Linear Equations

Linear equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + p(x)y = q(x)$$

The terms linear in y are the homogeneous part, the terms independent of y are the inhomogeneous terms.

Linear equations allow solutions to be added:

$$\left.\begin{array}{r} y_1' + p(x)y_1 = q_1(x) \\ y_2' + p(x)y_2 = q_2(x) \end{array}\right\}$$

$$\Longrightarrow \qquad (y_1 + y_2)' + p(x)\,(y_1 + y_2) = q_1(x) + q_2(x)$$

Solve the homogeneous equation first:

$$y' + py = 0$$

Separable:

$$\frac{\mathrm{d}y}{y} = -p\,\mathrm{d}x$$

$$y = Ce^{-\int p\,\mathrm{d}x}$$

Now solve the inhomogeneous equation:

Variation of parameter:

$$y = C(x)e^{-\int p\,\mathrm{d}x}$$

Put in the O.D.E. and solve for $C(x)$.