

5.34

1 5.34, §1 Asked

Solve:

$$y' + x^2y = x^2$$

2 5.34, §2 Solution

$$y' + x^2y = x^2$$

The equation is linear.

Solution of the homogeneous equation:

$$y' + x^2y = 0 \implies \frac{dy}{y} = -x^2 dx$$

$$\ln |y| = -\frac{1}{3}x^3 + C_1 \implies y = Ce^{-\frac{1}{3}x^3}$$

Solution of the inhomogeneous equation:

$$y = C(x)e^{-\frac{1}{3}x^3}$$

into

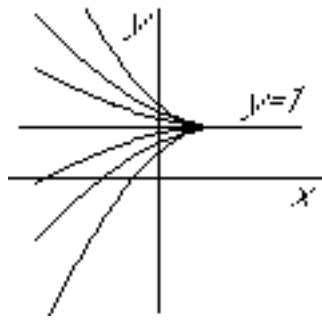
$$\begin{aligned} y' + x^2y &= x^2 \\ C'e^{-\frac{1}{3}x^3} - Ce^{-\frac{1}{3}x^3}x^2 + x^2Ce^{-\frac{1}{3}x^3} &= x^2 \\ C' &= x^2e^{\frac{1}{3}x^3} \implies C = e^{\frac{1}{3}x^3} + C_0 \end{aligned}$$

Solution:

$$y = C(x)e^{-\frac{1}{3}x^3} = 1 + C_0e^{-\frac{1}{3}x^3}$$

Note: function $y(x) = 1$ is called a particular solution. It is *one* solution that satisfies the inhomogeneous equation.

The general solution of linear equations is always: (any arbitrary particular solution) plus (the general solution of the homogeneous equation).



(What is wrong in the graph above)?