Model Predictive Control of Regenerative Dampers with Acceleration and Energy Harvesting Trade-Offs

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Regenerative suspensions are becoming a potential to achieve active suspensions without reducing fuel economy. In this paper, a model predictive controller (MPC) is developed to minimize sprung mass vertical acceleration or maximize the energy regenerated by the suspension. MPC has difficulties when harvested energy is the optimization function. This led to development of an MPC algorithm that includes Lyapunov constraints that stabilizes the system states. Simulation results show that optimizing the energy output of the system increases energy production but degrades the ride comfort. Implementation of the Lyapunov constraints help reduce ride degradation and increase ride while improving energy regeneration.

Model Predictive Control, Regenerative Suspensions, Nonconvex Relaxations, Bond Graph Modeling

1. INTRODUCTION

Active suspensions are not widely used in passenger vehicles due to the loss of fuel efficiency from powering the suspension. Regenerative dampers that also act as active force elements are one possibility to overcome this problem. It has been shown that regenerative dampers are capable of regenerating enough energy to power an active suspension [1]. The control methods to achieve an optimal balance between suspension performance and energy regeneration are currently attracting much interest in the research community [2,3]. In this paper, an active control methodology is designed for a quarter car, using a model predictive control (MPC) scheme to minimize sprung mass acceleration or maximize energy regeneration. This MPC is then frequency weighted in an attempt to combine the merits of optimizing with respect to either objectives, while mitigating their demerits as much as possible. Traditional MPC techniques are predicated on the use of convex cost functions enabling the optimization problems to be solved easily. Optimization of energy regeneration requires a nonconvex cost function which makes implementation of a predictive controller difficult. Further, optimization of energy regeneration does not guaranty stability in the system. The control algorithm that is presented in this paper utilizes Lyapunov stability to guaranty that the system states reach an equilibrium point while optimizing the cost functional. Simulations where the controller seeks to minimize sprung mass acceleration are compared to a power regenerative maximization controller. The results show that under the proposed MPC, maximizing the regenerated energy and minimizing the sprung mass acceleration are competing objectives and that minimizing acceleration results in a controller that has an overall better system performance when ride comfort is taken into account. Further, by frequency weighting the system, it is possible to implement a blended cost function without the Lyapunov stability constraints, making the implementation easier. Further, the cost function can be weighted towards acceleration reduction or energy regeneration.

2. PREVIOUS WORK

Model predictive control has been used in suspension control for many years. Earlier studies focused on using MPC coupled with road preview information to achieve good suspension performance in ride comfort and road holding [4, 5]. MPC has also been used a as a means to calculate control laws in suspensions using semi-active dampers [6]. A problem that MPC faces in implementation in real suspension control is computation time. This was addressed by the implementation of a "fast" algorithm by Canale, et al. [7], which proved to be both implementable and improve the ride and handling aspects of the car. Predictive control for regenerative dampers have not been investigated much in recent literature. Other control algorithms have been investigated such as multiobjective algorithms like that designed by Di Iorio, and Casavola [8]. This algorithm allows the designer to emphasize either energy harvesting or ride comfort.

3. SYSTEM MODEL

Suspension control is generally done by utilizing passive, semi-active, or active elements at the corners of a vehicle. The system model used in this study follows a standard quarter car but with an electrodynamic damper in place of the standard passive damper. A diagram of the model can be seen in Fig. 1. To obtain the equations of motion for the system, the bond graph [9] in Fig. 2 was created. The equations of motion are easily obtained and are shown in equations (1-5). This is done through standard bond graph equation of motion formulation. The only difficulty results from the derivative causality that occurs on the motor moment of inertia. This leads to slightly more complicated equations as seen below. This model assumes that the motor generates energy ideally. While this is not realistic in a real system, it is appropriate for this study because the main objective is to compare the performance of various cost functions in the MPC algorithm and how the two objectives compete or complement each other. The electrical components of the system have also been omitted and the study looks at the available mechanical power which is the product of the motor torque and speed.



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If the states are taken around the equilibrium point, the resulting equations are all linear and can be written in standard linear form. Further, dividing equations (2) and (4) by m_{us} and m_s respectively, yields the following equations

$$\dot{q}_t = v_i(t) - v_{us} \tag{1}$$

$$\dot{p}_{us} = \frac{(J_m + m_s r_m^2)k_t}{am_s r_m^2} q_t - \frac{k_s}{a} q_s - \frac{1}{ar_m} \tau_m$$

$$\dots - \frac{(J_m + m_s r_m^2)}{am_s r_m^2} m_{us} g - \frac{J_m}{am_s r_m^2} m_s g$$
(2)

$$\dot{y}_s = v_{us} - v_s \tag{3}$$

$$\dot{p}_s = \frac{J_m k_t}{a m_{us} r_m^2} q_t + \frac{k_s}{a} q_s + \frac{1}{a r_m} \tau_m \tag{4}$$

$$\dots - \frac{\sigma_{m}}{am_{us}r_{m}^{2}}m_{us}g - \frac{(\sigma_{m} + m_{us}r_{m})}{am_{us}r_{m}^{2}}m_{s}g$$

$$a = \frac{(m_{us} + m_{s})J_{m} + m_{us}m_{s}r_{m}^{2}}{m_{us}m_{s}r_{m}^{2}}$$
(5)

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{6}$$

Where

v

6

$$x = [q_t \ v_{\mu s} \ q_s \ v_s]^T \tag{8}$$

$$u = \tau_m$$
 (7)

$$v = v_i$$
 (9)



Fig. 2 - System Bond Graph

These equations were used with the following parameters for the simulations in the study.

Table 1 - System Parameters				
Unsprung Mass	m_{us}	66.3 kg		
Sprung Mass	m_s	331.4 kg		
Tire Stiffness	k_t	$1.67 \times 10^5 N/m$		
Suspension Stiffness	k_s	$2.04 \times 10^4 N/m$		
Pinion Radius	r_m	0.013 m		
Motor Moment of Inertia	J_m	$7.33 \times 10^{-5} kg \cdot m^2$		
Gravity	g	9.81 m/s^2		

 Table 2 - System Variables and Inputs

Tire Deflection	q_t	m
Suspension Deflection	q_s	m
Unsprung Mass Velocity	v_{us}	m/s
Sprung Mass Velocity	Vs	m/s
Motor Torque Input	τ_m	$N \cdot m$
Motor Angular Velocity	ω_m	rad/s
Road Disturbance	vi	m/s

4. CONTROL ALGORITHM

The control algorithms used in this study are all model predictive controllers starting with a standard MPC formulation in 4.1, adding additional stability constraints in 4.2, and finally blending the two cost functions using frequency shaping 4.3.

4.1 STANDARD MPC FORMULATION

Standard MPC with linear differential equations uses the following formulation:

$$\min_{u_0,...,u_{N-1}} J(X,U)$$

subject to $x_{k+1} = Ax_k + Bu_k$
 $x_0 = x(t)$
 $x_k \in X$
 $u_k \in \mathcal{U}$ (10)

Where $X = [x_0, ..., x_N]$ and $U = [u_0, ..., u_{N-1}]$. The two cost functions that the MPC in this study sought to optimize where

$$J_1 = x_N^T P x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + 2x_k^T N u_k + u_k^T R u_k$$
(11)

$$J_2 = \sum_{k=0}^{N} \omega_k \tau_k \tag{12}$$

Where J_1 is a standard LQR cost function and the matrices were formulated from minimizing the square of the sprung mass vertical acceleration. J₂ minimizes the power supplied to the system by the electrodynamic damper. By minimizing the power supplied by the electrodynamic damper, the power that is regenerated is maximized. It is obvious that J_1 is convex but J_2 is a nonconvex function. To compensate for the nonconvexity of J_2 , ω_k was replaced with $\omega(t)$, the initial condition at each optimization. This makes the power optimization a linear program and easily solvable. Further, a horizon of 1 was chosen as the basis for comparison of the two controllers. This horizon was chosen because when the terminal state cost is included in J_1 ($x_N^T P x_N$), and P is the solution of the infinite-horizon Discrete Algebraic Ricatti Equation, the MPC will converge to the LQR solution when there are no state or input constraints for a horizon of 1 [10]. This gives a more direct comparison of the two control objectives. An issue that arises is that J_2 does not guarantee asymptotic stability of the system states while J1 does under standard LQR formulation requirements [11].

4.2 MPC FORMULATION WITH LYAPUNOV STABILITY CONSTRAINTS

In order to compensate for the lack of guaranteed asymptotic stability when optimizing power output, an alternative formulation was developed. Including a constraint that the Lyapunov function is negative definite is sufficient to guarantee asymptotic stability [12]. The new formulation that is used in this study is the following:

$$\min_{u_0,...,u_{N-1}} J(X, U)$$

subject to $x_{k+1} = Ax_k + Bu_k$
 $x_0 = x(t)$
 $V_k = x_k^T P x_k$
 $V_{k+1} - V_k < 0$
 $x_k \in \mathcal{X}$
 $u_k \in \mathcal{U}$
(13)

Where the matrix P is found using the following offline linear matrix inequality (LMI):

$$\begin{bmatrix} -P^{-1} & (AP^{-1} + BL)^T & (C_y P^{-1} + DL)^T \\ (AP^{-1} + BL) & -P^{-1} + \gamma^{-2} B_w B_w^T & 0 \\ (C_y P^{-1} + DL) & 0 & -\mathbf{I} \end{bmatrix} \preceq 0 \quad (14)$$

And $L = KP^{-1}$ such that $A+BKP^{-1}$ is Hurwitz.

The Lyapunov condition is nonconvex so the program has to be further relaxed for implementation. Because of the single step horizon that is being optimized over, this was accomplished by requiring that

$$\|x_1\|_P^2 \le \beta \|x_0\|_P^2, \ 0 \le \beta \le 1$$
(15)

The notation $||x||_P^2 = x^T P x$ represents the set of Lyapunov candidate functions for a linear time invariant system parameterized by

$$P \in \mathbb{S}_{++} \tag{16}$$

Using equation 15 also gives the system robustness to the noise of the road disturbance [13] and results in a linear matrix inequality by requiring that the Schur complement be negative semidefinite. This constraint be directly implemented in the MPC as a convex stability constraint. The Schur complement matrix is

$$H = \begin{bmatrix} -\beta \|x_0\|_P^2 & x_1^T \\ x_1 & -P^{-1} \end{bmatrix}$$
(17)

Inserting equation 17 the MPC formulation of equation 13 instead of the Lyapunov constraint results in the following MPC formulation

$$\min_{u_0,...,u_{N-1}} J(X,U)$$

subject to $x_{k+1} = Ax_k + Bu_k$
 $x_0 = x(t)$
 $H \leq 0$
 $x_k \in X$
 $u_k \in \mathcal{U}$ (18)

The same cost functions and horizon are used as in 4.1.

4.3 MPC FORMULATION WITH LYAPUNOV STABILITY CONSTRAINTS AND COST FUNCTION BLENDING

As will be shown in section 5, maximizing power regeneration and reducing sprung mass acceleration are competing objectives. The final controller that was designed for this study seeks to blend the two cost functions by minimizing acceleration across the range of vibrations that humans are sensitive to and minimizing supply power outside of this bandwidth. The blending function that is used is based on ISO 2631-1. It was shown by Zuo and Nayfeh that this can be approximated in the frequency domain [14] as

$$W_c(s) = \frac{86.51s + 546.1}{s^2 + 82.17s + 1892} \tag{19}$$

The complement of the sensitivity function is found by

$$\bar{W}_c(s) = W_{c,max} - W_c(s) \tag{20}$$

And

$$W_{c,max} = \|W_c(s)\|_{\infty} \tag{21}$$

Implementing these in the cost functions and adding them together results in the following blended cost

$$J = \int_{t}^{t+T} (W_c(\sigma)a_f^2(\sigma) + \bar{W}_c(\sigma)P(\tau_m))d\sigma$$
(22)

Augmenting the state space with the frequency weighting and changing the MPC cost function to J_3 yields an implementable algorithm.

$$J_{3} = \rho x_{N}^{T} P x_{N} + \sum_{k=0}^{N-1} \omega_{0} \tau_{k} + \rho (x_{k}^{T} Q x_{k} + 2x_{k}^{T} N u_{k} + u_{k}^{T} R u_{k})$$
(23)

Where ρ is the optimization weighting parameter.

By weighting the acceleration appropriately, the controller is a hybrid controller that is capable of regenerating power outside of the desired bandwidth and minimizing acceleration across the appropriate bandwidth [15]. This system was implemented both with and without the Lyapunov stability constraint relaxation which will be discussed in section 5.

4.4 SKYHOOK DAMPING

As a basis of comparison, Skyhook damping was implemented in the model by solving for the desired damper forces

$$F_{skyhook} = -b_s v_s \tag{24}$$

These resulted in control laws for the motor of the form

$$\tau_m = -kr_m m_s b_s v_s \tag{25}$$

Because passive damping is not included in the system model, an effective damping coefficient of 1,550 N-s/m was used. This has been used as a basis of comparison in previous studies [16].

The above control damper force and control law does not reflect a theoretical Skyhook suspension because it is not realizable as an actual vehicle suspension. This control laws give a realizable suspension that is a good basis for comparison.

5. RESULTS

The system was simulated over multiple types of roads for the MPC formulation with Lyapunov stability constraints and cost function blending. The standard MPC with formulation was only simulated for one of the roads and will be explained below. For simulations no constraints were placed on the states but a maximum magnitude value was placed on the input. The input constraint is

$$|u_k| \le u_{max} \tag{26}$$

 u_{max} was taken as 20 N-m for this study. Saturation values of these actuators are not known at this time but are currently under investigation. All of the simulations were performed on random roadways at a forward speed of 50 mph for 10 seconds. The random ISO roadways were generated using the algorithm by Agostinacchio, et al [17].

5.1 STANDARD MPC FORMULATION

When the system was simulated without the stability constraints and subject to the cost functions J_1 and J_2 , equations 11 and 12, it was found that J_1 minimized the acceleration as expected and still regenerated energy. J_2 decreased the acceleration as well but regenerated approximately 416% more energy. The problem is that this resulted in a bang-bang solution for the control, which resulted in excessively acceleration frequencies, see Fig. 3. Bang-bang solutions are problematic due to the infinite bandwidth requirement and the peak acceleration of the energy optimization algorithm was only slightly lower than the uncontrolled system. Fig. 3 shows that energy optimization and sprung mass vertical acceleration are competing objectives * as expected [8]. Interestingly, when the energy is minimized, the damper effectively acts as a passive damper; dissipating energy most of the time.



Fig. 3 - Acceleration Response for Standard MPC Formulation



Fig. 4 – Acceleration Response for MPC with Lyapunov Stability Constraint

5.2 MPC FORMULATION WITH LYAPUNOV STABILITY CONSTRAINTS

For the reformulated system, multiple classes of roads were used for simulation to study the effect road quality has on the controller performance. The accelerations followed the general pattern seen in Fig. 4. In order to get an idea of the performance tradeoff between ride comfort and energy harvesting, the total regenerated energy

$$E = \int_{t_0}^{t_f} P(t)dt \tag{28}$$

Was compared to the variance gain of the sprung mass acceleration to the road input velocity

$$g_{var} = \frac{\int_{t_0}^{t_f} a_s^2(t) dt}{\int_{t_0}^{t_f} v_i^2(t) dt}$$
(29)

This resulted in the Table 3.

Table 3 - Harvested Energy and Sprung Mass Acceleration Variance Gain

Road Class	J ₁	J ₂	Skyhook
Α	$(0.59, 1.1 \times 10^{-5})$	(-2000, 882)	(-34.1, 2.87)
В	$(4.3, 1.2 \times 10^{-5})$	(-1784, 207)	(-90, 1.8)
С	$(15.8, 1.2 \times 10^{-2})$	(-2168, 64)	(-483.7, 2.7)
D	(-797.5,3.33)	(-5030, 20)	(-2546, 3.17)

This data was collected from 10 seconds of simulated data. As the simulation time increases, the amount of energy will increase but the trends remain the same. In this table if the sign is negative, that indicates that the system regenerated energy over the course of the simulation. Table 3 shows that an LQR solution to minimize sprung mass acceleration requires energy. For the class D road the reason that energy was regenerated was that the input would need more control authority to reduce the sprung mass acceleration to the levels seen in road classes A-C. Skyhook ends up being an intermediate solution to J₁ and J₂ showing that there will be a trade-off between energy harvesting and ride comfort which will mean that a regenerative suspension will end up with a trade-off surface weighing the performance of road holding, ride comfort, and energy regeneration.

It is important to note that while the inclusion of the Lyapunov stability constraints did help reduce the sprung mass acceleration, the control input still exhibited Bang-Bang qualities.

5.3 MPC FORMULATION WITH LYAPUNOV STABILITY AND COST FUNCTION BLENDING

When the frequency blended cost function was implemented as laid out in 4.3, the controller was able to implement a combination of the two controllers based

^{*} It should be noted that even though minimizing acceleration and maximizing power output are competing objectives, using active control to reduce sprung mass vertical acceleration in a quarter car generally results in a dissipative system [16, 18].

on the magnitude of ρ . As expected, when ρ is increased, the MPC tends towards the frequency weighted acceleration controller. Multiple benefits were found from implementing the frequency weighted controller.

Table 4 – Frequency Weighted Harvested Energy and Sprung Mass Acceleration Variance Gain

Road Class	J_3
А	(-78.9, 10.9)
В	(-215.4, 4.3)
С	(-837.0, 5.2)
D	(-3297, 2.9)

By setting $\rho = 200$, the controller harvested more energy but had a slightly higher variance gain than Skyhook. The benefit of the frequency weighting was seen as the road became rougher and on a class D road, the frequency weighted MPC out performed Skyhook in both energy harvesting and variance gain. Across all classes of road, the frequency weighted controller had better road holding characteristics, in smaller values of unsprung mass velocities and tire deformations than the standard acceleration minimization. Fig. 5 shows a set of acceleration curves with the frequency shaped controller showing more acceleration but also harvesting more energy.



Fig. 5 - Sprung Mass Acceleration over a Class C Road

Additional work could be done to find an optimum weighting for the cost function blending.

6. FUTURE WORK

There are many extensions of this project that will be included in future work. To begin with, inclusion of state constraints, such as rattlespace limits, will be included future studies. In order to get a more realistic idea of the effect of the bang-bang power optimization controllers, actuator rate limits will also be included in the MPC. The model should also be extended to a full or half car. It was found by Di Iorio and Casavola that pitch and specifically roll contribute significantly to the amount of harvested energy [8]. Expanding the model to a half or full car will help illuminate the effect that these additional degrees of freedom have on energy regeneration. The control algorithm that is implemented for this extension does not have to be MPC but it will be beneficial if implementation of an energy harvesting optimization is desired. In most studies of control for regenerative dampers, the main objective of the controller is to increase ride comfort. A major aspect of regenerative suspension design that needs to be addressed is including a road holding, ride comfort, and regeneration trade-off. Throughout this study, all of the efficiencies associated with the regenerative damper were assumed to be ideal. Taking efficiencies into account will be important to fully access the capabilities of the suspension to self-sustain itself. Because MPC requires full state knowledge, it will be important to include the effects of estimation in the controller. This will be very important given the fact that tire deformation can be very difficult to estimate. Further studies will also include studying how the forward speed affects the energy regeneration for different road grades. Also, since simpler algorithms, such as Skyhook, lead to dissipative systems. It would be interesting to look at how varying the damping coefficient affects the energy production and ride comfort.

7. CONCLUSION

In this paper, energy regeneration and ride comfort were used to design cost functions for model predictive controllers. Due to the lack of asymptotic stability guaranty while regenerating energy, a Lyapunov stability constraint was added to the MPC constraints. The results showed how much the objectives compete with each other and agreed with past works showing that the while minimizing acceleration the system maintains dissapavity. When the Lyapunov constraints were imposed on the MPC, the ride comfort increased when the energy output was optimized but still suffered from a bang-bang input solution. It was interesting to see that minimizing sprung mass acceleration out performed Skyhook from road classes A-C but Skyhook outperformed sprung mass vertical acceleration minimization for road classes D and E. This led to the conclusion that it could be beneficial to alter the control or system parameters over different road classes to optimize the suspension performance. Frequency weighting was implemented as well and had benefits over the Lyapunov constrained MPC algorithm. On rougher roads. Additional power was generated, tire deformation and unsprung mass velocity were lower, and the sprung mass acceleration approached but did not reach the acceleration minimization values. Future work was presented that included implementing more state and control rate constraints in the MPC as well as many others.

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