EML 3011C
Mechanics & Materials
Chapter1

Force

FAMU-FSU College of Engineering
Department of Mechanical Engineering
Spring 2007

Mechanics

- Concerned with the state of rest or motion of bodies
- Two Branches
  - Statics
  - Mechanics of Materials

Mechanics

- Statics
  - Equilibrium of Bodies
- Mechanics of Materials
  - Relationship between the external loads, the intensity of internal forces & its deformation response

Basic Concepts

Quantities

- Length (location, position, size)
- Time (succession of events)
- Force (Push, Pull)
- Mass (Properties of Matter)

Idealization

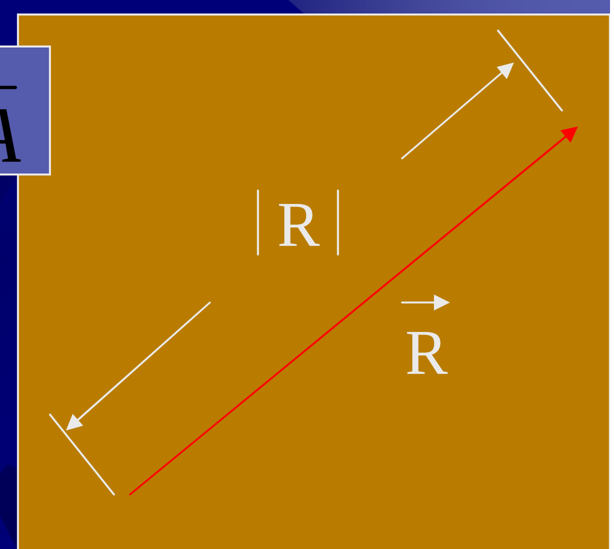
- Particle (neglect, size, geometry)
- Rigid Body (all points within remain in the same position, at fixed distances from each other)
- Concentrated Force (over a very small area, zero)

Basic Concepts
(Vector Operations)

- Physical Qualities
  - Mass, Force, time distance, density, Temperature, volume, area, length, displacement, velocity, acceleration, weight

Basic Concepts
(Vector Operations)

- Scalar Quantities
  - Described by their magnitude, mass
  - (*italic form*) or lower case (a for A)
- Vector Quantities
  - Described by a magnitude, a direction, and a point of application
  - (**Bold Face**) in the book
  - Bar or Arrow in handwritten work $\vec{A}, \bar{A}$
  - Magnitude $|A|$ or *A (italic)* or a = $|A|$



Basic Concepts

- Newton's First Law
  - A body at rest tends to remain at rest & a body in motion at a constant velocity will tend to maintain the velocity.
- Newton's Second Law
  - Change of motion is proportional to the moving force impressed and takes place in the direction of the straight line in which such force is impressed.

$$\overline{F} = m\overline{a}$$

Basic Concepts

- Newton's Third Law
  - When two bodies interact, a pair of equal and opposite reaction forces will exist at their contact point
  - This force pair will have the same magnitude and acts along the same direction, but have opposite <u>sense</u>
  - The mutual force of action and reaction between two bodies are equal, opposite, and collinear

Kg = mass

lbf =forces

Basic Concepts

- Gravitational Law

$$F = G\frac{m_1 m_2}{r^2}$$

G = universal constant of gravitation

$$= 66.73 \cdot 10^{-12}\ m^3 / kg \cdot s^2$$

m_1,m_2 = mass of each of the two particles

r= distance

Basic Concepts

- Weight

If m1 = mass of the particle

m2 = mass of the earth

r = distance to the earth's center

W = weight of the particle

$$W = G\frac{mm_2}{r^2}$$

if

$$g = \frac{Gm_2}{r^2} \rightarrow W = mg$$

Basic Concepts

- Units
  - Length, time, mass, force – basic quantities

$$\overline{F} = m\overline{a}$$

(Note: we use bars to denote forces or vectors)

- SI (International System of Units)
  - Meter (m)
  - Second (sec)
  - Kilogram (kg)
  - Newton (N) $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$

$$W = mg$$

$$g = 9.81 m/s^2$$

Ex : mass = 1kg

→ W = 9.81N

Basic Concepts

- US Customary (FPS : Feet Pounds Seconds)
  - feet (ft)
  - second (sec)
  - Pound (lb)
  - Slug $\left(\frac{\mathrm{lb} \cdot \mathrm{s}^2}{\mathrm{ft}}\right)$

F = ma

1 lb = 1 slug $\cdot 1\ ft/s^2 \Rightarrow slug = lb \cdot s^2/ft$

$$m = \frac{W}{g} \rightarrow 32.2\ ft/s^2$$

$$g = 9.81\, m/s^2 = 32.2\ ft/s^2$$

Basic Concepts

- Conversion of Units

	FPS		SI
Force	1 lb	=	4.4482 N
Mass	1 slug	=	14.5938 kg
Length	1 ft	=	0.3048 m

Ex:

$$2\,{}^{kN}\!/_{m} = \frac{2kN}{m}\left(\frac{1000N}{1kN}\right)\left(\frac{1lb}{4.4482N}\right)\left(\frac{0.3048m}{1ft}\right)$$

Chapter 2

- Force Vectors
  - Scalars : A quantities represented be a number (positive or negative)

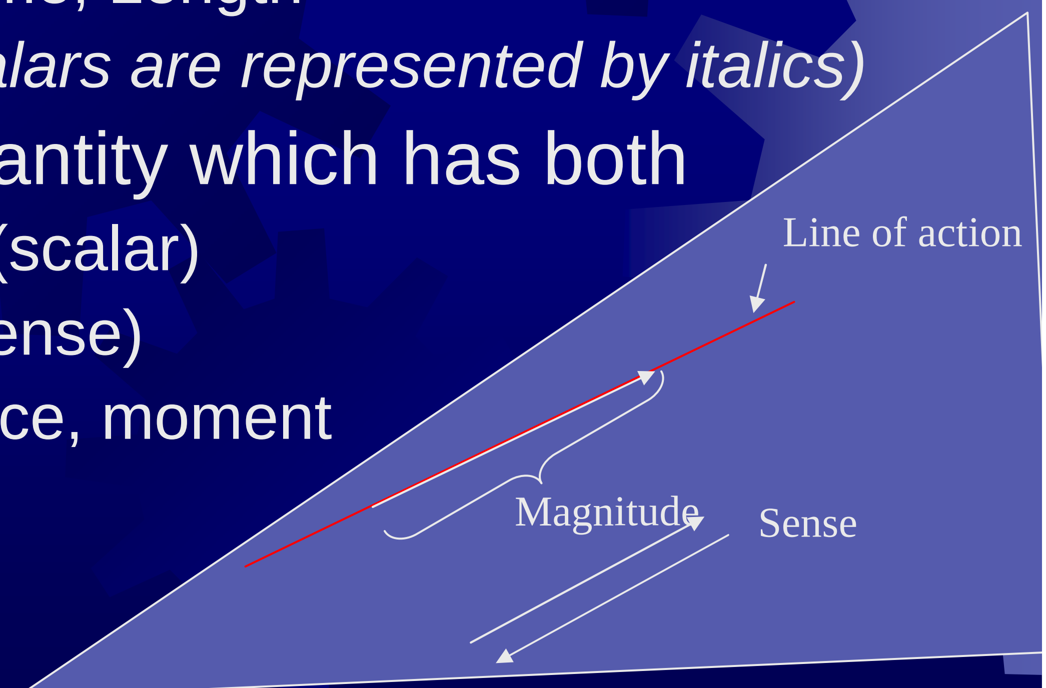
    Ex: Mass, Volume, Length

    (in the book scalars are represented by italics)
  - Vectors : A quantity which has both

    A – magnitude (scalar)

    B – direction (sense)

    Ex: position, force, moment

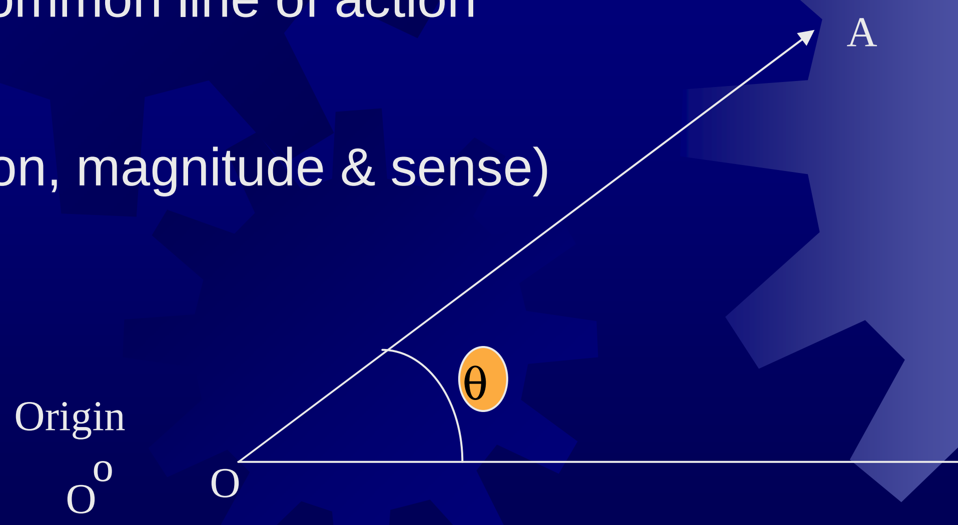


Chapter 2

- Classification of Forces
  - Contact
    - 1 – Contacting or surface forces (mechanical)
    - 2 – Non-Contacting or body forces (gravitational, weight)
  - Area
    - 1 – Distributed Force, uniform and non-uniform
    - 2 – Concentrated Force

Chapter 2

- Classification of Forces
  - Force System
    - 1 – Concurrent : all forces pass through a point
    - 2 – Coplanar : in the same plane
    - 3 – Parallel : parallel line of action
    - 4 – Collinear : common line of action
  - Three Types
    - 1 – Free (direction, magnitude & sense)
    - 2 – Sliding
    - 3 – Fixed

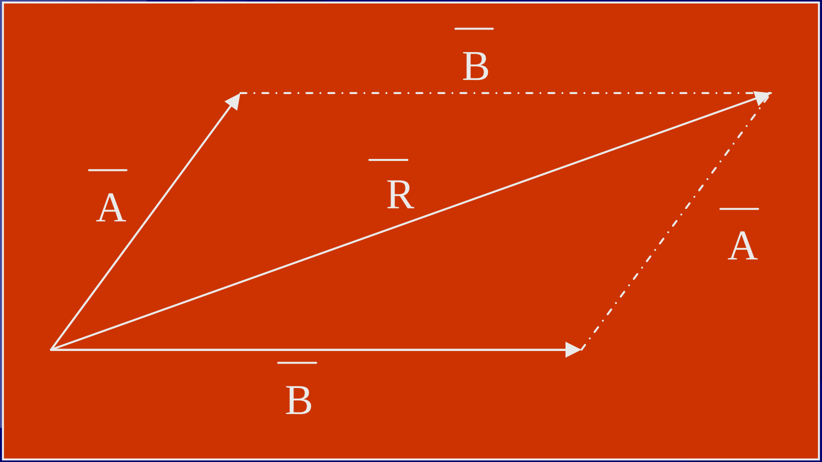


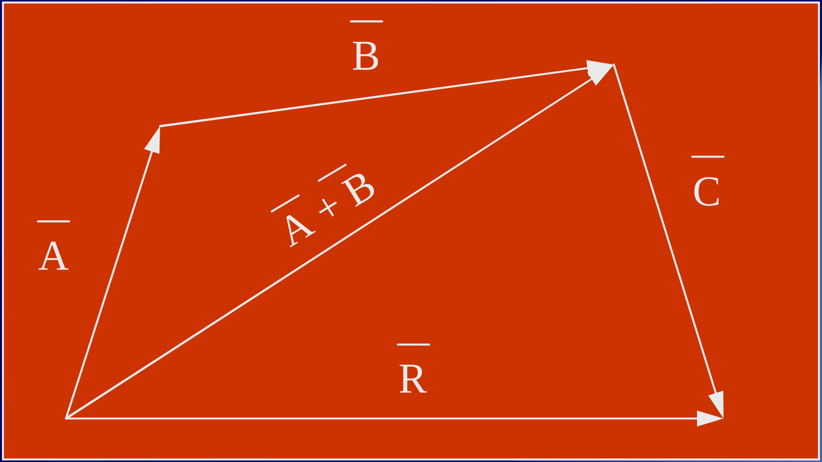
Chapter 2

- Properties of Vectors
  - 1 – Vector Addition
  - 2 – Vector Subtraction
  - 3 – Vector Multiplication

Chapter 2

Vector Addition





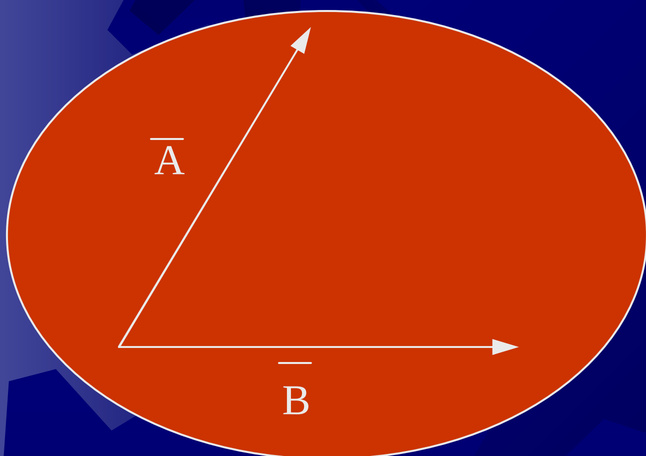
$$\overline{R} = \overline{A} + \overline{B} = \overline{B} + \overline{A}$$

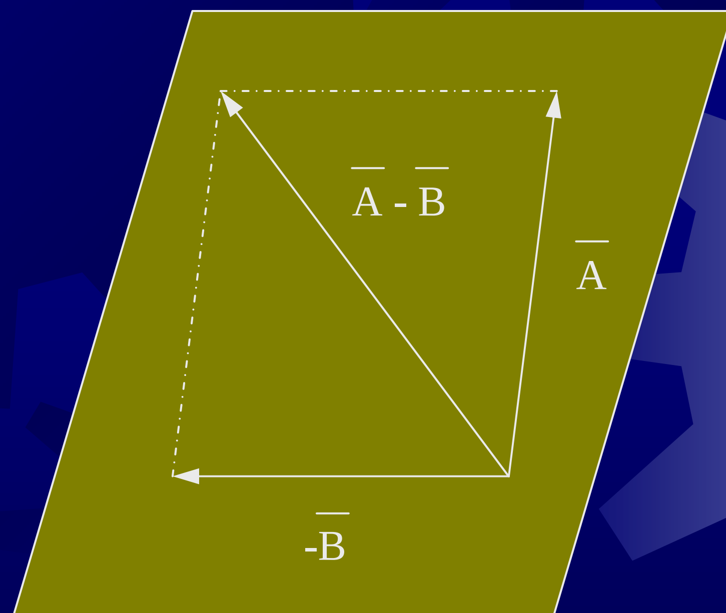
$$\overline{R} = \overline{A} + \overline{B} + \overline{C}$$

Chapter 2

Vector Subtraction

$$\overline{A} - \overline{B} = \overline{A} + \left(-\overline{B}\right)$$

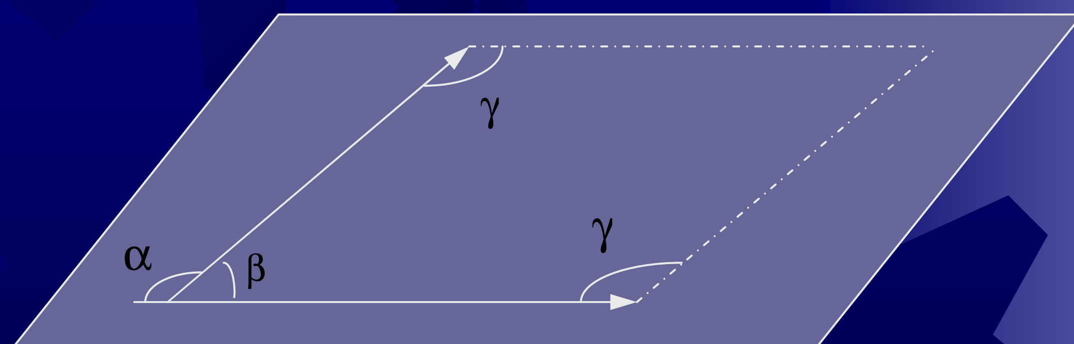




Methodology

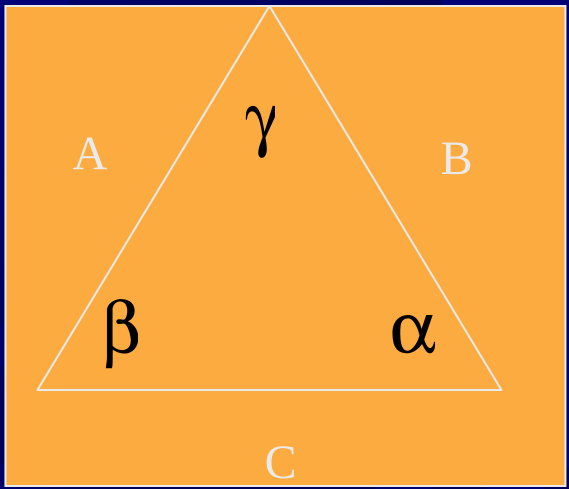
- Use of a Parallelogram
  - A – sum of the three angles is 180º
  - B – sum of the interior angles is 360º
  - C –

$$\alpha + \beta = 180$$
$$\alpha = \gamma$$



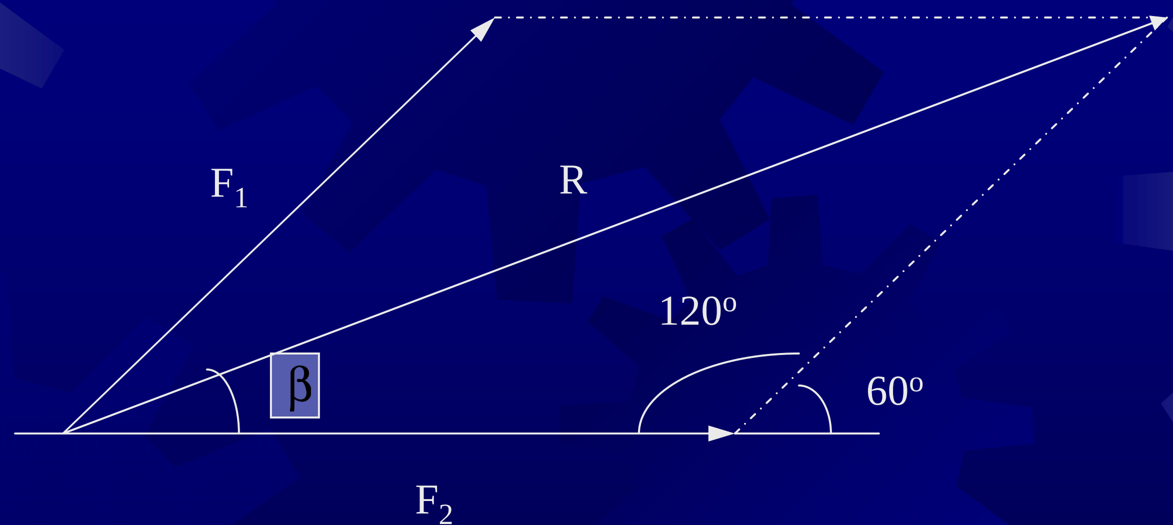
Methodology

- Trigonometry
  - A – Sin Law $\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$
  - B – Cosine Law $C^2 = A^2 + B^2 - 2AB\cos C$



Methodology

Ex: If the bottom angle between F_1 = 54N & F_2 = 60N is 60º. Find the Resultant force & the angle β.



Methodology

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\phi$$

$$R^2 = 60^2 + 54^2 - 2\cdot 60\cdot 54\cdot\cos(120)$$

$$|R| = 98.77 \approx 98.8N$$

$$\frac{\sin\beta}{F_1} = \frac{\sin 120}{R}$$

$$\sin\beta = \frac{F_1}{R}\sin 60$$

$$\beta = 28.26^o$$

Methodology

Vector Multiplication

$$(m+n)\vec{A} = m\vec{A} + n\vec{A}$$

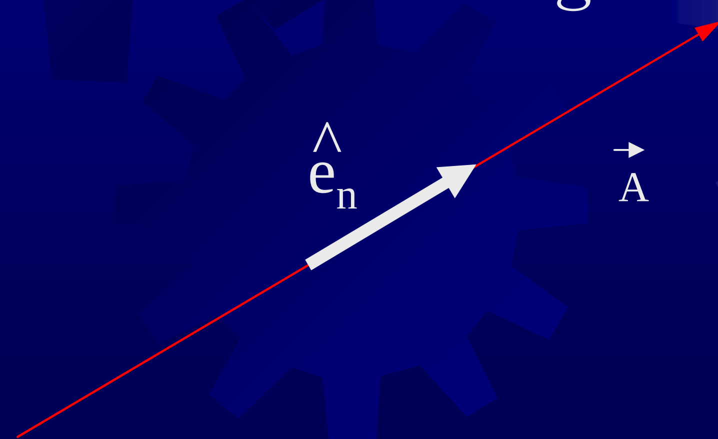
$$m(\vec{A}+\vec{B}) = m\vec{A} + m\vec{B}$$

$$m(n\vec{A}) = mn\vec{A}$$

Unit Vector : a vector with a unit magnitude

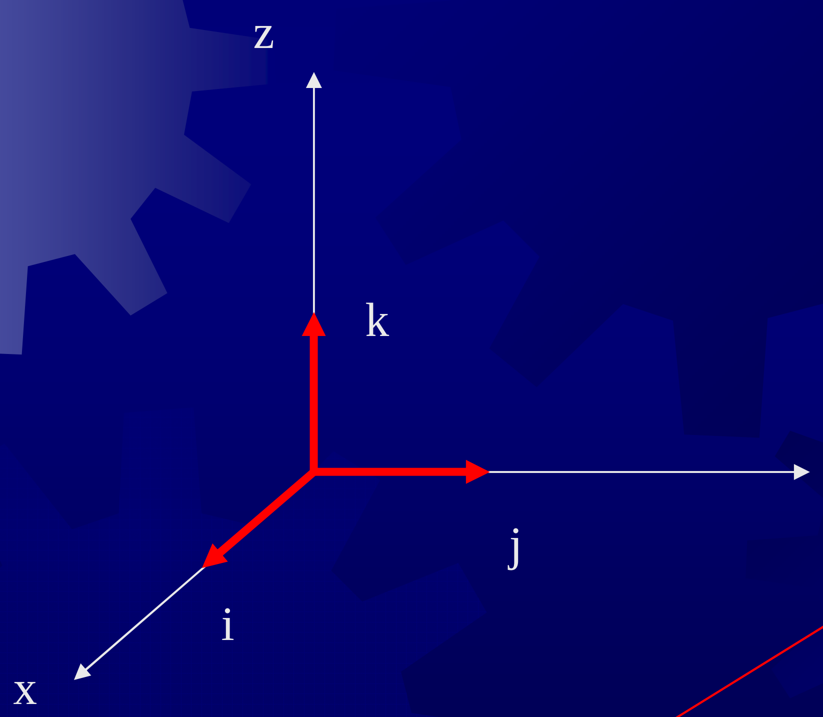
$$\vec{A} = |A|\hat{e}_n$$

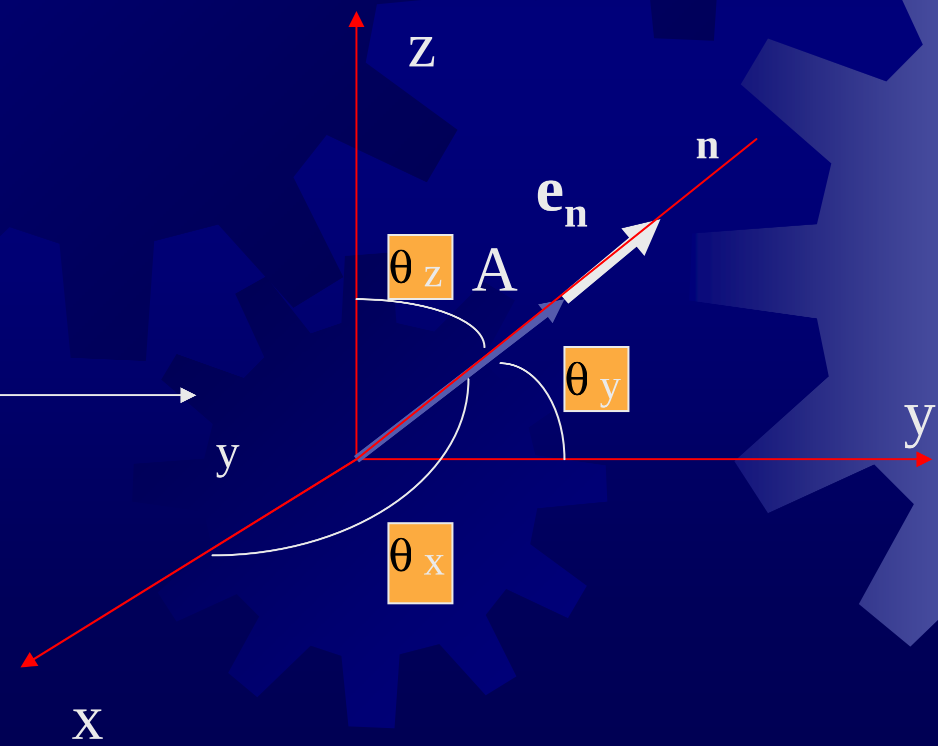
$$\hat{e}_n = \frac{\vec{A}}{|A|}$$



Coordinate Systems

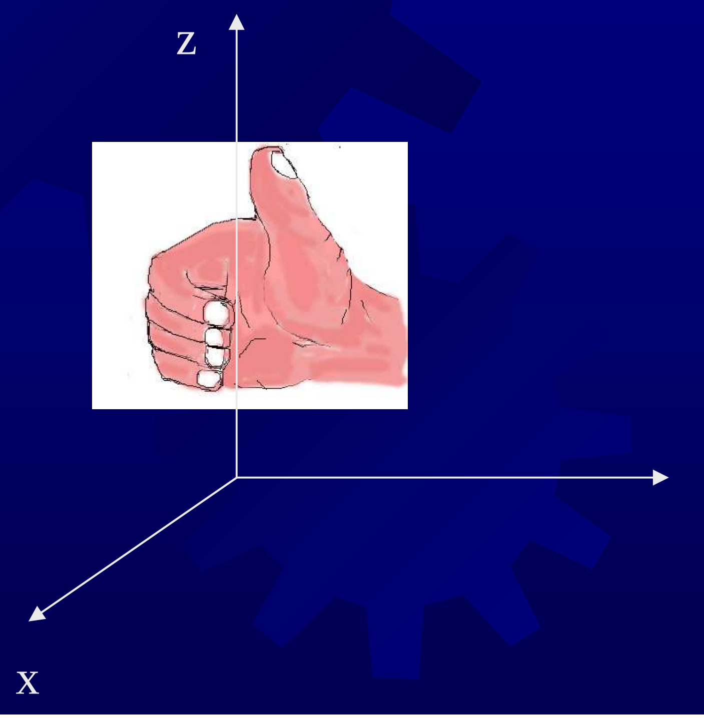
- Cartesian
  - Simplification of Vector Analysis





Right Handed Coordinate System

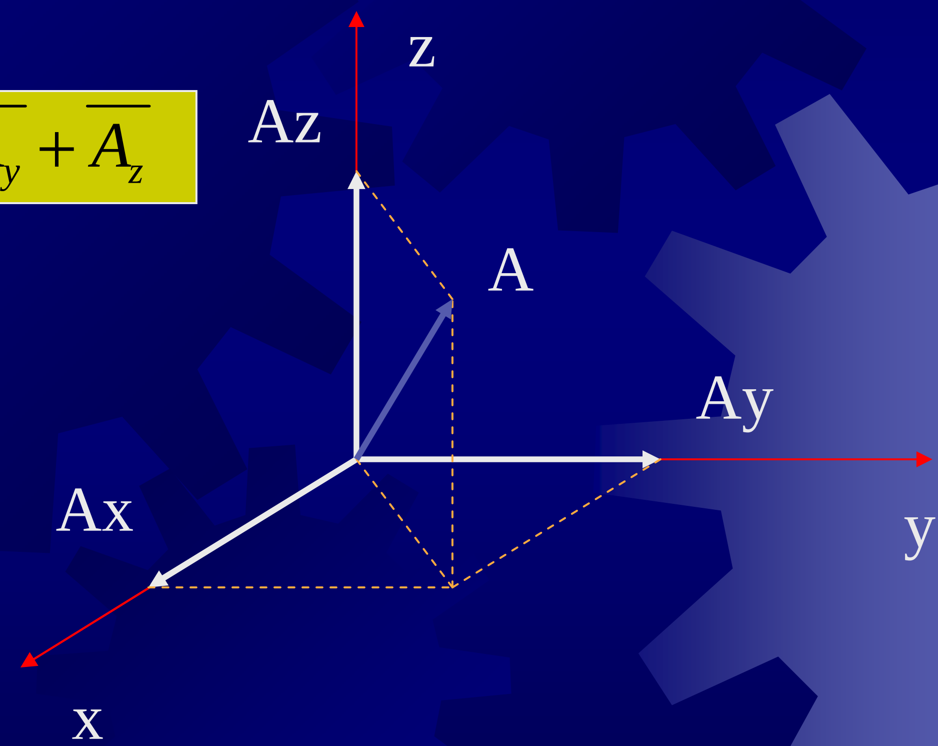
- If the thumb of the right hand points in the direction of the positive z-axis when the fingers are pointed in the x-direction & curled from the x-axis to the y-axis.
- Imagine pushing the x-axis into the y-axis

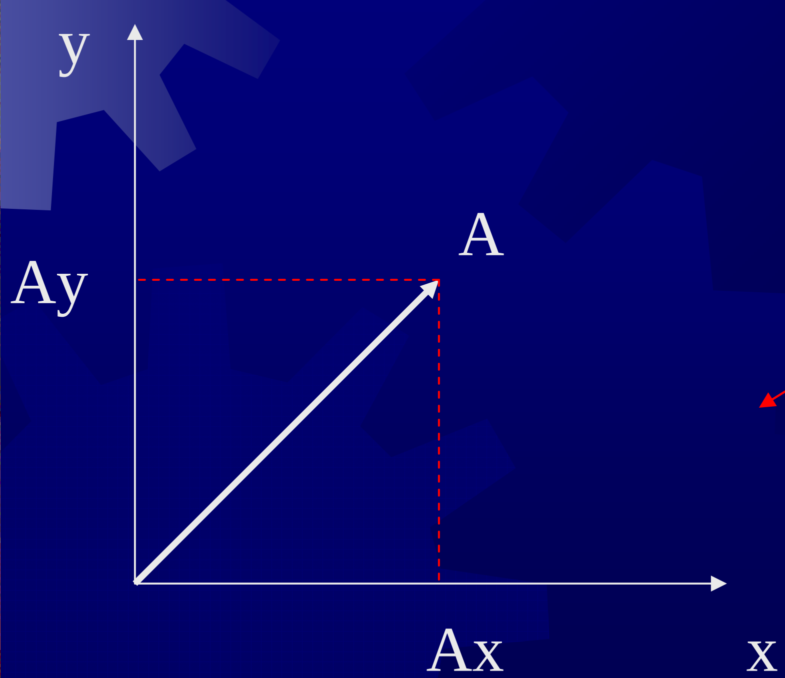


Coordinate Systems

- Cartesian (Rectangular) Components of a Vector

In 3-D $$\overline{A} = \overline{A_x} + \overline{A_y} + \overline{A_z}$$





In 2-D $$\overline{A} = \overline{A_x} + \overline{A_y}$$

Coordinate Systems

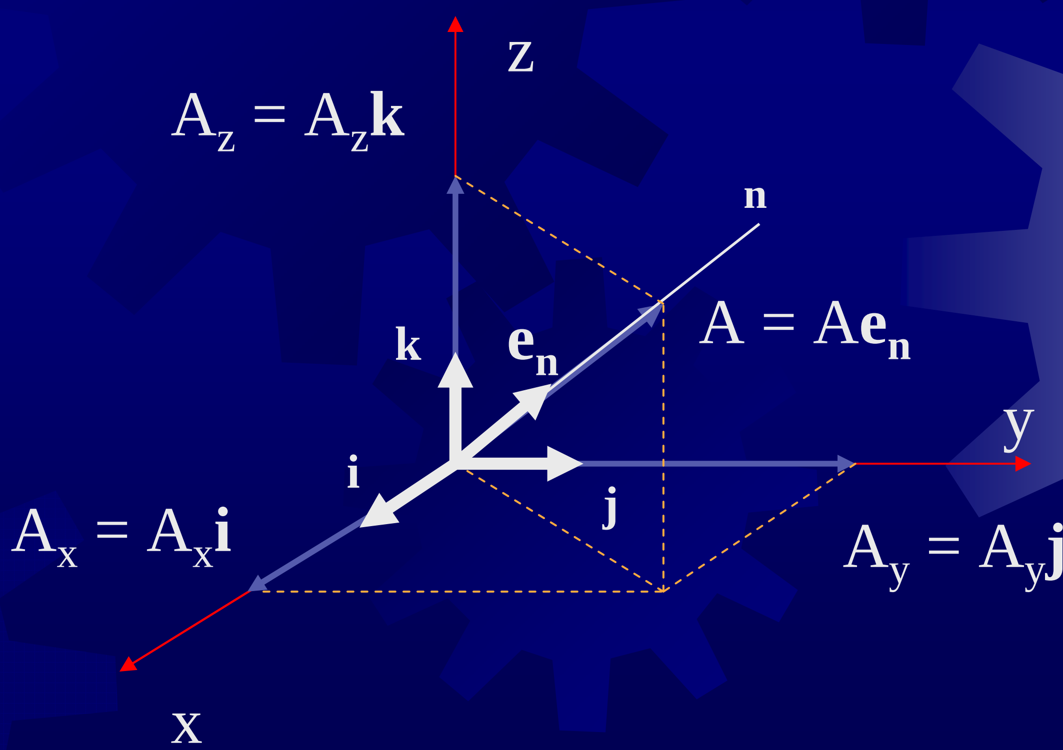
- Cartesian Unit Vectors

$$\overrightarrow{A_x} = |A_x|\hat{i}$$

$$\overrightarrow{A_y} = |A_y|\hat{j}$$

$$\overrightarrow{A_z} = |A_z|\hat{k}$$

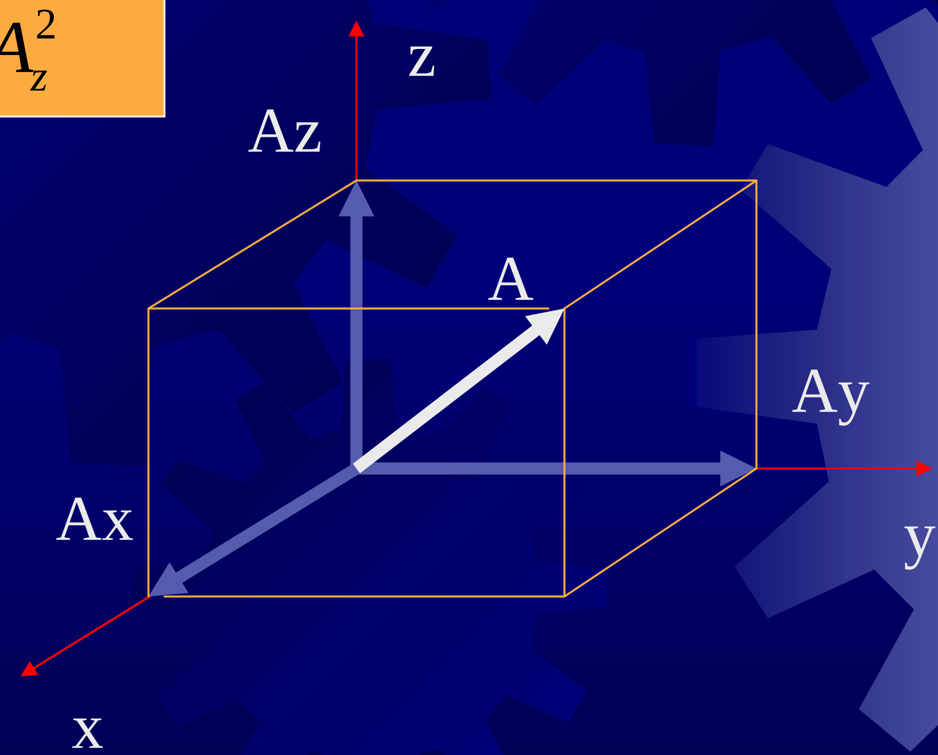
$$\Rightarrow \overrightarrow{A} = A_x\hat{i} + A_y\hat{j} + A_x\hat{k}$$



Coordinate Systems

- Magnitude of a Cartesian Vector

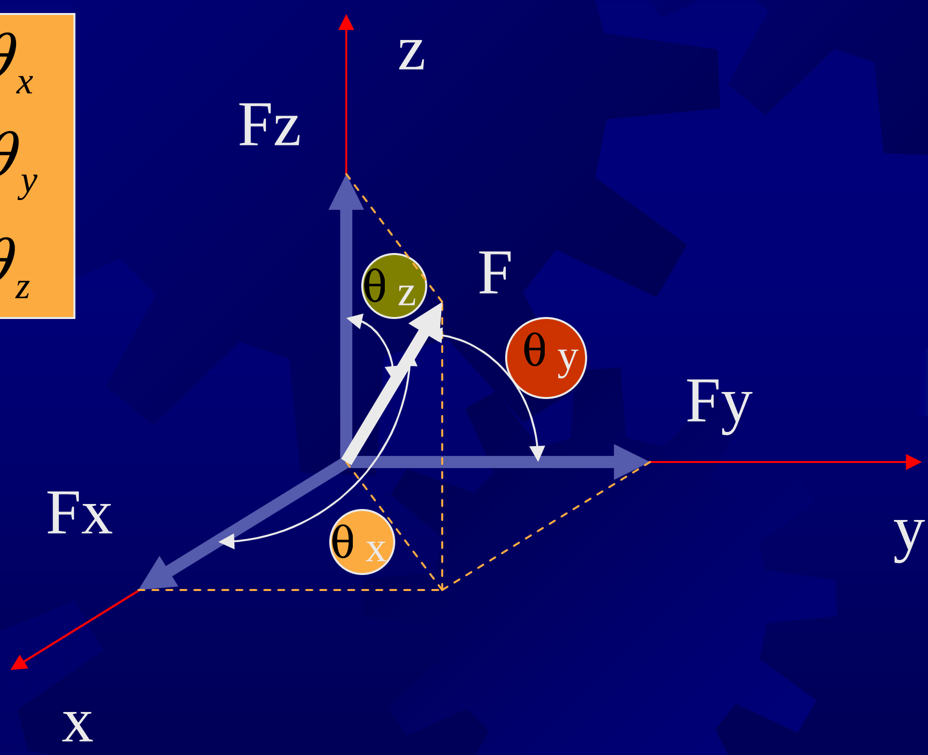
$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Force Analysis

- Force is treated like any vector!

$$F_x = F\cos\theta_x$$
$$F_y = F\cos\theta_y$$
$$F_z = F\cos\theta_z$$



$$\cos\theta_x = \frac{F_x}{F}$$
$$\cos\theta_y = \frac{F_y}{F}$$
$$\cos\theta_z = \frac{F_z}{F}$$

Force Analysis

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$ **magnitude**

$$\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$$ **Cartesian**

$$\vec{F} = F\cos\theta_x\hat{i} + F\cos\theta_y\hat{j} + F\cos\theta_z\hat{k}$$

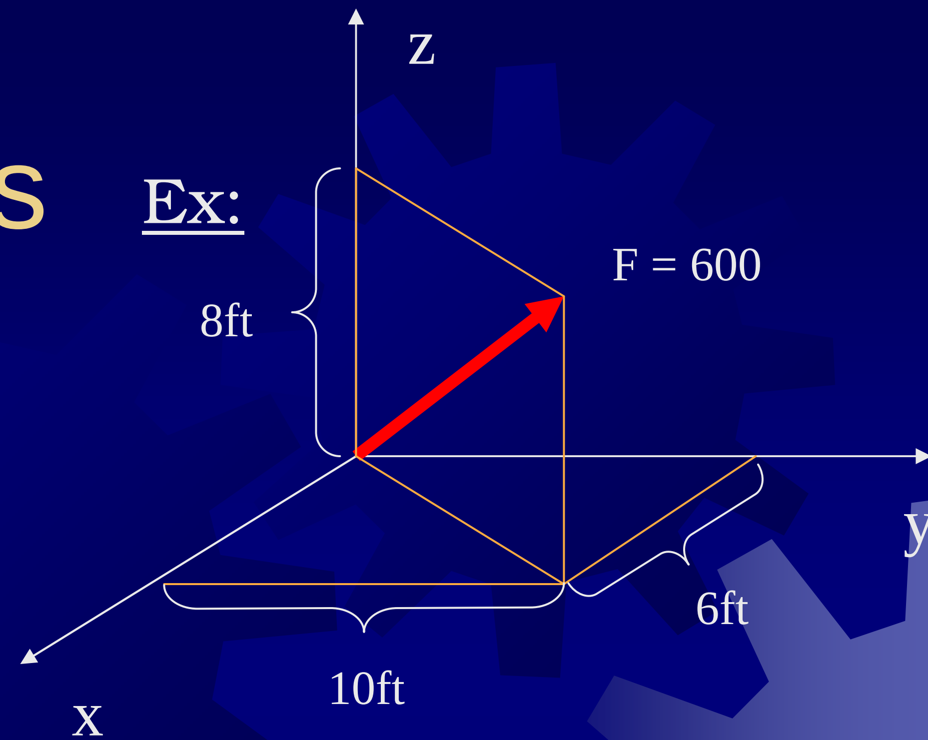
$$\vec{F} = F(\cos\theta_x\hat{i} + \cos\theta_y\hat{j} + \cos\theta_z\hat{k})$$

$$\hat{U}_f = \cos\theta_x\hat{i} + \cos\theta_y\hat{j} + \cos\theta_z\hat{k}$$ **direction**

$$\vec{F} = F\hat{U}_f$$ **magnitude and direction**

Force Analysis

Ex:



$$d = \sqrt{x^2 + y^2 + z^2}$$
$$= \sqrt{6^2 + 10^2 + 8^2}$$
$$= 14.14\ ft$$

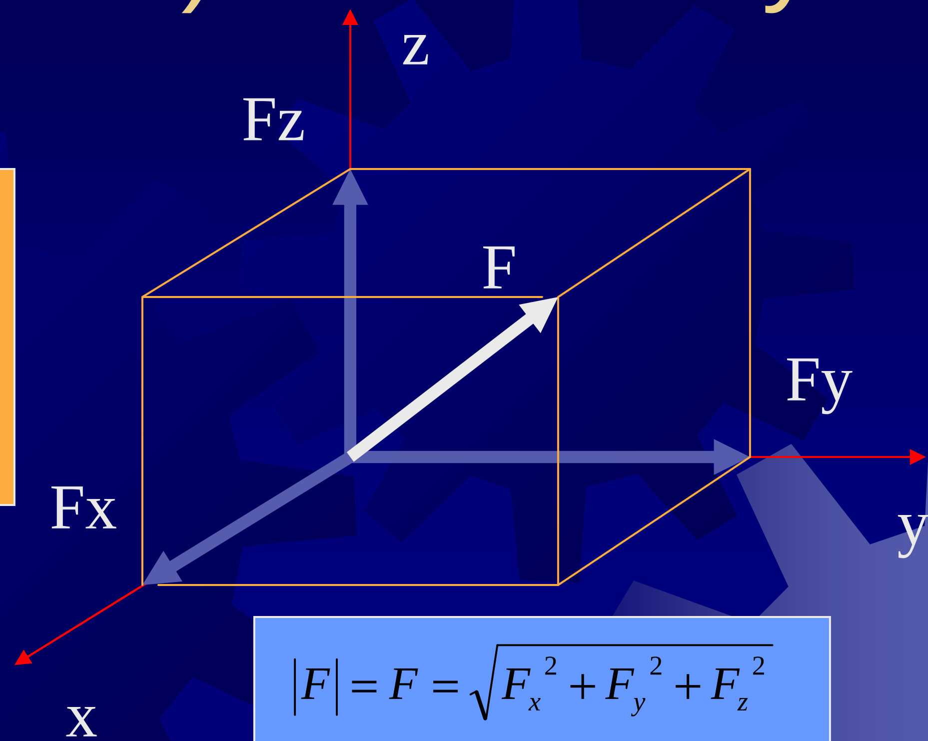
$$F_x = F\cos\theta_x = 600\frac{6}{14.14} = 254\ lb$$
$$F_y = F\cos\theta_y = 600\frac{10}{14.14} = 424\ lb$$
$$F_z = F\cos\theta_z = 600\frac{8}{14.14} = 339\ lb$$

$$F = (255\hat{i} + 424\hat{j} + 339\hat{k})lb$$

Use geometry to get direction !!

Summary of (vector) Force Analysis

$$\overline{F} = \overline{F_x} + \overline{F_y} + \overline{F_z}$$
$$= F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$
$$= F\cos\theta_x\hat{i} + F\cos\theta_y\hat{j} + F\cos\theta_z\hat{k}$$



$$F_x = F\cos\theta_x$$
$$F_y = F\cos\theta_y$$
$$F_z = F\cos\theta_z$$

$$|F| = F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

$$\theta_x = \cos^{-1}\frac{F_x}{F}$$

$$\theta_y = \cos^{-1}\frac{F_y}{F}$$

$$\theta_z = \cos^{-1}\frac{F_z}{F}$$

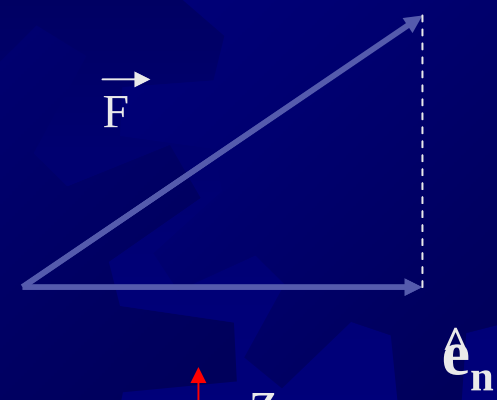
Remember :

$$Cos^2\theta_x + Cos^2\theta_y + Cos^2\theta_z = 1$$

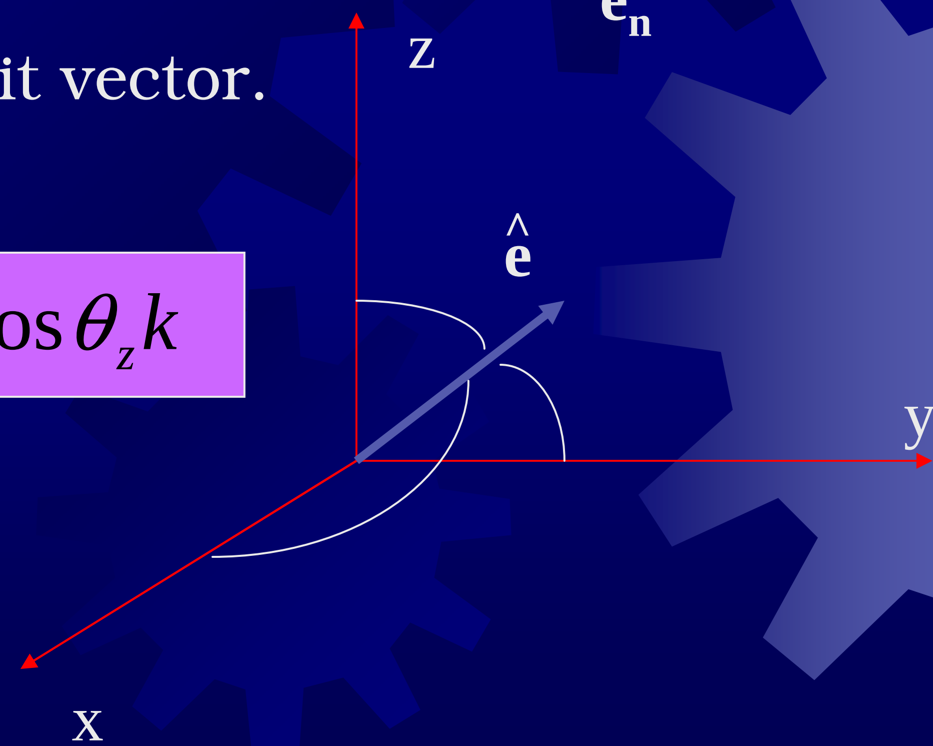
(Prove)

Components of the force

$$F_n = \overrightarrow{F} \cdot \hat{e}_n$$
$$= \left(F_x \hat{i} + F_y \hat{j} + F_z \hat{k}\right) \cdot \hat{e}_n$$



-make sure that e_n is a unit vector.



$$\hat{e}_n = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k$$

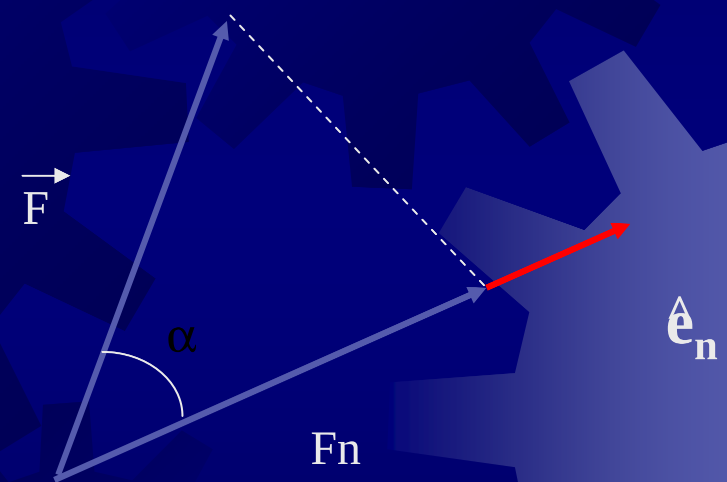
Components of the force

$$F_n = \vec{F}\cdot\hat{e}_n = F_x\cos\theta'_x + F_y\cos\theta'_y + F_z\cos\theta'_z$$

$$\overline{F_n} = \left(\overline{F}\cdot\hat{e}_n\right)\hat{e}_n$$

Magnitude

Direction



$$= F_n\left(\cos\theta'_x\hat{i} + \cos\theta'_y\hat{j} + \cos\theta'_z\hat{k}\right)$$

Question

What is the angle between $\overline{F}\ \&\ \hat{e}_n$???

$$\overline{F}\cdot\hat{e}_n = \left|F\right|\left|1\right|\cos\alpha \Rightarrow \alpha = \cos^{-1}\frac{\left|\overline{F}\right|\cdot\hat{e}_n}{\left|F\right|}$$

$$F\cdot\hat{e}_n = F\cos\alpha \Rightarrow \alpha = \cos^{-1}\frac{F_n}{F}$$

Dealing with many forces … why we need vector operations!

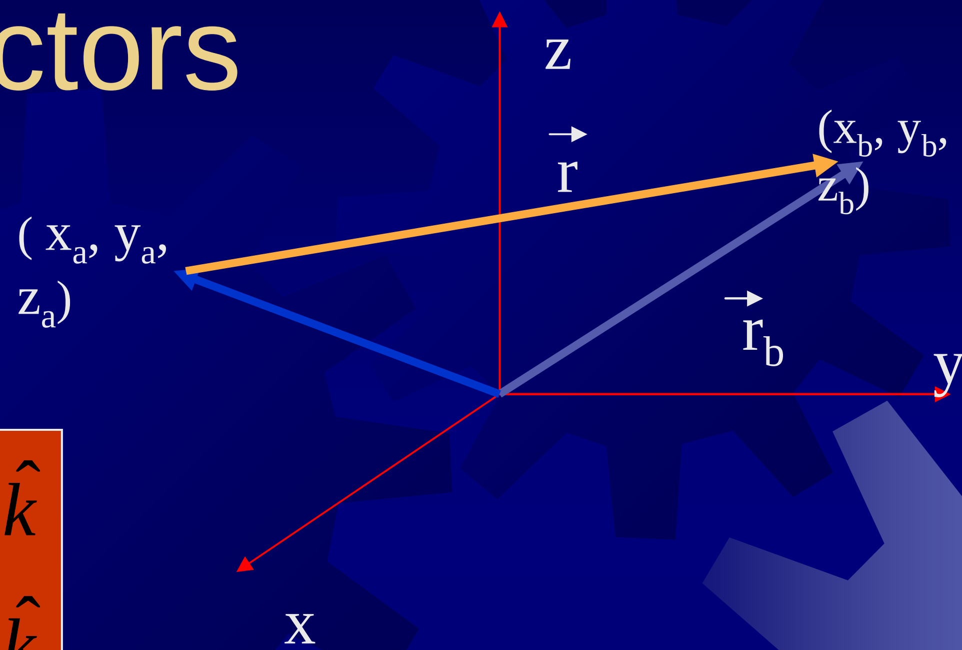
$$\vec{R}_x = \sum F_x = R_x \hat{i}$$

$$\vec{R}_y = \sum F_y = R_y \hat{j}$$

$$\vec{R}_z = \sum F_z = R_z \hat{k}$$

Position Vectors

$$\vec{r} = \vec{r}_B - \vec{r}_A$$



$$\vec{r}_B = x_B\hat{i} + y_B\hat{j} + z_B\hat{k}$$

$$\vec{r}_A = x_A\hat{i} + y_A\hat{j} + z_A\hat{k}$$

$$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

- Position vectors can be determined using the coordinates of the end & beginning of the vector. Note: from A to B is r_B-r_A

Force Vector Along a Line

A force may be represented by a magnitude & a position

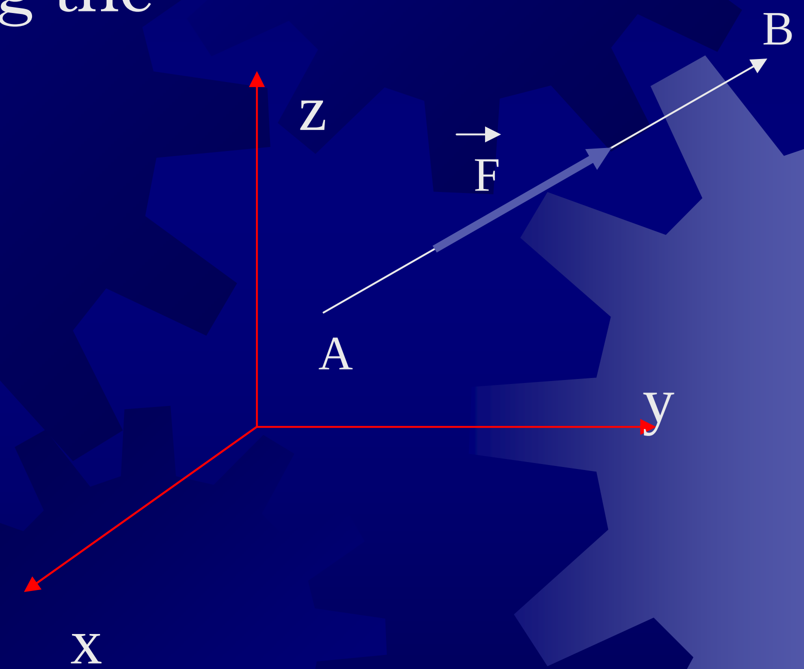
Force $\vec{F}$ is oriented along the vector AB (line AB)

$$\vec{F} = |F| u_{AB}$$

u_{AB} = Unit vector along the line AB

$$= \frac{\overrightarrow{AB}}{|AB|}$$

$$\vec{F} = |F| \frac{\overrightarrow{AB}}{|AB|}$$

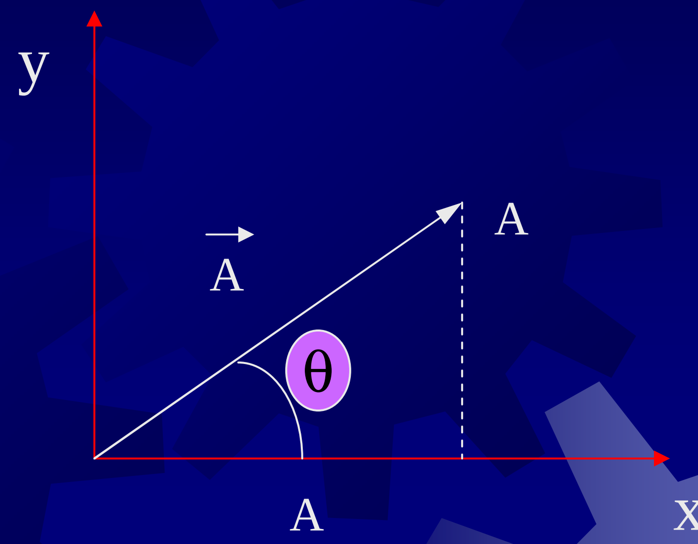


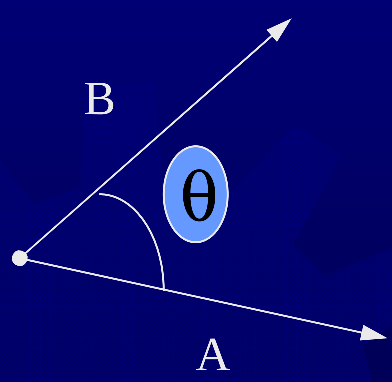
Vector Multiplication (Cartesian)

Dot Product (Scalar Product)

$$\overline{A} \cdot \overline{B} = \overline{B} \cdot \overline{A} = AB\cos\theta$$

$$A_x = A \cdot i = A(1)\cos\theta$$





$$\overline{A} = A_x \overline{i} + A_y \overline{j} + A_z \overline{k}$$

$$\overline{B} = B_x \overline{i} + B_y \overline{j} + B_z \overline{k}$$

$$\overline{A} \cdot \overline{B} = A_x B_x + A_y B_y + A_z B_z$$

Problem

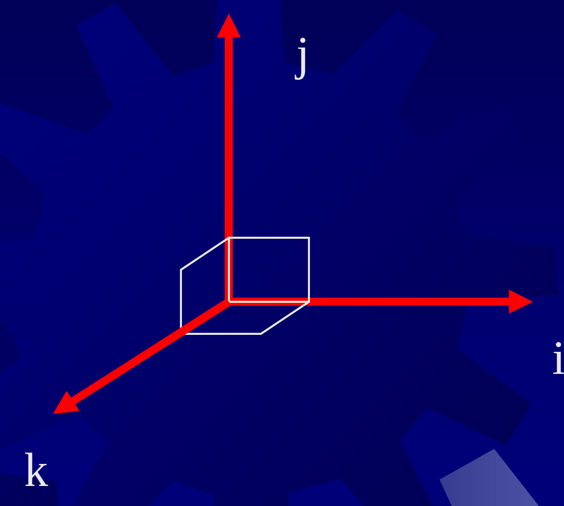
- Using the equation

A.B=ABcosθ

If

$$\overline{A} = 2i + 2j + k$$

$$\overline{B} = -i + j + k$$



- Find

a)

$\bar{i} \cdot \bar{j} =$

$\bar{i} \cdot \bar{k} =$

$\bar{j} \cdot \bar{k} =$

$\bar{j} \cdot \bar{j} =$

$\bar{i} \cdot \bar{i} =$

$\bar{k} \cdot \bar{k} =$

$b)\overline{A} \cdot \overline{A}$

c) Angle θ

Solution

$$\overline{A}\cdot\overline{B}=(-2+2+1)=(4+4+1)^{1/2}(1+1+1)^{1/2}\cos\theta$$

$$+1=3\sqrt{3}\cos\theta$$

$$\cos\theta=\frac{1}{3\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{9}$$

$$\theta=\cos^{-1}\frac{\sqrt{3}}{9}$$

Application of Dot Product

Component of a Vector along a line

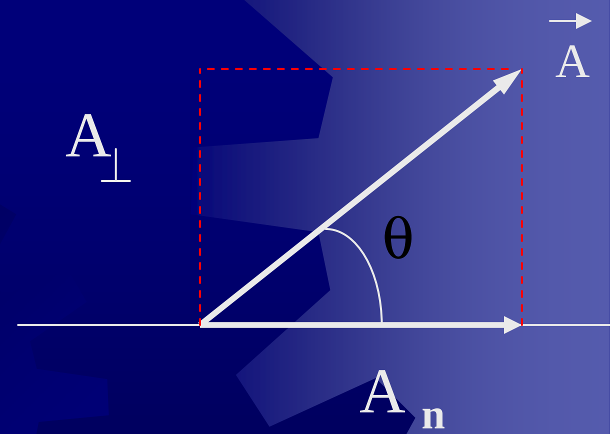
$$A_{||} = A \cos \theta$$

$$= \overline{A} \cdot \hat{U}$$

$$\overline{A}_{||} = A \hat{U} = (\overline{A} \cdot \hat{U})\hat{U}$$

$$\overline{A}_{\perp} = \overline{A} - \overline{A}_{||}$$

$$A_{\perp} = A \sin \theta$$



Application of dot product

Angle between two vectors

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\overline{A}\,\overline{B}}{AB}\right)$$