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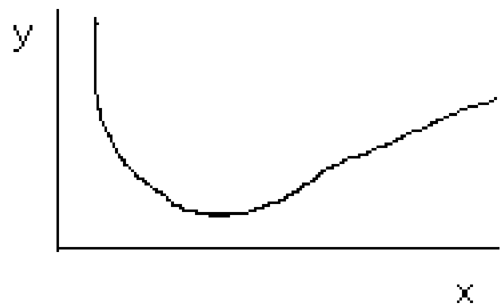
1 p369, #14, §1 Asked

Given: A particle moves along a curve described by

$$x = \tfrac{1}{2}t^2 \qquad y = \tfrac{1}{2}x^2 - \tfrac{1}{4}\ln x \tag{1}$$

Asked: The velocity and acceleration at $t = 1$

2 p369, #14, §2 Graphically



3 p369, #14, §3 Position

At $t = 1$:

$$x = \tfrac{1}{2}t^2 = \tfrac{1}{2} \qquad y = \tfrac{1}{2}x^2 - \tfrac{1}{4}\ln x = 0.298 \tag{2}$$

hence

$$\vec{r} = \begin{pmatrix} 0.5 \\ 0.298 \end{pmatrix} = 0.5\hat{\imath} + 0.298\hat{\jmath} \tag{3}$$

4 p369, #14, §4 Velocity

Velocity:

$$\vec{v} = \begin{pmatrix} \dfrac{\mathrm{d}x}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} \dfrac{\mathrm{d}x}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}x}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} t \\ (x - \frac{1}{4}x^{-1})t \end{pmatrix} = \begin{pmatrix} t \\ \frac{1}{2}t^3 - \frac{1}{2}t^{-1} \end{pmatrix} \tag{4}$$

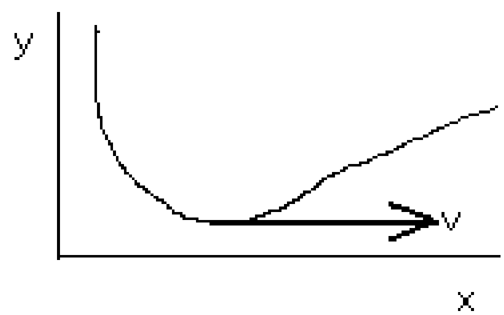
Velocity at $t = 1$:

$$\vec{v}(1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1\hat{\imath} + 0\hat{\jmath} = \hat{\imath} \tag{5}$$

Components at $t = 1$:

$$v_x \equiv \frac{\mathrm{d}x}{\mathrm{d}t} = 1 \qquad v_y \equiv \frac{\mathrm{d}y}{\mathrm{d}t} = 0 \tag{6}$$

5 p369, #14, §5 Graphically



6 p369, #14, §6 Properties

Magnitude at $t = 1$:

$$|\vec{v}| = v \equiv \frac{\mathrm{d}s}{\mathrm{d}t} = \sqrt{v_x^2 + v_y^2} = 1 \tag{7}$$

Angle with the positive x-axis at $t = 1$:

$$\tau = \arctan \frac{v_y}{v_x} = 0 \text{ (not } \pi). \tag{8}$$

7 p369, #14, §7 Acceleration

Acceleration:

$$\vec{a} = \begin{pmatrix} \frac{\mathrm{d}v_x}{\mathrm{d}t} \\ \frac{\mathrm{d}v_y}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{3}{2}t^2 + \frac{1}{2}t^{-2} \end{pmatrix} \tag{9}$$

from (4).

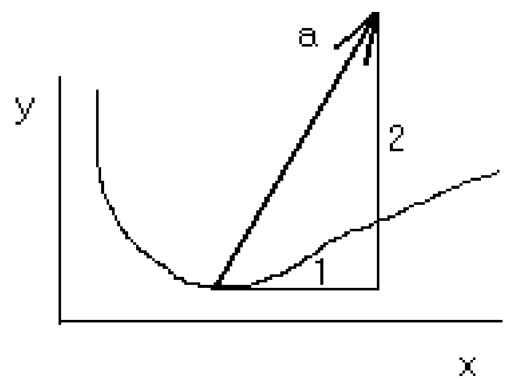
Acceleration at $t = 1$:

$$\vec{a}(1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1\hat{\imath} + 2\hat{\jmath} \tag{10}$$

Components at $t = 1$:

$$a_x \equiv \frac{\mathrm{d}v_x}{\mathrm{d}t} = 1 \qquad a_y \equiv \frac{\mathrm{d}v_y}{\mathrm{d}t} = 2 \tag{11}$$

8 p369, #14, §8 Graphically



9 p369, #14, §9 Properties

Magnitude at $t = 1$:

$$|\vec{a}| = a = \sqrt{a_x^2 + a_y^2} = \sqrt{5} \tag{12}$$

Angle with the positive x-axis at $t = 1$:

$$\phi = \arctan \frac{a_y}{a_x} = 63^\circ \text{ (not } 243^\circ\text{)}. \tag{13}$$

Component tangential to the motion:

$$a_t \equiv \frac{\mathrm{d}v}{\mathrm{d}t} \equiv \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{\vec{a} \cdot \vec{v}}{|\vec{v}|} = \frac{a_x v_x + a_y v_y}{|\vec{v}|} = 1 \tag{14}$$

Component normal to the motion:

$$a_n \equiv \frac{v^2}{R} = \sqrt{a^2 - a_t^2} = 2 \tag{15}$$