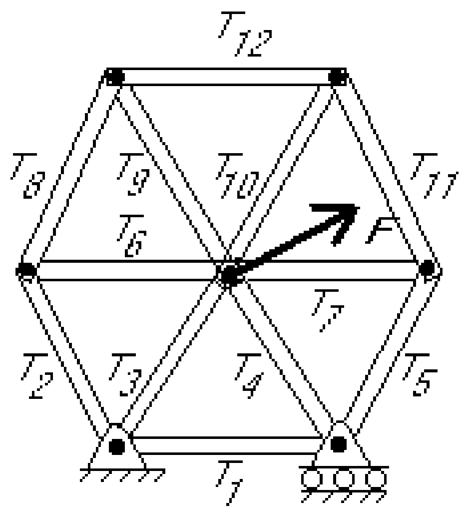
Null spaces

1 Null spaces

The null space of a matrix A are all vectors $\vec{x}$ so that $A\vec{x} = 0$. If A is square and $|A|$ is nonzero, the null space is simply $\vec{x} = 0$ and has dimension 0.



Nontrivial null spaces may correspond to internal stresses in structures, connectivity problems, vibrational mode shape, buckling shapes, eigenvectors corresponding to a given eigenvalue, etcetera.

You typically want to describe the null spaces as simply as possible. Defining a basis for the null space allows you to do so.

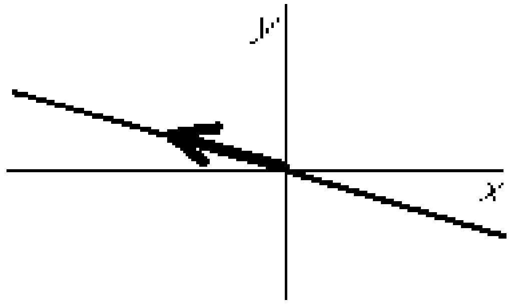
2 2D Example

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right) \qquad \begin{array}{c} (1) \\ (2) \end{array}$$

Forward elimination:

$$\left(\begin{array}{cc|c} \boxed{1} & 2 & \\ 0 & 0 & \end{array}\right) \qquad \begin{array}{c} (1) \\ (2') \end{array}$$

Back substitution: $x = -2y$. So, the null space is a line through the origin:



Vector form:

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} y$$

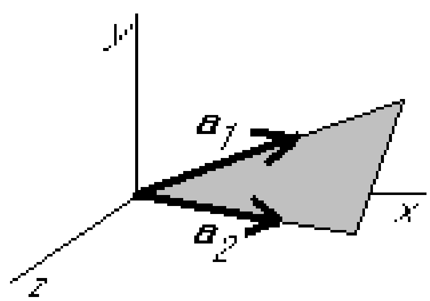
so that $(-2, 1)$ is one possible basis vector for this line. A line is a one-dimensional space, so it needs exactly one basis vector.

3 3D Example

$$\left(\begin{array}{ccc|c} \boxed{1} & -2 & -3 & 0 \end{array} \right) \qquad \begin{array}{c} (1) \\ (2) \end{array}$$

Forward elimination is trivial.

Back substitution: $x = 2y + 3z$. The solution space is a plane through the origin:



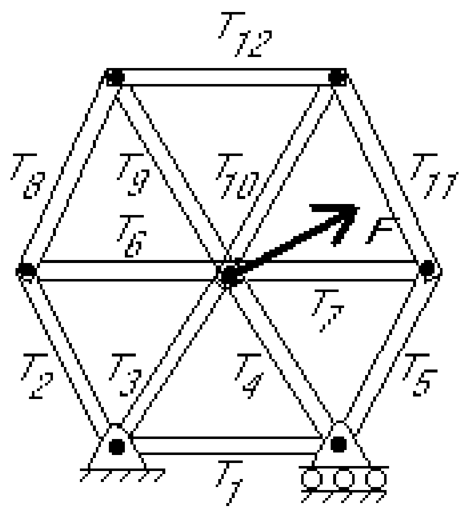
Vector form:

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} y + \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} z$$

so that $(2, 1, 0)$ and $(3, 0, 1)$ are one possible set of two basis vector for this plane. A plane is a 2D space.

4 12D Example

Assuming there is no external force, i.e. $\vec{F} = 0$ in the truss below,



the solution of the homogeneous equilibrium equations is

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} T_{12}$$

All bars in the outer ring have the same tension force T_{12}, while the spokes have an opposite compressive force.