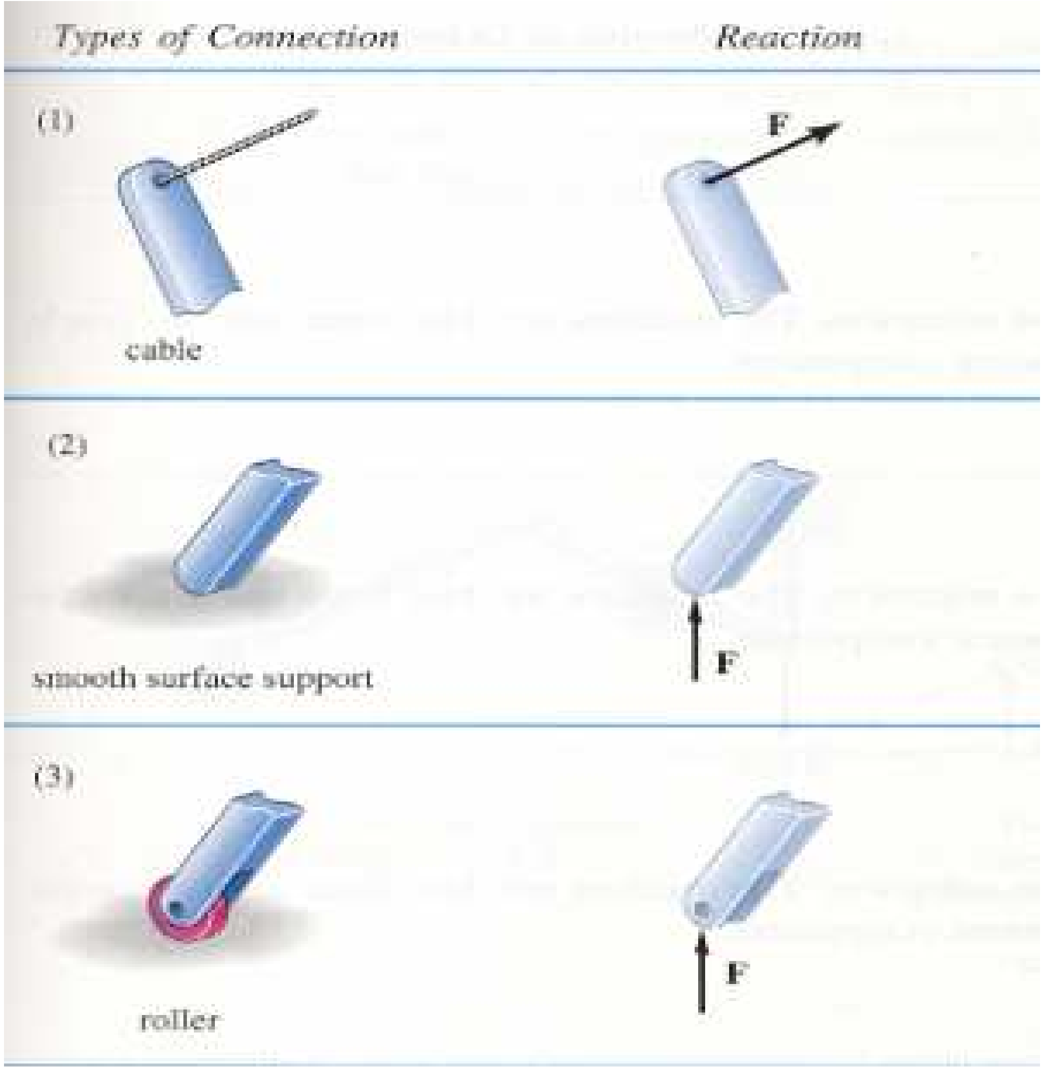
$$R = \sum F_x i + \sum F_y j + \sum F_z k = 0$$

$$C = \sum M_x i + \sum M_y j + \sum M_z k = 0$$

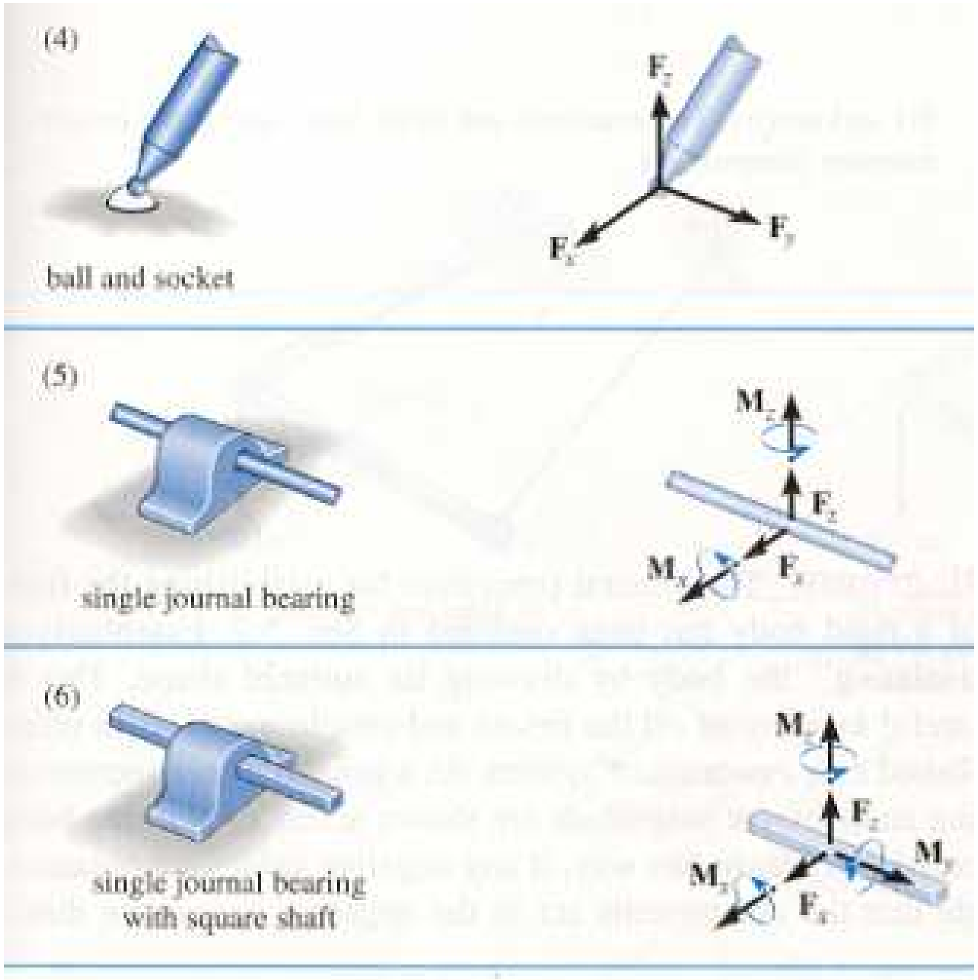
$$\sum F_x i = 0\,, \quad \sum F_y j = 0\,, \quad \sum F_z k = 0$$

$$\sum M_x i = 0, \quad \sum M_y j = 0, \quad \sum M_z k = 0$$

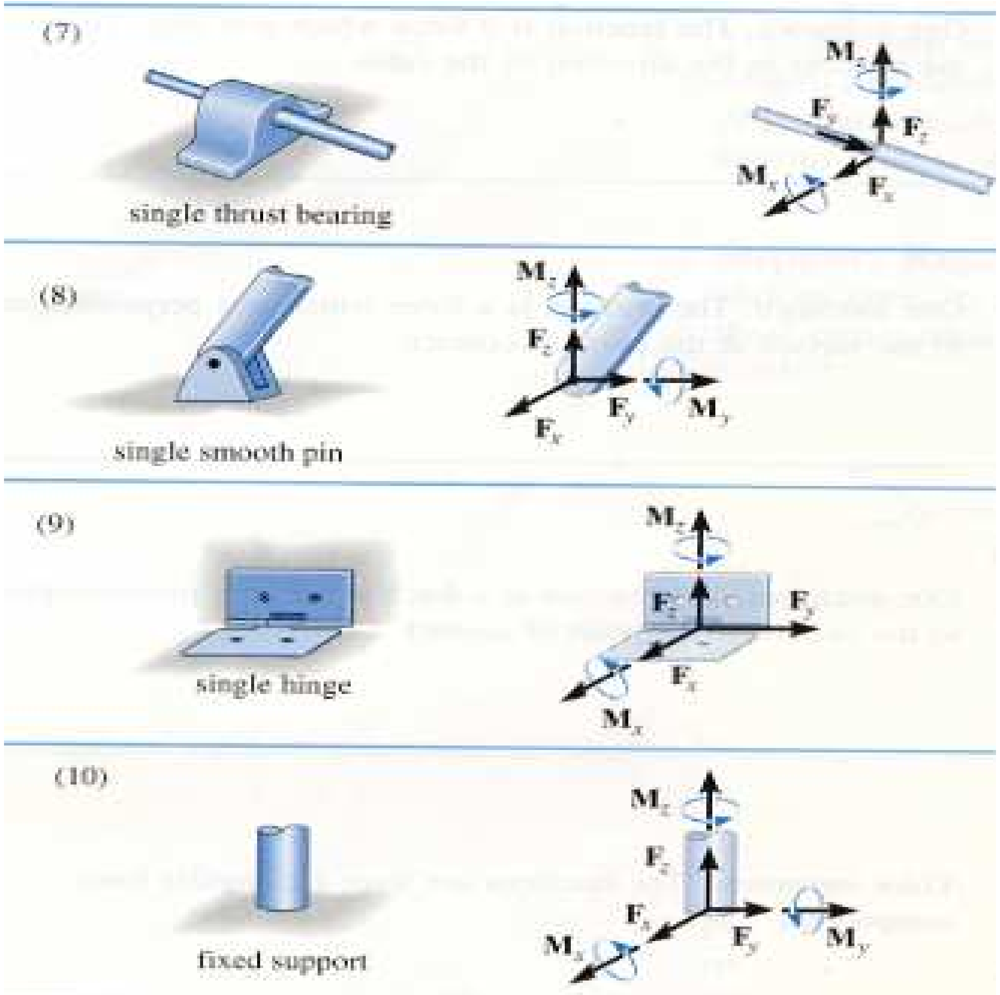
Supports for Rigid Bodies Subjected to 3-D Force Systems-1



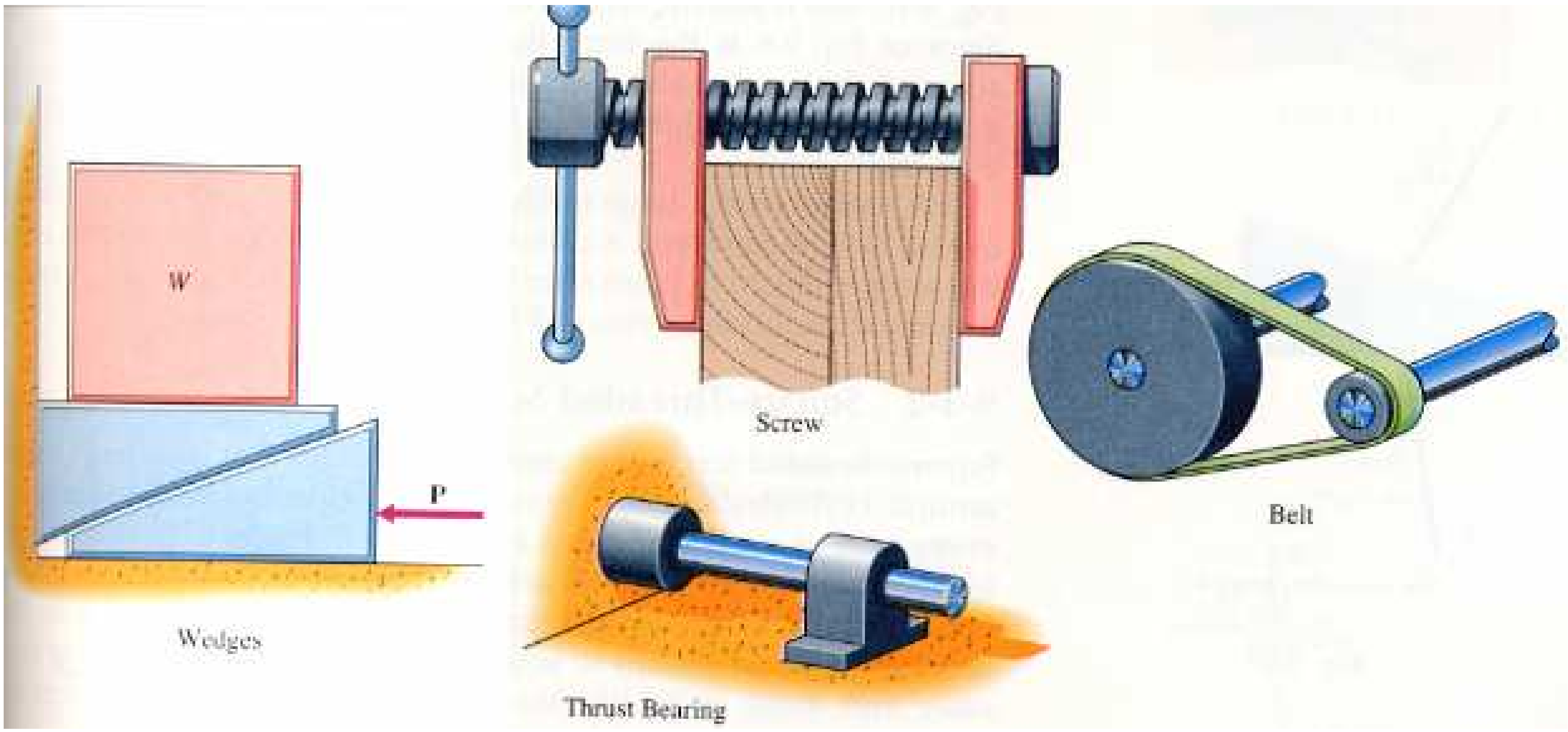
Supports for Rigid Bodies Subjected to 3-D Force Systems-2



Supports for Rigid Bodies Subjected to 3-D Force Systems-3



Friction Force: Is it Useful?



Friction Force

In classical mechanics, two types of friction are commonly encountered: fluid friction and dry, or Coulomb friction. Here we are concerned with *dry friction*.

Figure A

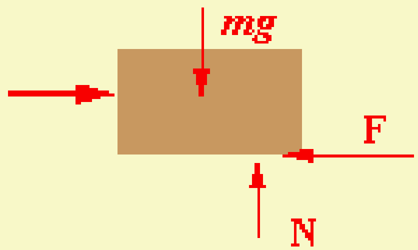
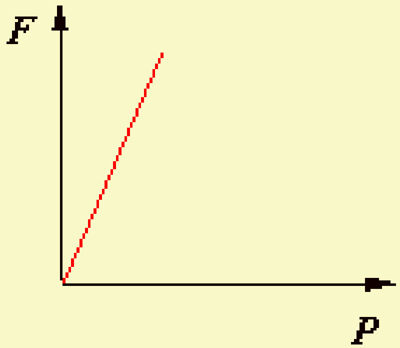


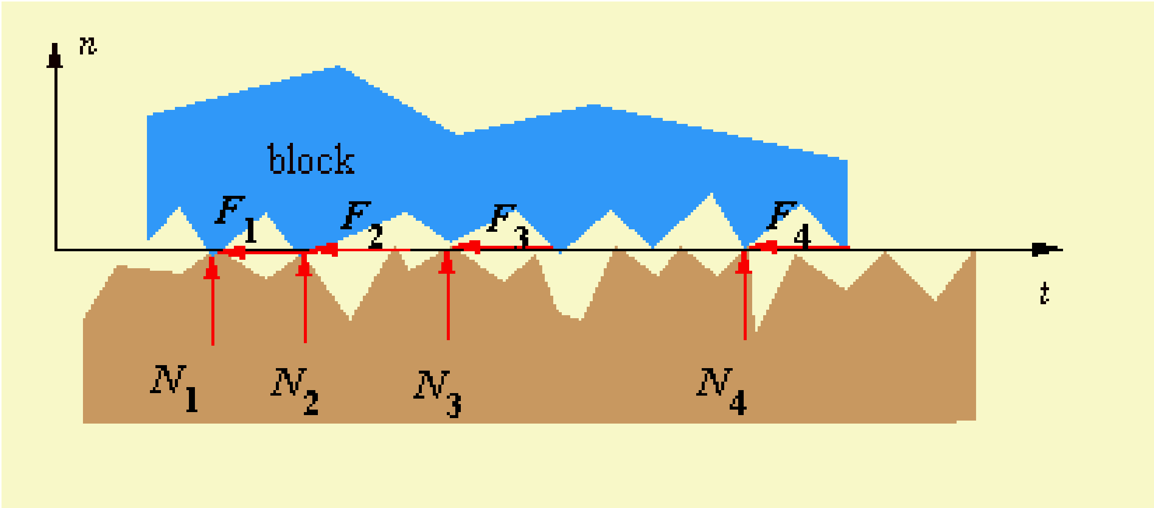
Figure B



Obviously, force **F** cannot continue to balance the increasing load **P** indefinitely. There must be some maximum friction force $\mathbf{F}_{\mathbf{max}}$ above which the block will slip. The maximum friction force is related to the normal force through the coefficient of static friction (μ_s) by $F_{max} = \mu_s N$. After slipping occurs, the friction force decreases, and the coefficient of kinetic friction should be used, $F = \mu_k N$.

Friction Force

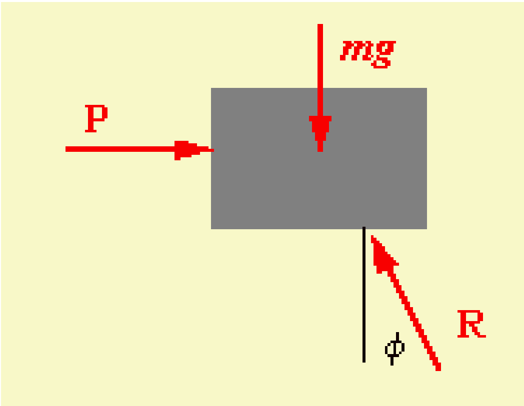
- In reality:

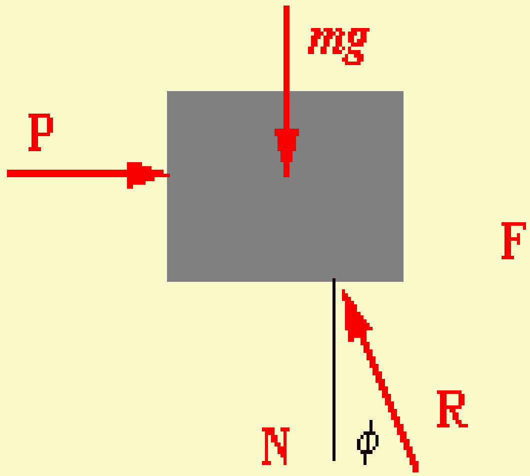


Friction Force

- Instead:

Use of resultant force R to contain both the normal and the shear force





The resultant force can be resolved into **two** components which are normal and tangent to the contacting surfaces. The tangential force is the friction force (F).

$F = R\sin\phi$ $\qquad$ $N = R\cos\phi$

The ratio of the friction force to the normal force is the tangent of the angle θ.

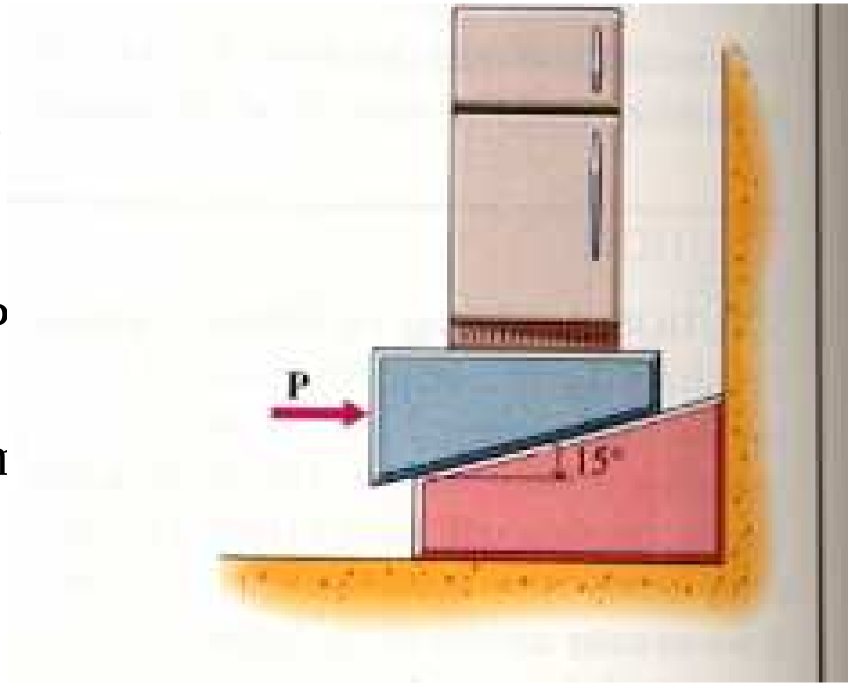
$$\frac{F}{N} = \frac{R\sin\phi}{R\cos\phi} = \tan\phi$$

Since $F = \mu_s N$, it is obvious that $\tan\phi = \mu_s$.

The angle ϕ is called the *angle of friction*. In statics, it is often given a subscript s, and called the angle of static friction. In kinetics, the subscript k is used, and ϕ is called the angle of kinetic friction.

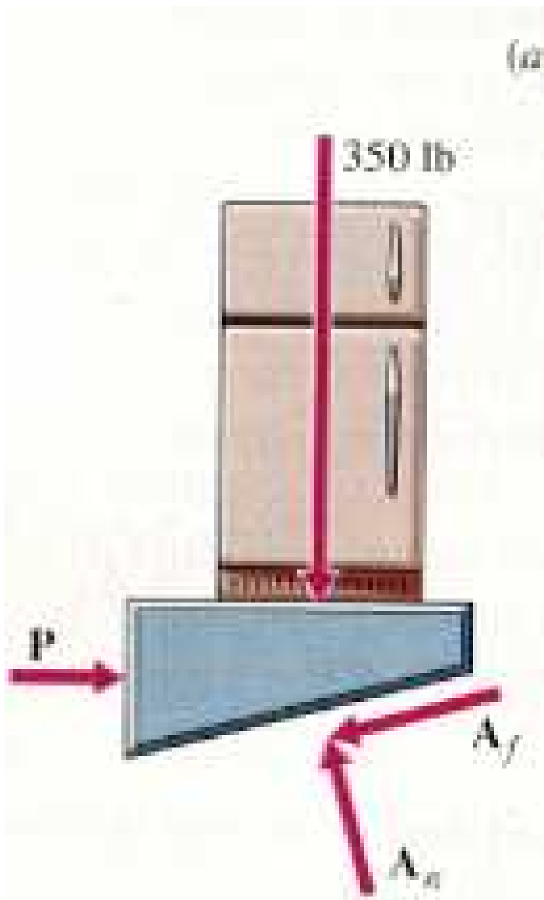
Example: Friction Force

- **A wedge is used to raise a 350-lb refrigerator. The coefficient of friction is 0.2 at all surfaces**
- 1.Determine the minimum force P needed to insert the wedge.
- 2.Will the system be in equilibrium P=0?
- If the system is not in equilibrium when P=0 determine the force necessary to keep the wedge in place, or if the system is in equilibrium when P=0, determine the force necessary to remove the wedge.



Solution of Example

- First Step:
- Draw Free Body Diagram of the upper wedge and the refrigerator



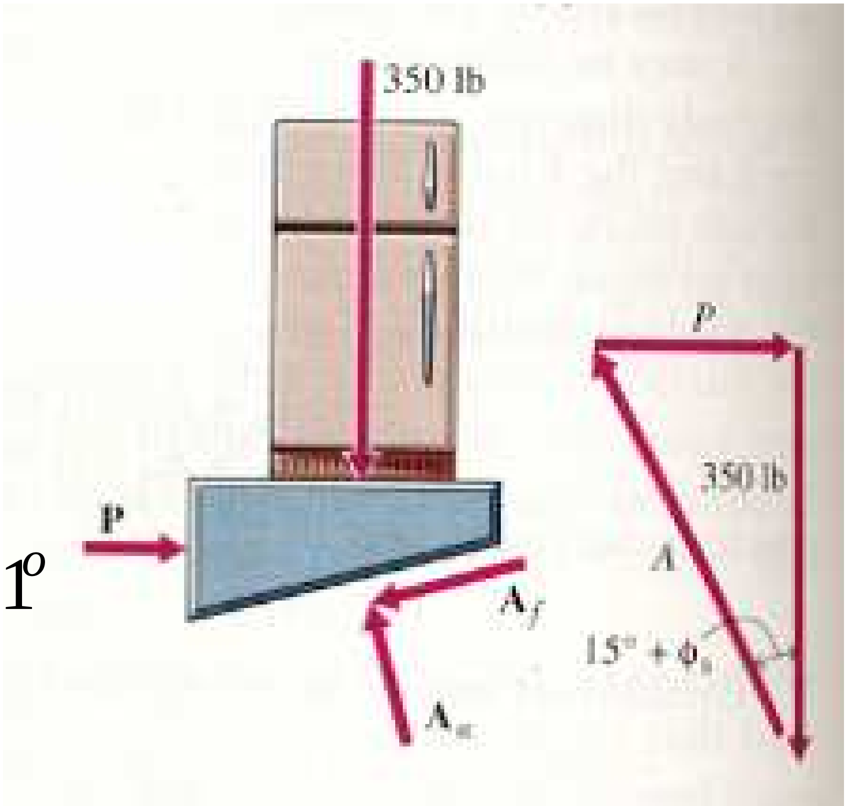
Solution: Part A

- Second Step: Force analysis
- The normal and the friction forces are combined into a single resultant force acting at the angle of static friction

$$\phi_s = \tan^{-1}(\mu) = \tan^{-1}(0.2) = 11.31^o$$

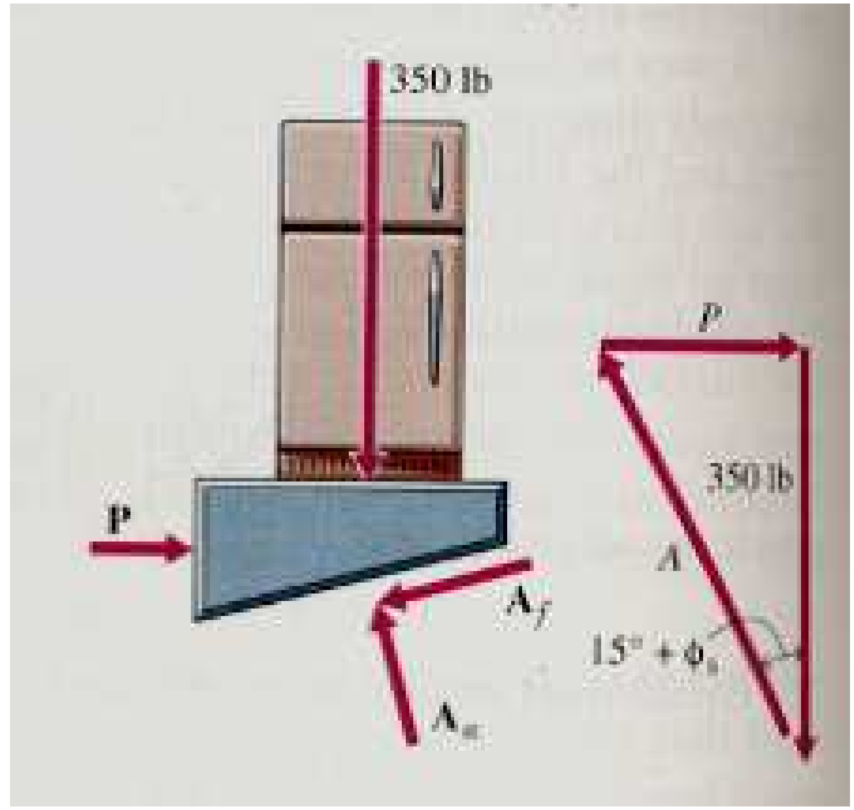
$$\tan(15^o + 11.31^o) = \frac{P}{350}$$

Hence $\quad P = 173.1$



Solution: Part B

- For P very small, the wedge will tend to move to the left and the friction will have to act to the right to oppose this motion.
- The resultant force is drawn at angle ϕ relative to the normal force or 15^0-ϕ relative to the vertical direction



From equilibrium force traingle, when P = 0

$$15^o - \phi = \tan^{-1}\left(\frac{P}{350}\right) = 0 \Rightarrow \phi = 15^o$$

But the angle of resultant ϕ can never be greater than the angle of static friction $\phi_s = 11.31^o$

Therefore $\Rightarrow$ the wedge will not be in equilibrium if P = 0; the force P is removed

Solution : Part C

- Since the wedge will not stay in place by itself, force P acting to the right, is necessary to hold the wedge in place

The minimum force necessary to hold the wedge in place is attained when

$\phi = \phi_s = 11.31^o$

From the force equilibrium triangle

$$\tan(15^0 - 11.31^o) = \frac{P}{350}$$

$Hence, \Rightarrow P = 22.6\ lb$