

1.57

1 1.57, §1 Asked

Given:

$$\vec{r} = t^3\hat{i} - t^2\hat{j} + (2t - 3)\hat{k}$$

for $0 \leq t \leq 5$.

Asked: (a) Find the point P on the curve corresponding to $t = 2$. (b) Find the initial point Q and the terminal point Q'. (c) Find the unit tangent vector \vec{T} to the curve when $t = 2$.

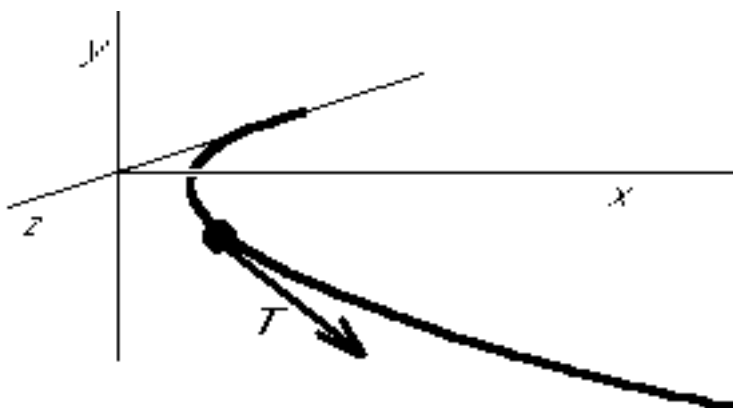
2 1.57, §2 Solution

$$\vec{r} = t^3\hat{i} - t^2\hat{j} + (2t - 3)\hat{k}$$

for $0 \leq t \leq 5$.

Point P: When $t = 2$, $\vec{r} = (8, -4, 1)$.

At end point Q, $t = 0$, $\vec{r} = (0, 0, -3)$; at end point Q', $t = 5$, $\vec{r} = (125, -25, 7)$.



Vector \vec{T} is proportional to

$$\frac{d\vec{r}}{dt} = \begin{pmatrix} 3t^2 \\ -2t \\ 2 \end{pmatrix}.$$

Then

$$\vec{T} = \frac{d\vec{r}}{dt} / \left| \frac{d\vec{r}}{dt} \right| = \begin{pmatrix} 3t^2 \\ -2t \\ 2 \end{pmatrix} / \sqrt{9t^4 + 4t^2 + 4}.$$

At $t = 2$

$$\vec{T} = \begin{pmatrix} 12 \\ -4 \\ 2 \end{pmatrix} / \sqrt{144 + 16 + 4} = \begin{pmatrix} 6/\sqrt{41} \\ -2/\sqrt{41} \\ 1/\sqrt{41} \end{pmatrix}$$