

Homogeneous Equations

Homogeneous equation:

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

Solution: define a new unknown

$$u = \frac{y}{x},$$

so replace y by xu . The equation for $u(x)$ will be separable.

Note: Do not confuse this use of the word “homogeneous” for first order equations with the use of the same word for “homogeneous” for linear ODEs!

Note: To check exactness, you can replace x by tx and y by ty and see whether the ts cancel.