Mechanics & Materials 1

Chapter 14

Combined Loadings

FAMU-FSU College of Engineering

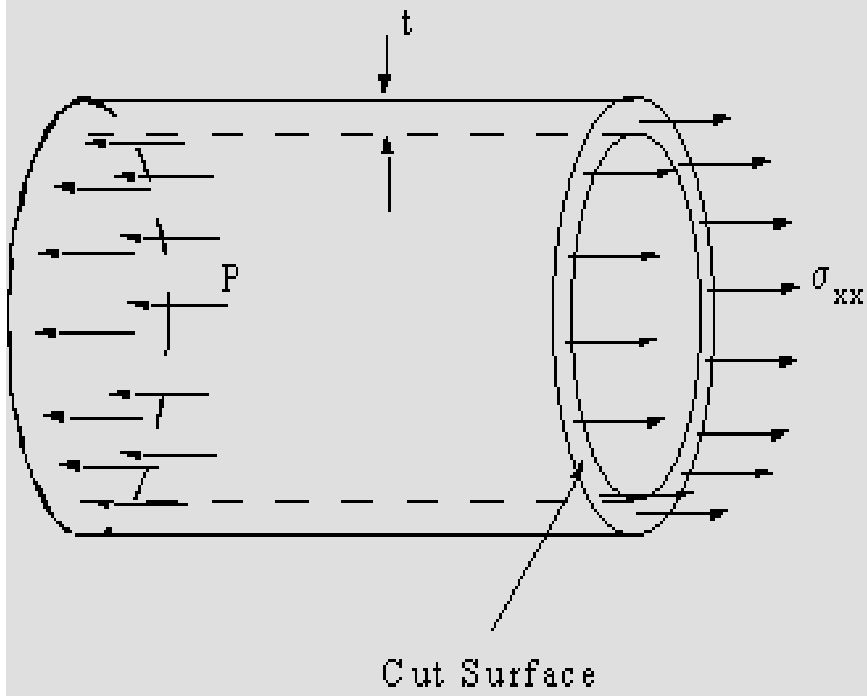
Department of Mechanical Engineering

Thin Walled Pressure Vessels

- Cylindrical or spherical pressure vessels are used in industry as tanks, boilers or containers.
- When under pressure the material is subjected to loadings in all directions.
- In general thin wall refers to an inner radius to wall thickness ratio $r_i/t \geq 10$, in most cases it is actually > 50.
- If the vessels wall is thin, the stress distribution through the thickness can be assumed to be uniform or constant.

Cylindrical Pressure Vessels

- Consider tubes with an internal pressure and closed ends
- $\sigma_{xx} \rightarrow$ axial stress due to the pressure on the end walls
- $\sigma_{\theta\theta} \rightarrow$ "Hoop" stress due to the pressure acting on the curved surface.
- P is the internal pressure
- Look at a FBD of the axial section



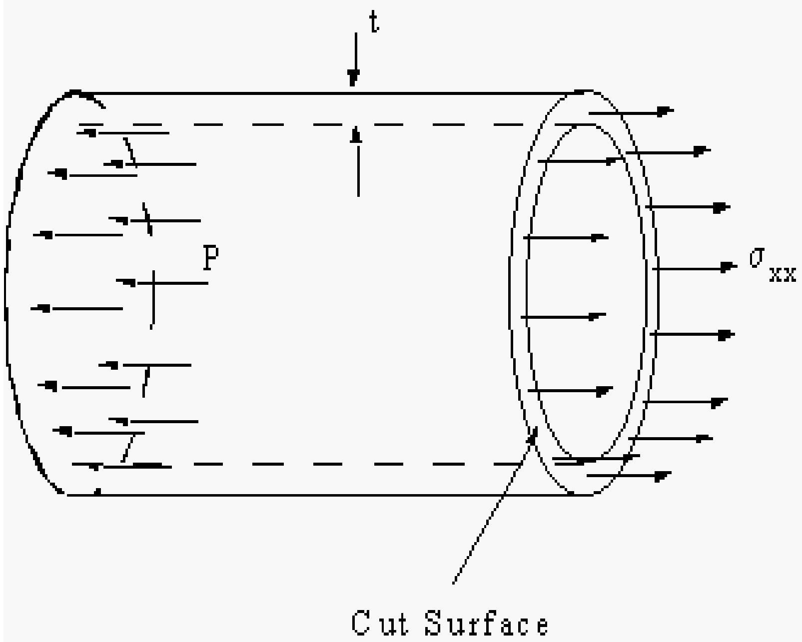
FBD of axial section of a cylindrical pressure vessel

Cylindrical Pressure Vessels: Axial Stress

- Consider the axial equilibrium on this structure.

$$\sum F_x = 0 = -P\pi r^2 + 2\pi r t \sigma_{xx}$$

$$\sigma_{xx} * (2\pi r t) = P\pi r^2$$



FBD of axial section of a cylindrical pressure vessel

- Which gives the equation for Axial Stress:

$$\sigma_{xx} = \sigma_2 = \frac{\mathrm{Pr}}{2t}$$

Cylindrical Pressure Vessels: Hoop Stress

Look now at a FBD of the circumferential section

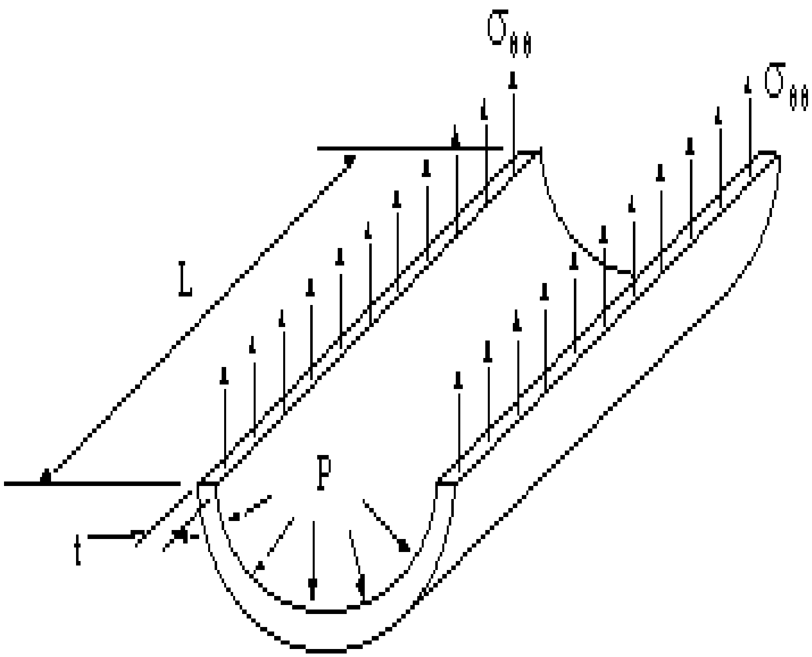
- Equating the forces vertically gives:

$$\sum F_y = 0 = -P*R*2L + \sigma_{\theta\theta}*2Lt$$

$$2\sigma_{\theta\theta}(Lt) = 2rLP$$

- Which simplifies to give the equation for Hoop Stress:

$$\sigma_{\theta\theta} = \sigma_1 = \frac{pr}{t}$$



FBD of circumferential section of the cylindrical pressure vessel

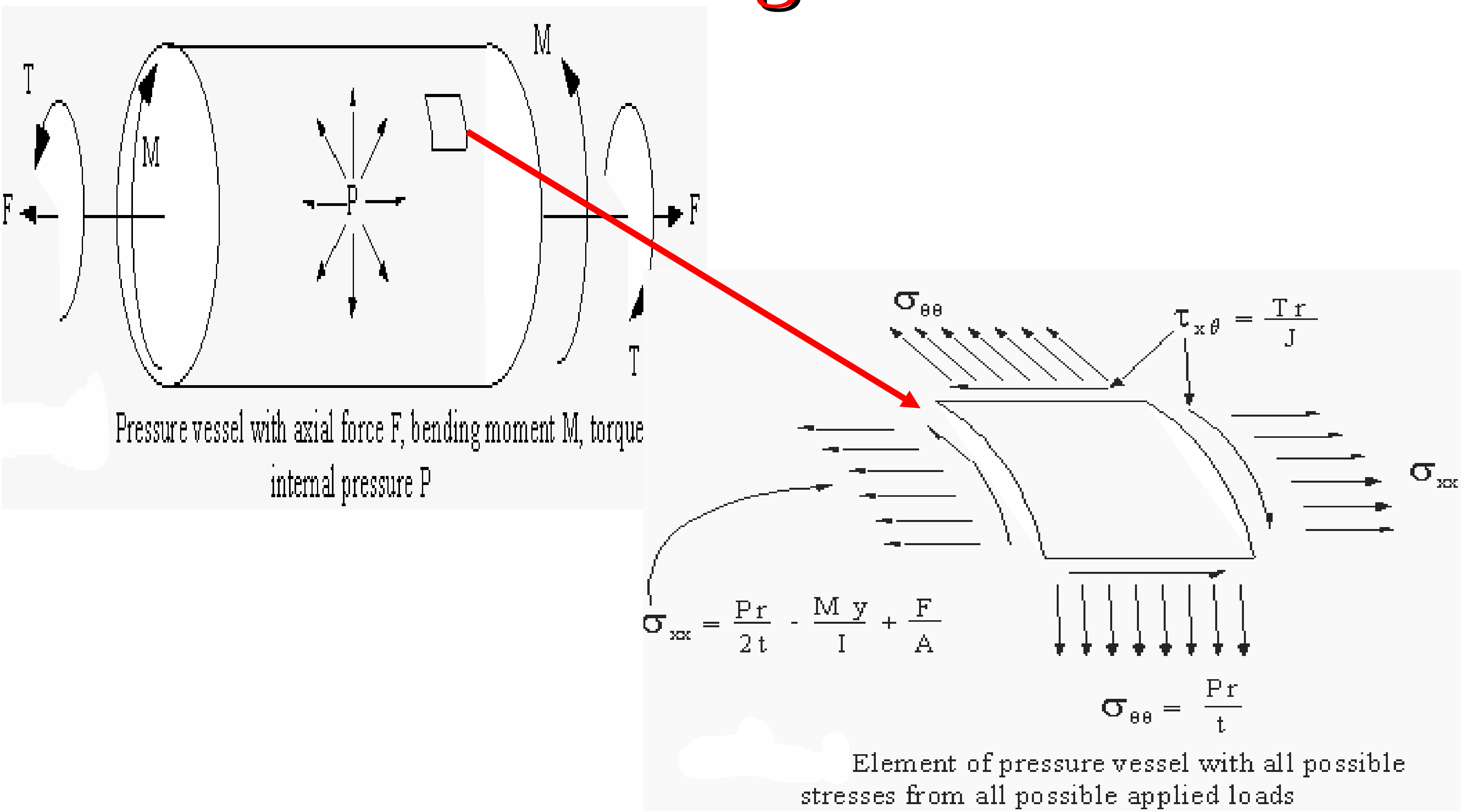
Cylindrical Pressure Vessels

- Pressure vessels also have stresses created by the weight of the pressurized fluid inside, its own weight, externally applied loads and by an applied torque.

- To analyze this, each loading condition is analyzed individually and the stresses are then combined along their respective axes by the superposition method.

- Pressure vessel with ALL possible loading conditions:

Pressure Vessel With ALL Possible Loading Conditions:



Pressure vessel with axial force F, bending moment M, torque internal pressure P

Element of pressure vessel with all possible stresses from all possible applied loads

Spherical Pressure Vessels

- Common for storing liquid gases (maximum storage volume for the least volume of material)

- Equating the forces vertically:

$$\sum F_y = 0 = -P * \pi r^2 + 2\pi r t \sigma$$

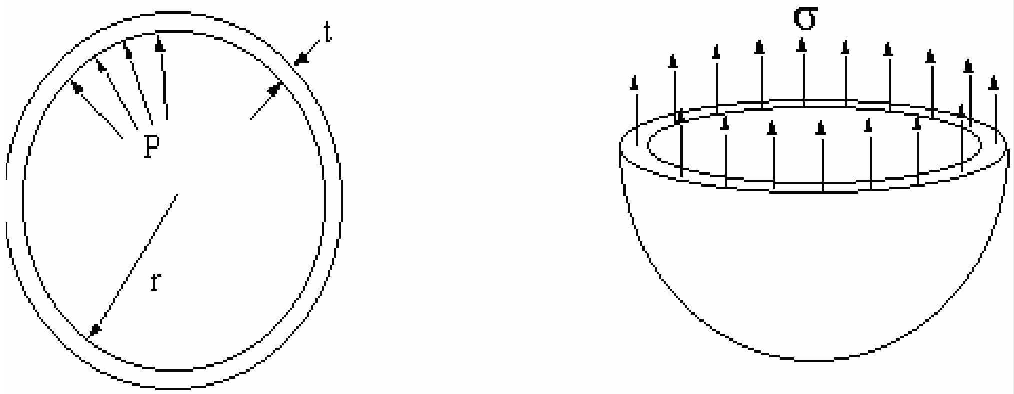


Diagram of spherical pressure vessel with internal pressure P

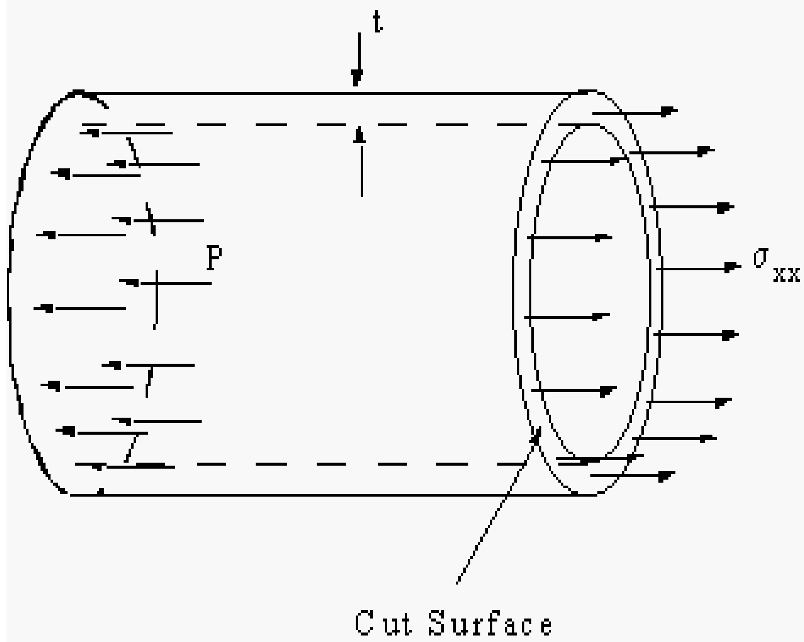
- Which simplifies to:

$$\sigma = \frac{\text{Pr}}{2t}$$

- This is not dependent on the orientation of our FBD, so this stress acts on both surfaces of the element simultaneously.

Example

- A steel gas bottle 2 m long with a diameter of 250 mm and a 3 mm wall thickness is pressurized to 3 MPa.
- Determine the stresses



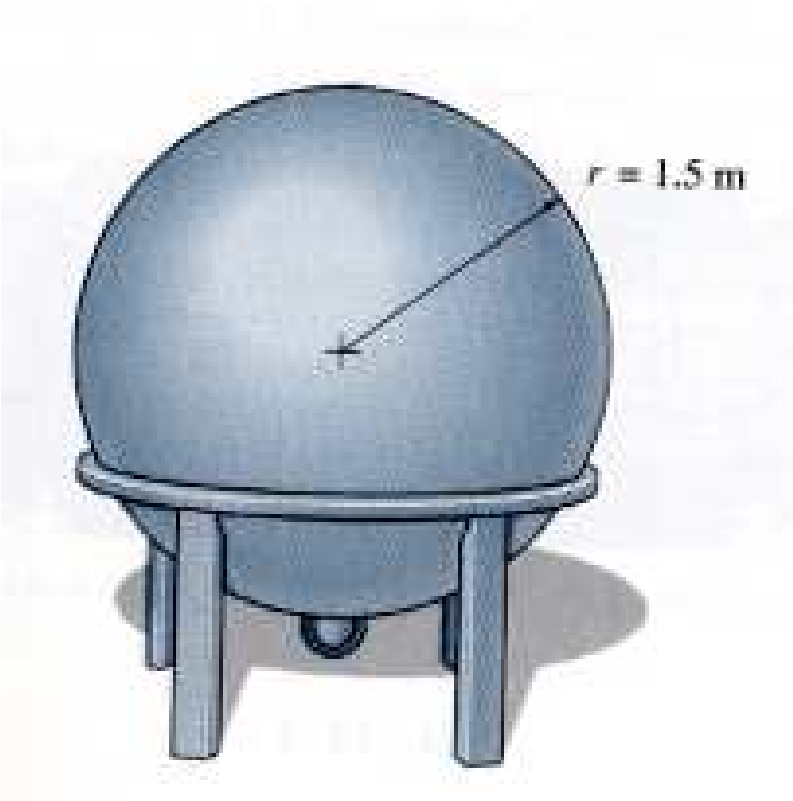
FBD of axial section of a cylindrical pressure vessel

Solution

$$\sigma_{xx} = \frac{Pr}{2t} = \frac{3x10^6 x 0.125}{2x0.003} = 62.5\ MPa$$

$$\sigma_{\theta\theta} = \frac{Pr}{t} = \frac{3x10^6 x 0.125}{0.003} = 125\ MPa$$

Problem 14.1



- The spherical gas tank has an inner radius of r = 1.5m.(If it is subjected to an internal pressure of p=300kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

Solution

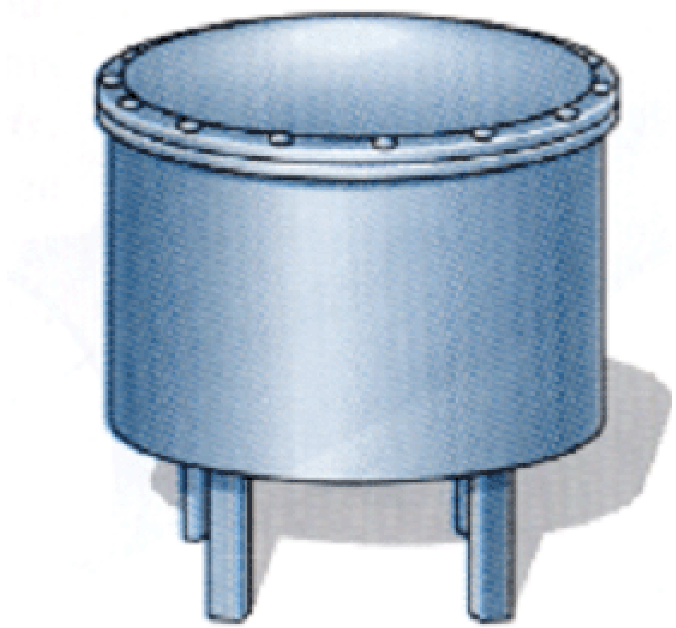
$$\sigma_z = \sigma_{allow} = \frac{\Pr}{2t}$$

$$12(10^6) = \frac{300(10^3)(1.5)}{2t}$$

$$t = 0.01875m = 18.8mm$$

Example 14.3

- The cap of the cylindrical tank is bolted along the flanges. The tank has an inner diameter of 1.5m and a wall thickness of 18mm.
- If the largest normal stress is not to exceed 150MPa, determine the maximum pressure the tank can sustain .
- Also compute the number of bolts required to attach the cap to the tank if each bolt has a diameter of 20mm.
- The allowable stress for the bolt is 180MPa

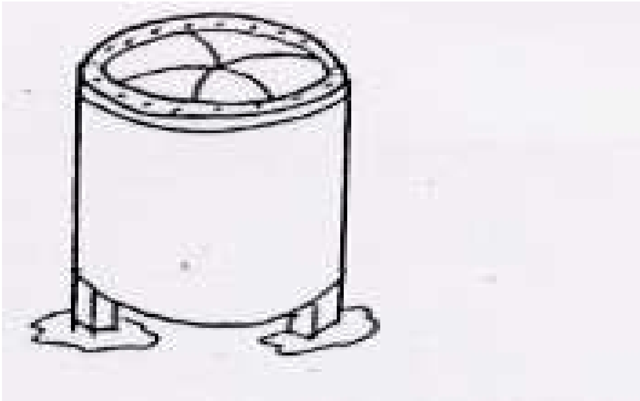


Solution

$$\sigma_1 = \frac{\rho r}{t} \qquad \text{here} \quad r = 0.759\text{m}$$

$$150(10^6) = \frac{\rho(0.75)}{0.018}$$

$$\rho = 3{,}600{,}000 \; N/m^2 = 3.60 \; MPa$$



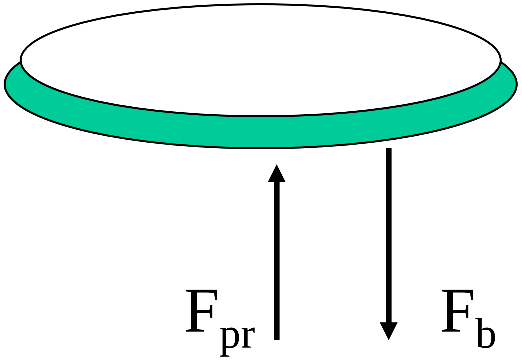
$$F_{pr} = F_b = 3.60(10^6)(\pi/4)(1.5^2)$$

$$= 6361.73 kN$$

$$\sigma_{allow} = \frac{F_b}{nA_b}$$

$$180(10^6) = \frac{6361.73(10^3)}{n(\pi/4)(0.02)^2}$$

$$n = 112.5 \Rightarrow 113$$



Analysis of Stress in Combined Loading

- Normal Stress Sources(σ)
  - Axial Load :- The uniform normal stress distribution due to the normal force.

$$\sigma = \frac{F}{A}$$

  - Bending Moment :- Normal stress distribution due to the bending moment.

$$\sigma = \frac{My}{I}$$

  - Pressure Cylinder :- Biaxial state of stress in the material due to pressure in a thin-walled cylinder.

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_1 = \frac{pr}{t}$$

  - Thermal :- Normal stress distribution due to temperature change.

$$\sigma = \alpha \Delta T E$$

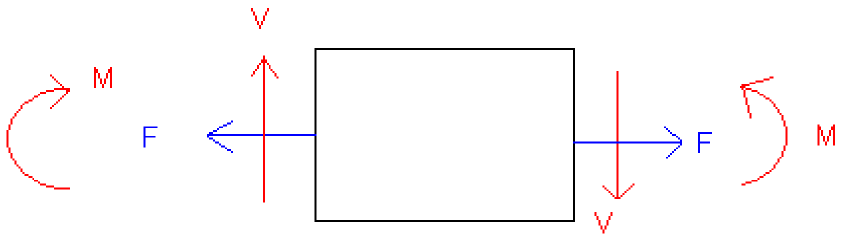
Analysis of Stress in Combined Loading

- Shear Stress Sources (τ):
  - Torsion:- Shear-stress distribution that varies linearly from the central axis to the maximum at the shaft's outer boundary.

$$\tau = \frac{T\rho}{J}$$

  - Transverse Shear Stress :- Shear stress distribution that acts over the cross section.

$$\tau = \frac{VQ}{It}$$



Analysis of Stress in Combined Loading

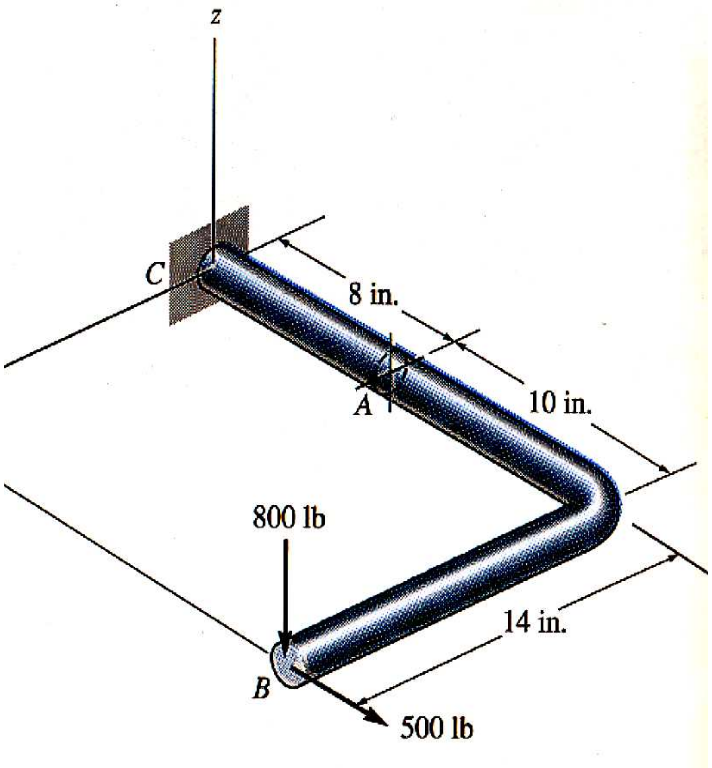
- After finding each component of the stress we
  - add (algebraically) all those components which contribute normal stress to get the total normal stress
  - add (algebraically) the components which give a shear stress to get the total shear stress.
  - We can’t add a shear component to a normal stress component, because stresses are tensorial quantities which need at least 2 directions rather than one as in the vectors.

Analysis of Stress in Combined Loading

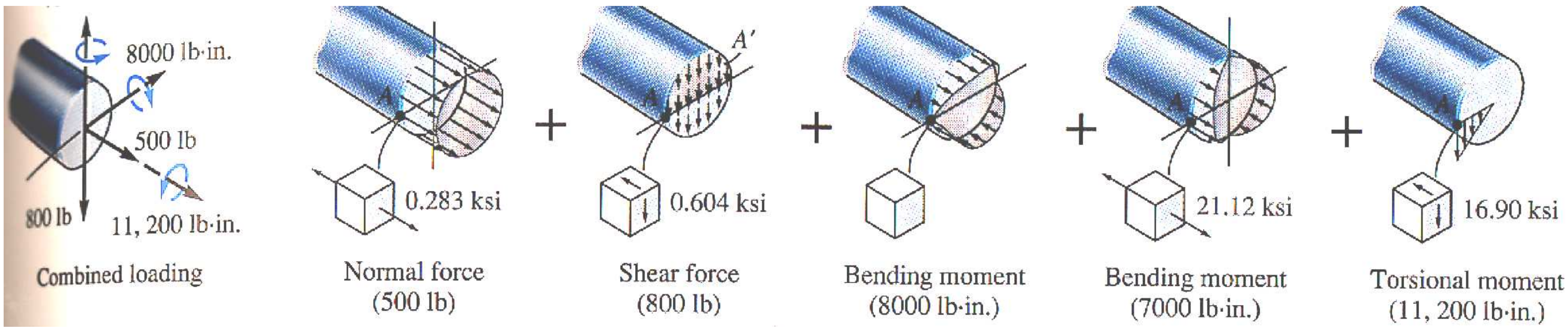
Remember to assign positive or negative signs for the stresses according to

- Axial Loading σ + (tension), σ(-) compression
- Bending: σ sign depends on both M sign and c sign, M sign from Moment diagram, c sign from N.A location
- Pressure: Inside pressure σ +, outside pressure σ -
- Thermal: Heating: expansion +σ, cooling: contraction: - σ
- Torsion: τ sign depends on torque (T) sign (Right hand rule)
- Transverse Shear: τ sign depends on Q sign (N.A location and section of interest) and shear force sign V, from shear diagram)

Combined Loading

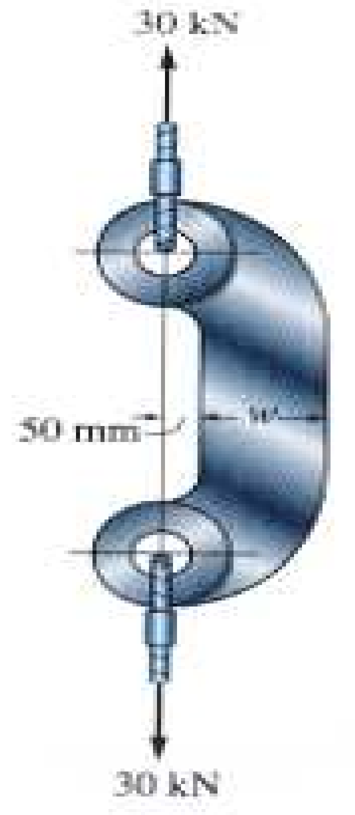


- This example has multiple stress components. The stresses on the rod are axial, shear, two bending, and a torsional stress.
- NOTE THAT ONLY 2 LOADS LEADS TO 5 STRESS COMPONENTS !!
- When found, each stress value is understood as shown in the diagram below.



Problem 14.11

- The offset link supports the loading shown. Determine its required width w if the allowable normal stress is 73MPa. The link thickness is 40mm.



Prob. 14–11

Solution

σ due to axial force

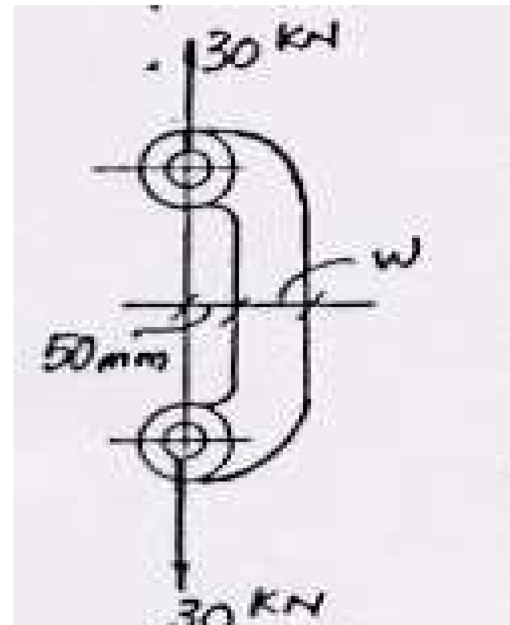
$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{w(0.04)} = \frac{750(10^3)}{w}$$

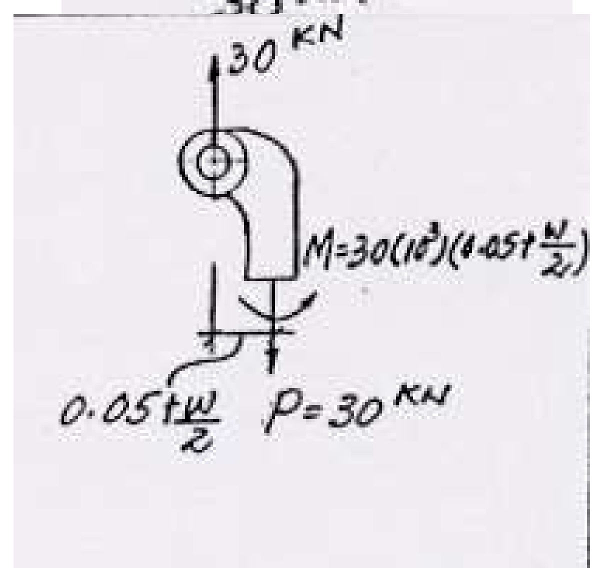
σ due to bending

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)\left(0.05+\frac{w}{2}\right)\left(\frac{w}{2}\right)}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500(10^3)\left(0.05+\frac{w}{2}\right)}{w^2}$$

$$\sigma_{\max} = \sigma_{allow} = \sigma_a + \sigma_b$$





Solution

$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)\left(0.05 + \frac{w}{2}\right)}{w^2}$$

$$73w^2 = 0.75w + 0.225 + 2.25w$$

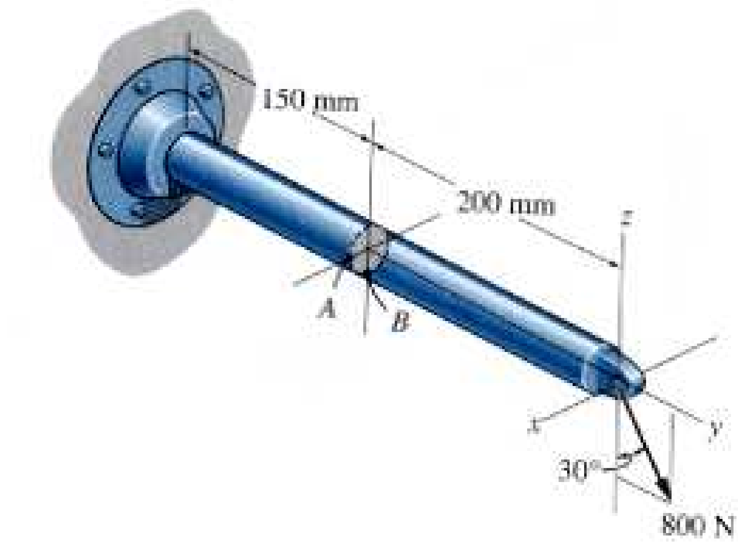
$$73w^2 - 3w - 0.225 = 0$$

$$w = 0.0797m$$

$$= 79.7mm$$

Problem 14.20

- The bar has a diameter of 40mm. If it is subjected to a force of 800N as shown, determine the stress components that act at points A and B and show results on volume elements located at these points.



Prob. 14–20

Solution

$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4)$$

$$= 0.1256637\ (10^{-6})m^4$$

$$A = \pi r^2 = \pi(0.02^2)$$

$$= 1.256637\ (10^{-3})$$

$$Q_A = \bar{y}'A' = \left(\frac{4(0.02)}{3\pi}\right)\left(\frac{\pi(0.02)^2}{2}\right)$$

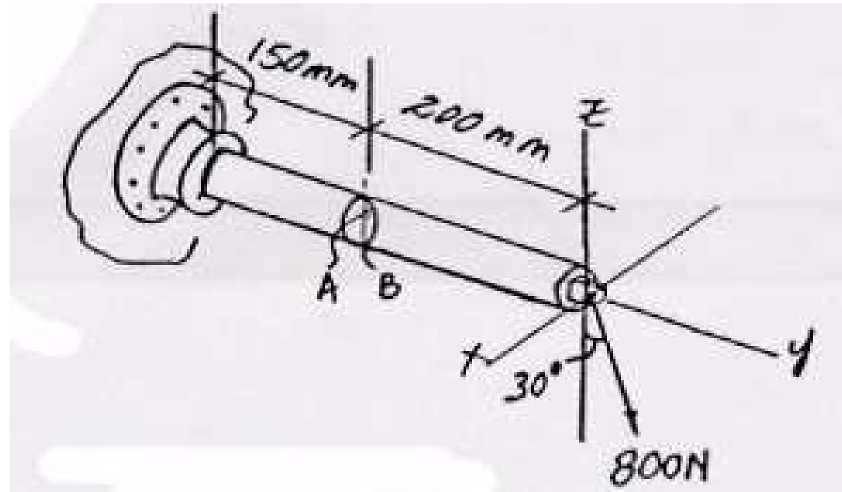
$$= 5.3333\,(10^{-6})m^3$$

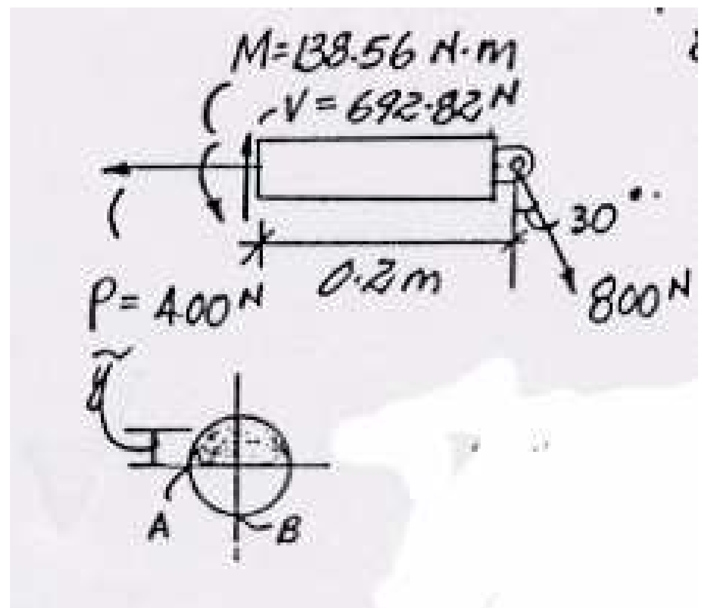
$$Q_B = 0$$

Axial stress : Axial extension + Bending

$$\sigma_A = \frac{P}{A} + \frac{Mz}{I} = \frac{400}{1.256637\ (10^{-3})} + 0$$

$$= 0.318\ MPa$$





Solution

Shear Stress: Only transverse shear

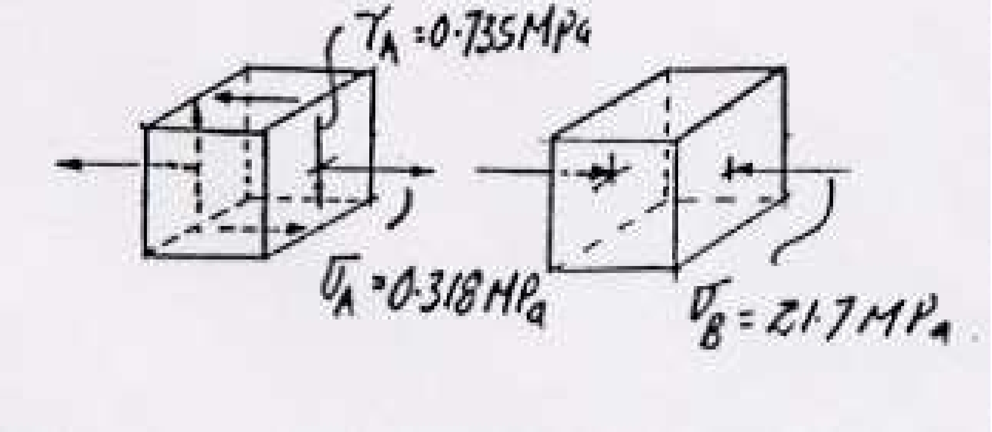
$$\tau_A = \frac{VQ_A}{It} = \frac{692.82(5.3333)(10^{-6})}{0.1256637(10^{-6})(0.04)}$$

$$= 0.735 MPa$$

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{0.1256637(10^{-6})} - \frac{138.56(0.02)}{0.1256637(10^{-6})}$$

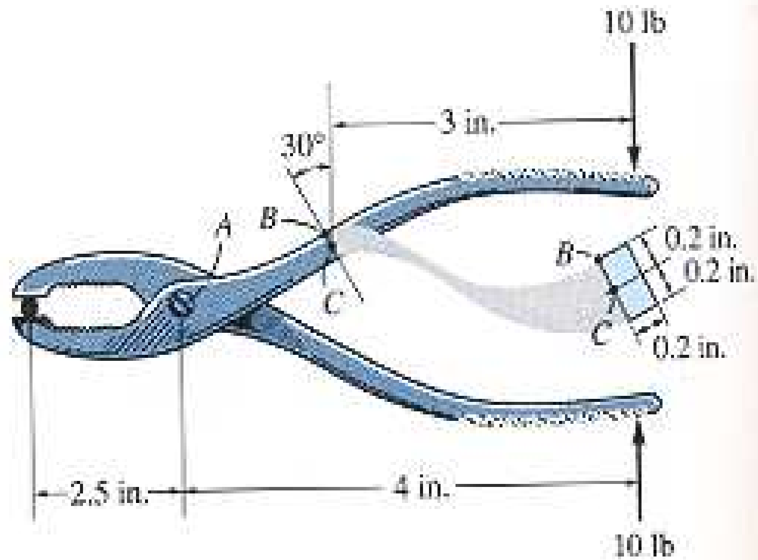
$$= -21.7 MPa$$

$$\tau_B = 0$$



Problem 14.25

- The pliers are made from two steel parts pinned together at A.
- If a smooth bolt is held in the jaws and a gripping force of 10lb is applied at the handles, determine the stress components in the pliers at points B and C.
- The cross section is rectangular having dimensions as shown.



Prob. 14–25

Solution

$\nwarrow \quad \sum F_X = 0 \qquad V - 10\cos 30 = 0$

$V = 8.660\,lb$

$\nearrow \quad \sum F_y = 0 \qquad N - 10\sin 30 = 0$

$N = 5lb$

$\sum M_c = 0 \qquad M - 10(3) = 0$

$M = 30\,lb$

$$I = \frac{1}{12}(0.2)(0.4^3)$$

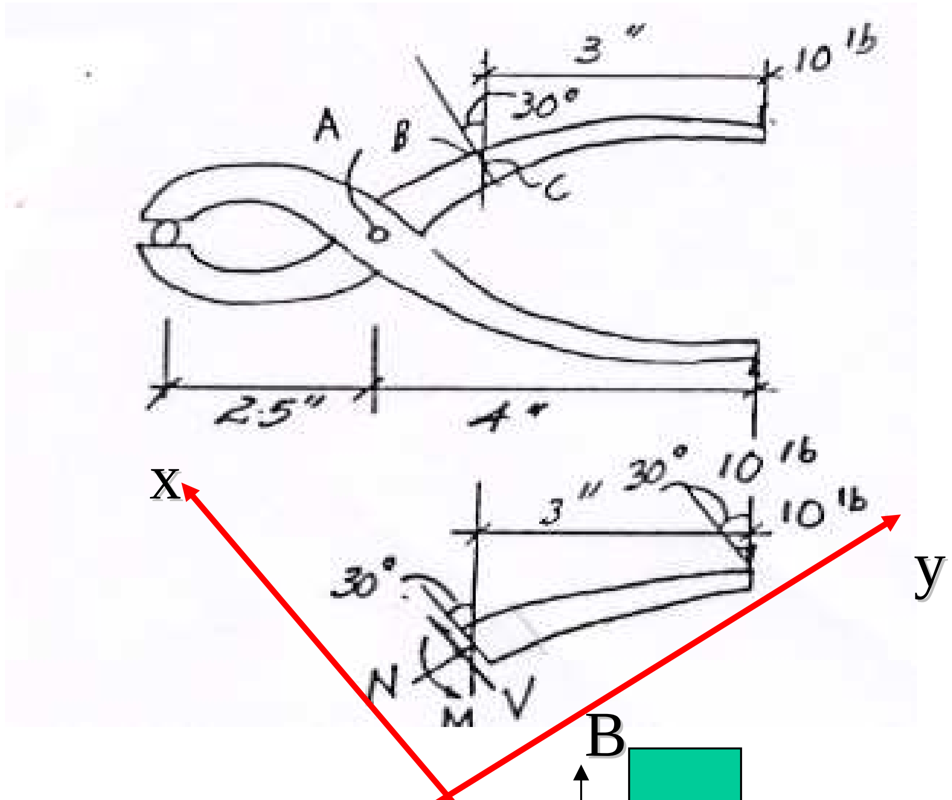
$$= 1.0667\,(10^{-3})$$

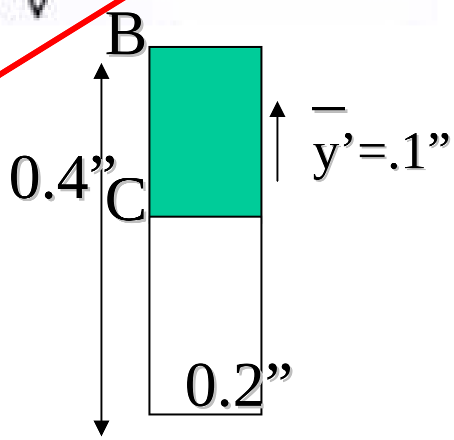
$$Q_c = \bar{y}'A' = (0.1)(0.2)(0.2)$$

$$= 4(10^{-3})in^3$$

$$Q_B = 0$$

$$A = 0.2(0.4) = 0.08\,in^2$$



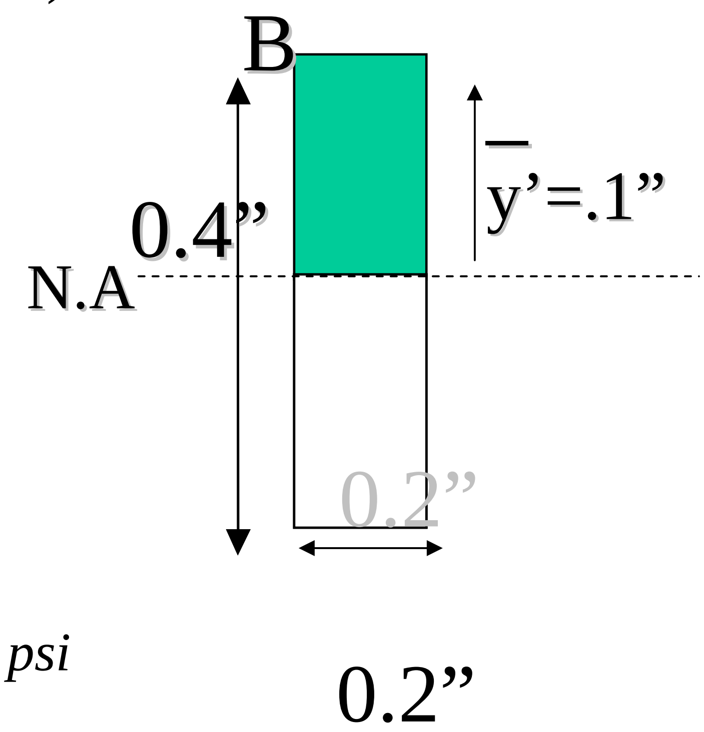


Solution

$$\sigma_B = \frac{-N}{A} + \frac{My}{I} = \frac{-5}{0.08}\frac{30(0.2)}{1.0666(10^{-3})}$$

$$= 5562\ psi$$

$$= 5.56\ ksi$$



$$\tau_B = 0$$

$$\sigma_c = \frac{-N}{A} + \frac{My}{I} = \frac{-5}{0.08} = -62.5\ psi$$

$$\tau_c = \frac{VQ_c}{It} = \frac{8.660(4)(10^{-3})}{1.0667(10^{-3})(0.2)} = 162\ psi$$