3.42

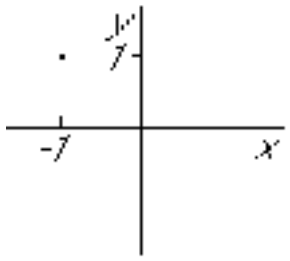
1 3.42, §1 Asked

Solve:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -(x^2+1)y \qquad y = 1 \text{ at } x = -1$$

2 3.42, §2 Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -(x^2+1)y \qquad y = 1 \text{ at } x = -1$$



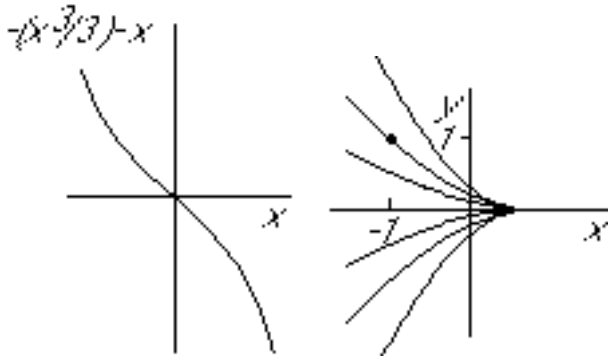
Solve the O.D.E. first:

$$\frac{\mathrm{d}y}{y} = -(x^2+1)\,\mathrm{d}x$$

$$\ln|y| = -\tfrac{1}{3}x^3 - x + C$$

$$y = \pm e^C e^{-\frac{1}{3}x^3 - x}$$

$$y = De^{-\frac{1}{3}x^3 - x}$$



Since the additional condition is $y = 1$ at $x = -1$, substitute in $y = 1$ and $x = -1$ to get D:

$$1 = De^{\frac{1}{3}+1}$$

So, at any x:

$$y = e^{-\frac{1}{3}x^3 - x - \frac{4}{3}}$$