Page 139, #13(a)

1 p139, #13(a), §1 Asked

Asked: Draw the graph of

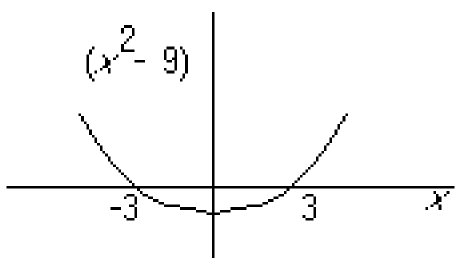
$$xy = \left(x^2 - 9\right)^2 \tag{1}$$

2 p139, #13(a), §2 Graph

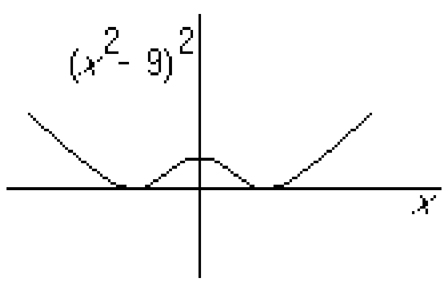
$$xy = \left(x^2 - 9\right)^2 \tag{2}$$

Instead of starting to crunch numbers, look at the pieces first:

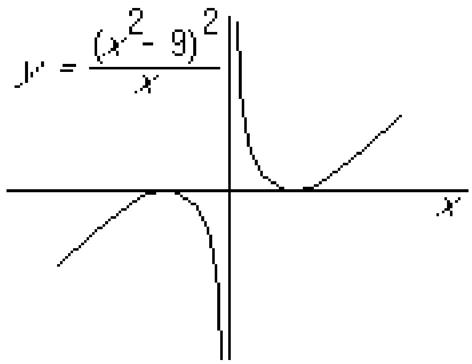
Factor $x^2 - 9 = (x-3)(x+3)$ is a parabola with zeros at $x = \pm 3$:



Squaring gives a quartic with double zeros at $x = \pm 3$:



Dividing by x will produce a simple pole at $x = 0$ and also a sign change at negative x:



Function $y(x)$:

- has an x-extent $x \neq 0$ and a y-extend $-\infty < y < \infty$;
- is odd (symmetric with respect to the origin);
- has a relative maximum at -3 of finite curvature: $y \propto (x+3)^2$;
- has a relative minimum at 3 of finite curvature: $y \propto (x-3)^2$;
- has a vertical asymptote at $x = 0$ with asymptotic behavior: $y \sim 81/x$ for $|x| \to 0$;
- behaves asymptotically as $y \sim x^3$ for $|x| \to \infty$;
- is concave up for $x > 0$, down for $x < 0$

3 p139, #13(a), §3 Alternate

$$y = \frac{(x^2 - 9)^2}{x}$$

Hence

- intercepts with x-axis are at $x = \pm 3$;
- no intercepts with the y axis;
- y is an odd function of x (symmetric about the origin);
- for $x \downarrow 0$, $y \to \infty$ (vertical asymptote);

- for $x \uparrow 0$, $y \to -\infty$ (singularity is an odd, simple pole);
- for $x \to \pm\infty$, $y \sim x^3 \to \pm\infty$.

$$y' \equiv \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2-9)(3x^2+9)}{x^2}$$

Hence,

- $y' > 0$ for $-\infty < x < -3$ (y increases from $-\infty$);
- $y' = 0$ for $x = -3$ (local maximum, $y = 0$);
- $y' < 0$ for $-3 < x < 0$ (y decreases towards $-\infty$);
- $y' = -\infty$ for $x = 0$ (singular point, vertical asymptote);
- $y' < 0$ for $0 < x < -3$ (decreases from ∞);
- $y' = 0$ for $x = 3$ (local minimum, $y = 0$);
- $y' > 0$ for $3 < x < \infty$ (increases to ∞).

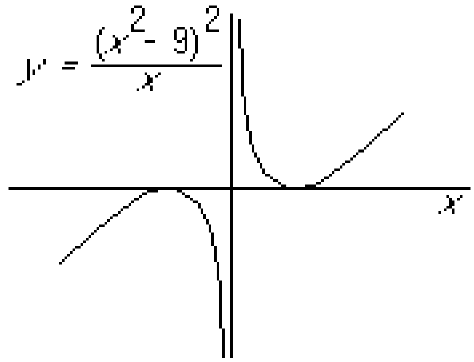
Also,

- $y' \to \infty$ when $x \to \pm\infty$ (no horizontal or oblique asymptotes);
- all derivatives exist, except at $x = 0$, which has no point on the curve (no corners, cusps, infinite curvature, or other singular points);
- probably no inflection points.

$$y'' = \frac{6x^4 + 162}{x^3}$$

Hence

- really no inflection points (since there is no point at $x = 0$);
- cocave downward for $x < 0$, upward for $x > 0$.



Hence the x- and y-extends as before.