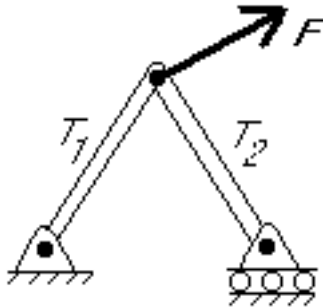


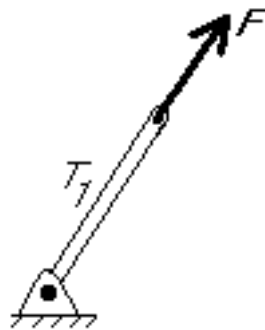
Gaussian elimination

1 Rectangular systems

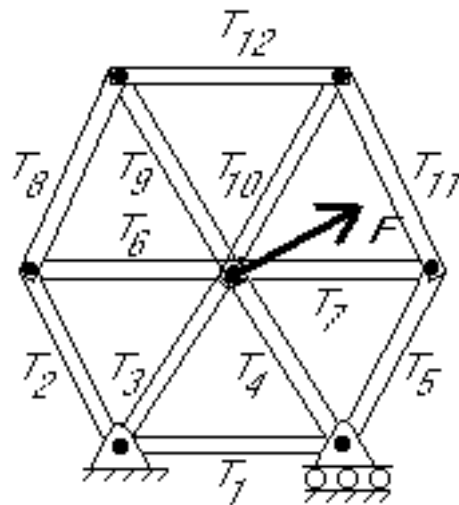
- Misdesigned systems;



- Systems with limited forces;



- Statically indeterminate systems;



- Measured data;

- Compression;
- Redistribution;
- ...

Matrix shapes:

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$$

There may be no solution, a unique solution, or infinitely many solutions, depending on circumstances.

2 Partial Pivoting

$$\begin{pmatrix} \boxed{1} & 2 & 3 & 4 & | & 5 \\ 2 & 4 & 6 & 9 & | & 10 \\ 1 & 2 & 4 & 5 & | & 6 \\ 1 & 2 & 4 & 6 & | & 7 \end{pmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

$$\begin{pmatrix} \boxed{1} & 2 & 3 & 4 & | & 5 \\ 0 & \bar{0} & \bar{0} & \bar{1} & | & \bar{0} \\ 0 & |0 & \boxed{1} & 1 & | & 1 \\ 0 & |0 & 1 & 2 & | & 2 \end{pmatrix} \begin{matrix} (1) \\ (2') \\ (3') \\ (4') \end{matrix}$$

I must interchange (2') and (3') before proceeding.

$$\begin{pmatrix} \boxed{1} & 2 & 3 & 4 & | & 5 \\ 0 & 0 & \boxed{1} & 1 & | & 1 \\ 0 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & 1 & 2 & | & 2 \end{pmatrix} \begin{matrix} (1) \\ (3') \\ (2') \\ (4') \end{matrix}$$

$$\begin{pmatrix} \boxed{1} & 2 & 3 & 4 & | & 5 \\ 0 & 0 & \boxed{1} & 1 & | & 1 \\ 0 & 0 & 0 & \bar{1} & | & \bar{0} \\ 0 & 0 & 0 & |1 & | & 1 \end{pmatrix} \begin{matrix} (1) \\ (3') \\ (2') \\ (4') - (3') \end{matrix}$$

For best accuracy: always use the pivot with the largest absolute magnitude that you can.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & | & 5 \\ \boxed{2} & 4 & 6 & 9 & | & 10 \\ 1 & 2 & 4 & 5 & | & 6 \\ 1 & 2 & 4 & 6 & | & 7 \end{pmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 1 & 2 & 3 & 4 & | & 5 \\ 1 & 2 & 4 & 5 & | & 6 \\ 1 & 2 & 4 & 6 & | & 7 \end{pmatrix} \begin{matrix} (2) \\ (1) \\ (3) \\ (4) \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 0 & 0 & 0 & -\frac{1}{2} & | & 0 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & | & 1 \\ 0 & 0 & 1 & 1\frac{1}{2} & | & 2 \end{pmatrix} \begin{matrix} (2) \\ (1') \\ (3') \\ (4') \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & | & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & | & 0 \\ 0 & 0 & 1 & 1\frac{1}{2} & | & 2 \end{pmatrix} \begin{matrix} (2) \\ (3') \\ (1') \\ (4') \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & | & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & | & 0 \\ 0 & 0 & 0 & \boxed{1} & | & 1 \end{pmatrix} \begin{matrix} (2) \\ (3') \\ (1') \\ (4'') \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & | & 1 \\ 0 & 0 & 0 & \boxed{1} & | & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & | & 0 \end{pmatrix} \begin{matrix} (2) \\ (3') \\ (4'') \\ (1') \end{matrix}$$

$$\begin{pmatrix} \boxed{2} & 4 & 6 & 9 & | & 10 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & | & 1 \\ 0 & 0 & 0 & \boxed{1} & | & 1 \\ 0 & 0 & 0 & 0 & | & \frac{1}{2} \end{pmatrix} \begin{matrix} (2) \\ (3') \\ (4'') \\ (1'') \end{matrix}$$

3 Reduction Algorithm

The following algorithm corrects the one in section 3.6 in the book:

1. Start assuming that the pivot will be the coefficient of the first unknown in the first equation.
2. If that coefficient is zero, look below the coefficient for one that is nonzero and swap equations to replace the zero coefficient with the nonzero one.
3. If there are no nonzero coefficients below either, go to the next unknown, i.e. move one place to the right in the matrix and repeat the previous step. (If there are no more unknowns, you are done.)
4. With the obtained nonzero pivot, create zeros below it.
5. Go process the submatrix consisting of the remaining equations below the pivot and the remaining unknowns beyond the pivot in the same way.

What is wrong with the book: Step 1 in the book procedure is impossible if the first row is zero. *Deleting equations or unknowns is a no-no with your instructor.* And step 4 may not be possible since there may not be any equations left after step 3 following the book.