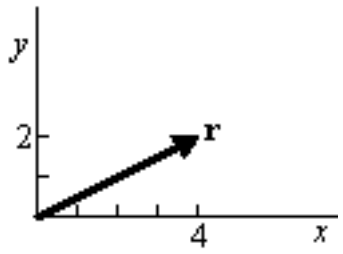
Introduction

Vectors:

- Statics and Dynamics
- Physics
- Geometry
- Computer solutions
- ...



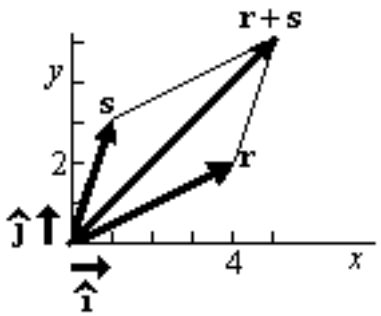
Properties:

- The vector above as a list of numbers:

$$\vec{r} = \vec{x} = \mathbf{x} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \text{ or } \vec{r}^T = \vec{x}^T = \mathbf{x}^T = (4, 2)$$

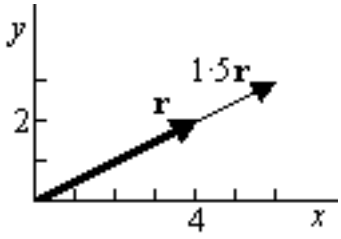
  If the list is written vertically, the vector is called a column vector; if it is written horizontally, it is a row vector. An n-dimensional row vector is equivalent to a $1 \times n$ size matrix; an n-dimensional column vector to a $n \times 1$ matrix,

- Components of the vector above: $r_1 = r_x = x_1 = x = 4, r_2 = r_y = x_2 = y = 2$.
- Addition of vectors:



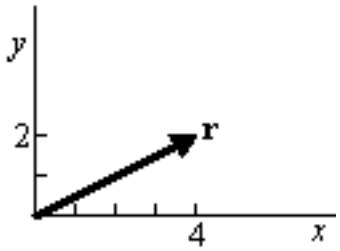
  $(4, 2) + (1, 3) = (4 + 1, 2 + 3) = (5, 5)$.

- Multiplication of a vector by a scalar:

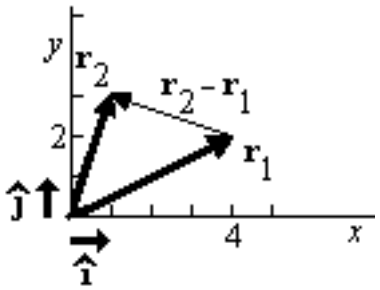


  $1.5(4,2) = (1.5\ 4, 1.5\ 2) = (6,3)$.

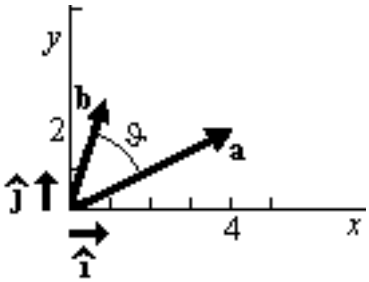
- Length or norm of a vector:



  - Definition:
$$||\vec{a}|| = |\vec{a}| = a = \sqrt{a_x^2 + a_y^2 + a_z^2 + \ldots}$$
  - Unit vectors: Unit vectors have length one.
  - Distance: The distance between two points $\vec{r}_1$ and $\vec{r}_2$ is by definition $||\vec{r}_2 - \vec{r}_1||$:



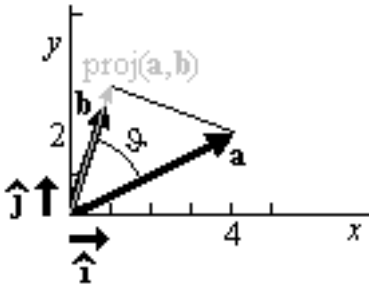
- Dot (scalar) product:



  - Definition:
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z + ... = ||\vec{a}||\ ||\vec{b}|| \cos\vartheta$$

  - Orthogonality: If the dot product is zero, the vectors are by definition orthogonal to each other.
  - Length: $||\vec{a}|| = \sqrt{\vec{a} \cdot \vec{a}}$.

- Projection:



The magnitude of the (orthogonal) component (or coordinate) of $\vec{a}$ in the direction of $\vec{b}$ is:

$$a_b = a\cos(\vartheta) = \vec{a} \cdot \hat{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||}$$

The projection of $\vec{a}$ onto $\vec{b}$ is

$$\text{proj}(\vec{a}, \vec{b}) = \vec{a}_b = a_b \hat{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||} \frac{\vec{b}}{||\vec{b}||}$$