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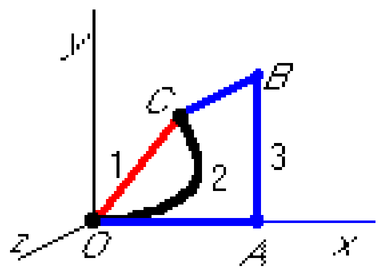
1 p510, #24(a), §1 Asked

Given: $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3x\hat{k}$



Asked: The work done by this force going from O to C along (1) the connecting line; (2) the curve $x = t$, $y = t^2$, $z = t^3$; (3) path OABC.

2 p510, #24(a), §2 Identification

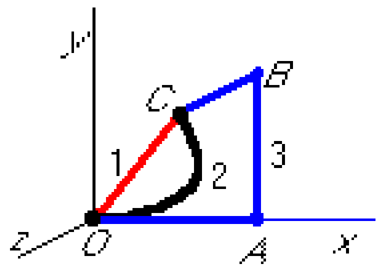
Find the curl of the vector to see whether the three integrals are going to be the same:

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3x \end{vmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$$

Nonzero, so the integrals along the three paths need not be the same.

3 p510, #24(a), §3 Solution

$$\int_O^C \vec{F}\,\mathrm{d}\vec{r} = \int_O^C x\,\mathrm{d}x + 2y\,\mathrm{d}y + 3x\,\mathrm{d}z$$



1. Along the line $y = x$ and $z = x$:

$$\int_{x=0}^{1} 6x \, \mathrm{d}x = 3$$

2. Along the curve $x = t$, $y = t^2$, $z = t^3$:

$$\int_{t=0}^{1} F_x \frac{\mathrm{d}x}{\mathrm{d}t} + F_y \frac{\mathrm{d}y}{\mathrm{d}t} + F_z \frac{\mathrm{d}z}{\mathrm{d}t} = \int_0^1 t \, \mathrm{d}t + 2t^2 \, 2t \, \mathrm{d}t + 3t \, 3t^2 \, \mathrm{d}t = \frac{15}{4}$$

3. Along OABC:

$$\int_{x=0}^{1} x \, \mathrm{d}x + \int_{y=0}^{1} 2y \, \mathrm{d}y + \int_{z=0}^{1} 3 \, 1 \, \mathrm{d}z = \frac{9}{2}$$