Introduction

1 General

The most usual representation of systems on computers and elsewhere is using matrices. Finite element problems, dynamics, fluid mechanics, ..., are almost always matrix problems for the computer.

A matrix A is a table of numbers:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$

An $m \times n$ matrix consists of n column vectors (the columns), or equivalently of m row vectors (the rows).

Conversely, a column vector is equivalent to a matrix with only one column and a row vector is a matrix with only one row.

Square matrices are matrices with the same number of rows as columns.

Index notation:

$$A = \{a_{ij}\} \qquad (i = 1, \dots, m; j = 1, \dots, n)$$

where $\{\cdot\}$ indicates "the collection of values" or "set of values".

2 Scalar multiplication

Multiplying a matrix by a scalar (i.e. a number) means multiplying each coefficient by that scalar:

$$kA = \begin{pmatrix} ka_{11} & ka_{12} & ka_{13} & \dots & ka_{1n} \\ ka_{21} & ka_{22} & ka_{23} & \dots & ka_{2n} \\ ka_{31} & ka_{32} & ka_{33} & \dots & ka_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ka_{m1} & ka_{m2} & ka_{m3} & \dots & ka_{mn} \end{pmatrix}$$

Just like for vectors.

Scalar multiplication in index notation:

$$B = kA \qquad \Longrightarrow \qquad b_{ij} = ka_{ij} \text{ for all } i \text{ and } j$$

3 Addition

Summation of two matrices adds corresponding coefficients:

$$A + B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} & \dots & a_{2n}+b_{2n} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} & \dots & a_{3n}+b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & a_{m3}+b_{m3} & \dots & a_{mn}+b_{mn} \end{pmatrix}$$

(just like for vectors.) The matrices *must* be of the same size.

Summation in index notation:

$$C = A + B \qquad \Longrightarrow \qquad c_{ij} = a_{ij} + b_{ij} \text{ for all } i \text{ and } j$$

4 Zero matrices

Zero matrices have *all* coefficients zero. Adding a zero matrix to a matrix does not change the matrix.