Transforming Matrices

We saw that a transformation matrix P from an old basis S to new basis S' transforms between $\vec{v}\ (= \vec{v}\big|_S)$ and $\vec{v}'\ (= \vec{v}\big|_{S'})$ as:

$$\boxed{\vec{v} = P\vec{v}' \text{ or } \vec{v}' = P^{-1}\vec{v}}$$

A square matrix A transforms similarly, but has in addition the inverse of the transformation matrix at the far right:

$$\boxed{A = PA'P^{-1} \text{ or } A' = P^{-1}AP}$$

The need for two transformation matrices comes from the fact that a matrix provides a transformation of vectors. Given an "original vector" $\vec{x}$, multiplying by matrix A produces an "image vector" $\vec{y} = A\vec{x}$. When we change coordinates, one transformation matrix is needed to transform $\vec{x}$, the other to transform $\vec{y}$:

$$\vec{y}' = P^{-1}\vec{y} = P^{-1}(A\vec{x}) = P^{-1}AP\vec{x}'$$

So the matrix that transforms $\vec{x}'$ into $\vec{y}'$ is $P^{-1}AP$.