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1 p438, #10(b), §1 Asked

Asked: The Maclaurin series of $\sin^2 x$.

2 p438, #10(b), §2 Identification

General Taylor series:

$$\begin{aligned} f(x) &= f(a) + f'(a)\frac{x-a}{1!} + f''(a)\frac{(x-a)^2}{2!} + \ldots \\ &= \sum_{n=0}^{\infty} f^{(n)}(a)\frac{(x-a)^n}{n!} \end{aligned}$$

This is a power series (a is a given constant.) Maclaurin series: $a = 0$.

Approach:

- note that $a = 0$;
- identify the derivatives;
- evaluate them at $a = 0$;
- put in the formula;
- identify the terms for any value of n.

3 p438, #10(b), §3 Results

$$\begin{array}{ll} f(x) = \sin^2 x & f(0) = 0 \\ f'(x) = 2\sin x \cos x & f'(0) = 0 \\ f''(x) = 2\cos^2 x - 2\sin^2 x = 2 - 4\sin^2 x & f''(0) = 2 \\ f'''(x) = -8\sin x \cos x = -4f'(x) & f'''(0) = 0 \\ f''''(x) = -4f''(x) & f''''(0) = -8 \\ f^{(5)}(x) = -4f'''(x) & f^{(5)}(0) = 0 \\ f^{(6)}(x) = -4f''''(x) = (-4)^2 f''(x) & f^{(6)}(0) = 32 \\ \vdots & \vdots \end{array}$$

$$\begin{aligned}\sin^2 x &= f(0) + f'(0)\frac{x-a}{1!} + f''(0)\frac{(x-a)^2}{2!} + \dots \\ &= 2\frac{x^2}{2!} - 8\frac{x^4}{4!} + 32\frac{x^6}{6!} + \dots\end{aligned}$$

General expression:

When $n = 2k$ with $k \geq 1$: $f^{(n)} = 2(-4)^{k-1}$ Otherwise: $f^{(n)} = 0$

$$\sin^2 x = \sum_{k=1}^{\infty} 2(-4)^{k-1} \frac{x^{2k}}{(2k)!}$$

4 p438, #10(b), §4 Other way

Write $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$ and look up the Maclaurin series for the cosine. (No fair.)