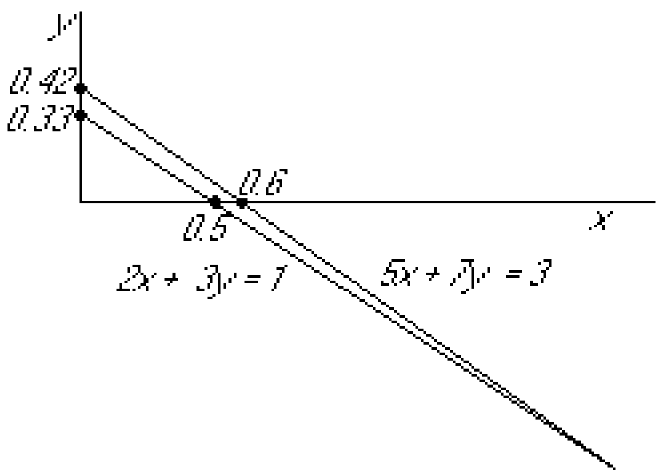
3.51(a)

1 3.51(a), §1 Asked

Asked: Solve

$$\begin{array}{rcrcrl} 2x & + & 3y & = & 1 & \quad (1) \\ 5x & + & 7y & = & 3 & \quad (2) \end{array}$$

2 3.51(a), §2 Graphically



One unique solution point $(x, y) = (2, -1)$

3 3.51(a), §3 Elimination

Gaussian elimination:

$$\begin{array}{rcrcrl} 2x & + & 3y & = & 1 & \quad (1) \\ 5x & + & 7y & = & 3 & \quad (2) \end{array}$$

A. Forward Elimination:

Use (1) to eliminate x from (2):

$$\begin{array}{rcrcrl} 2x & + & 3y & = & 1 & \quad (1) \\ & - & y & = & 1 & \quad (2') = 2(2) - 5(1) \end{array}$$

Note: you always must keep at least some of the original equation.

B. Back Substitution:

Solve (2') to find $y = -1$. Then use that value in (1) to find $x = 2$.

4 3.51(a), §4 Matrix Form

$$\begin{array}{ccccc} 2x & + & 3y & = & 1 \\ 5x & + & 7y & = & 3 \end{array} \qquad \begin{array}{c} (1) \\ (2) \end{array}$$

This can be written as

$$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \begin{array}{c} (1) \\ (2) \end{array}$$

or $A\vec{x} = \vec{b}$ where

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

More concisely, only write the augmented matrix:

$$\left(\begin{array}{cc|c} 2 & 3 & 1 \\ 5 & 7 & 3 \end{array}\right) \qquad \begin{array}{c} (1) \\ (2) \end{array}$$

After elimination:

$$\left(\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & -1 & 1 \end{array}\right) \qquad \begin{array}{l} (1) \\ (2') = 2(2) - 5(1) \end{array}$$

5 3.51(a), §5 Determinant

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = 2\,7 - 5\,3 = -1$$