

8.21

1 8.21, §1 Asked

Solve:

$$y'' + 2y' + 2y = 0$$

2 8.21, §2 Solution

$$y'' + 2y' + 2y = 0$$

Characteristic polynomial:

$$\begin{aligned}\lambda^2 + 2\lambda + 2 &= 0 \\ \lambda &= \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i \quad (i = \sqrt{-1})\end{aligned}$$

General solution:

$$y = C_1 e^{(-1+i)x} + C_2 e^{(-1-i)x}$$

Cleanup of complex exponentials is required in this class:

$$y = e^{-x} (C_1 e^{ix} + C_2 e^{-ix})$$

Euler:

$$\begin{aligned}e^{i\alpha} &= \cos(\alpha) + i \sin(\alpha) \\ y &= e^{-x} (C_1 [\cos x + i \sin x] + C_2 [\cos x - i \sin x]) \\ y &= e^{-x} ([C_1 + C_2] \cos x + i[C_1 - C_2] \sin x)\end{aligned}$$

Cleaned up solution:

$$y = e^{-x} (D_1 \cos x + D_2 \sin x)$$

