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1   p461, #27(a), §1 Asked

Given:

$$\omega = \sqrt[3]{\frac{g}{b}}$$

The maximum error in g is 1%, the maximum error in b is 0.5%.

Asked: The maximum percentage error in ω.

2   p461, #27(a), §2 Identification

Given are the relative errors:

$$\frac{\delta g}{g} = 0.01 \qquad \frac{\delta b}{b} = 0.005$$

Error manipulation rules:

1. During addition and substraction, add absolute errors;

2. During multiplication and division, add relative errors;

3. During exponentiation, multiply the relative error by the power.

3   p461, #27(a), §3 Results

$$\frac{\mathrm{d}(g/b)}{(g/b)} = \frac{b}{g}\left(\frac{b\mathrm{d}g - g\mathrm{d}b}{b^2}\right) = \frac{\mathrm{d}g}{g} - \frac{\mathrm{d}b}{b}$$

Hence the greatest possible relative error in (g/b) is:

$$\frac{\delta(g/b)}{(g/b)} = 0.01 + 0.005$$

(or use rule 2)

$$\frac{\mathrm{d}\left(\sqrt[3]{g/b}\right)}{\left(\sqrt[3]{g/b}\right)} = \tfrac{1}{3}\frac{\mathrm{d}(g/b)}{(g/b)}$$

(or use rule 3)

Hence

$$\frac{\delta\omega}{\omega} = 0.005 = 0.5\%$$